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SCIENTIFIC PAPER

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ITERATIVE LEARNING CONTROL WITH UNKNOWN TIME–DELAY

The paper presents a new algorithm for iterative learning control (ILC) called "natural" ILC. ILC is developed on the basis of a biological analog – the principle of self–adaptability. Sufficient conditions for the convergence of a new type of learning control algorithm for a class of time–varying delayed uncertain, non–linear systems – a process plant with unknown pure time–delay are presented. A new algorithm has benefits which include control of an object with unknown time–delay, improving the properties of tracking, as well as the speed of convergence of ILC.

Recently, there have been extensive research activities in the field of learning control for controlling dynamic systems in an iterative manner. The learning control concept differs from conventional control methodologies in that the control input can be appropriately adjusted to improve its future performance by learning from past experimental information as the operation is repeated. The common observation that human beings can learn perfect skills through repeated trials, motivates the idea of iterative learning control for systems performing repetitive tasks. Therefore, iterative learning control requires less *a priori* knowledge about the controlled system in the controller design phase and also less computational effort than many other kinds of control and has received a great deal of attention from many researchers in recent years [1, 2]. Iterative learding control is a technique to control systems operating in a repetitive mode with the additional requirement that a specified output trajectory $y_d(t)$ in a interval [0, t] be followed to a high precision and in order to improve performance from trial to trial in the sense that the tracking error is sequentially reduced.

Examples for such systems are more generally the class of tracking systems, also process plants that are required to repeat a given task to desired precision and where a partially known or unknown time – delay appears. In other words, if the required task is repetitive in nature, as in many industrial operations, iterative learning control is found to be a good alternative, especially when detailed knowledge about the plant is not available.

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PRELIMINARIES

A dynamic model of a process plant with unknown pure time–delay with uncertainties can be presented in the form of state space and output equations as a class of time–varying, non–linear systems [3, 4]:

$$\dot{x}_i = f(x_i,\ x_i\,(t-\tau),\ t) + B(x_i)u_i + \nu_i$$

$$y_i = g\,(x_i) \tag{1}$$

In these equations t denotes time, $t \in [0, T]$, $t \in \Re$, where T presents the terminal time which in known; x_i the state vector, $x_i \in \Re^n$, u_i the control vector, ν_i the vector bounded uncertainties or disturbances to the system, $u_i \in \Re^m$, y_i the output vector of the system, $y_i(t) \in R_r$ and i denotes the i–th repetitive operation of the system. Also, τ denotes the unknown pure time–delay and $\tau \le T$. For later use in proving the convergence of the proposed learning control, the following norms are introduced [5]:

– the vector norm as

$$||x|| = \max_{1\le i\le n} |x_i| \quad x = [x_1, x_2, \ldots x_n]^T \tag{2a}$$

– the matrix norm as

$$||A|| = \max_{1\le i\le m} \left(\sum_{j=1}^{n} |g_{i,j}| \right) \quad A = [a_{i,j}]_{m\times n} \tag{2b}$$

– the λ–norm for a real function:

$$h(t),\ (t \in [0,T]),\ h:[0,T] \rightarrow \Re^n \text{ as} \tag{2c}$$

$$||h(t)||_\lambda = \sup_{t\in[0,T]} e^{-\lambda t}\,||h(t)||, \quad \lambda > 0$$

Before presenting a new iterative learning control algorithm, the following assumptions on the system are imposed.

P1. Functions $f(.,.)$, $B(.,.)$:R^n x [0, T] $\rightarrow$ R^n and $g(.,.)$:R^n x [0, T] $\rightarrow$ R^r, belonging to a set of functions defined on [0, T], are piecewise continuous and satisfy the Lupschitzian continuity conditions, i.e.

$$\forall\, t \in [0,T] \tag{3}$$

$$\|f(x_{i+1}(t),x_{i+1}(t-\tau),t) - f(x_i(t), x_i(t-\tau), t)\| \le k_f(\| x_{i+1}(t) - x_i(t)\|) + k_{f\tau}(\|x_{j+1}(t-\tau) - x_j(t-\tau)\|)$$

$$\|B(x_{i+1}(t)) - B(x_i(t))\| \le k_B(\|x_{i+1}(t) - x_i(t)\|)$$

$$\|g(x_{i+1}(t)) - g(x_i(t))\| \le k_g(\|x_{i+1}(t) - x_i(t)\|)$$

$$\|g_x(x_{i+1}(t)) - g_x(x_i(t))\| \le k_{gx}(\|x_{i+1}(t) - x_i(t)\|)$$

where k_f, k_B, k_g, k_{gx}, $k_{f\tau} > 0$ are the Lipschitzian constants and the partial derivative $g_{xi}(.,.) = \partial g(x_i(t)) / \partial x_i(t)$ is differentiable in x.

P2. The system is causal. Furthermore, for a given bounded output there exists a unique bounded input and a unique bounded state $u_d(t)$, $x_d(t)$, $t \in [0, T]$ such that:

$$\dot{x}_d(t) = f(x_d(t), x_d(t-\tau), t) + B(x_d(t))u_d(t) \tag{4}$$

$$y_d(t) = g(x_d(t))$$

P3. Functions g_x, $B(.,.)$, $\dot{x}_d$, u_d are uniformly bounded:

$$a_g = \sup_{t\in[0,T]} \|g_{xi}\|, \forall x_i(t) \in R^n,\ a_B = \sup_{t\in[0,T]} \|B(x_i(t))\|, \forall x_i(t) \in R^n$$
$$a_{U_d} = \sup_{t\in[0,T]} \|u_d\|,\ a_{\dot{x}_d} = \sup_{t\in[0,T]} \|\dot{x}_d\|,\ a_v = \sup_{t\in[0,T]} \|v\| \tag{5}$$

P4. It is assumed that when $t < 0$, $x_i(t) = 0$.

The learning controller for generating the present control input is based on the previous control history and a learning mechanism. Motivated by human learning, the basic idea of iterative learning control is to use information from previous execution of the task in order to improve performance from trial to trial in the sense that the tracking error is sequentially reduced. The task is synthesis control u(t) applying the learning concept. For a given output trajectory $y_d(t)$, the control objective is to find a control input $u_i(t)$ such that when $i \to \infty$, the system output $y_i(t)$ will track the desired output trajectory as close as possible. A learning control law based on iterative learning can be found in the literature in the following manner [5, 6]:

$$u_{i+1}(t) = L(t)\, u_i(t) + K(t)\, \dot{e}_i(t) \tag{6}$$

Also the tracking errors and their first derivative are defined as:

$$e_i(t) = y_d(t) - y_i(t)$$

$$\dot{e}_i(t) = de_i(t)/dt = \dot{y}_d(t) - \dot{y}_i(t) \tag{7}$$

and L(t), K(t), are the gain matrices of appropriate dimensions.

MAIN RESULTS

In this paper a new algorithm for iterative learning control is suggested which differs from existing learning algorithms. For improving the properties of tracking, as well as the speed of convergence and, especially, control of a process plant with unknown time–delay, it is proposed to apply a biological analog – the principle of self–adaptability [7] which introduces local negative feedback on the control with great amplification. In the simples case the learning control law can be shown as (Fig. 1):

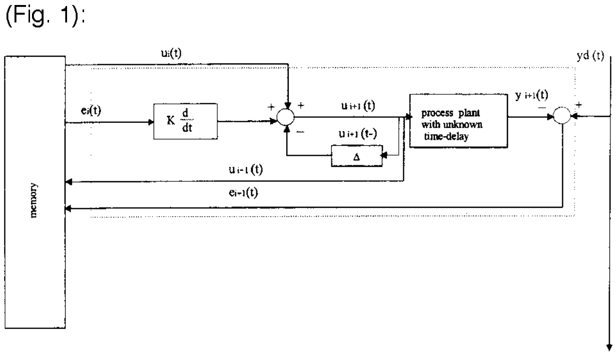

Figure 1. Block diagram of natural iterative learning control for a process plant with unknown time-delay

$$u_{i+1}(t) = -\Delta u_{i+1}(t^-) + u_i(t) + K(t)\, \dot{e}_i(t) \tag{8}$$

where Δ, K denotes matrices of appropriate dimensions; u(t) the value of the function at time t and $u(t^-) = u(t-\varepsilon)$, $\varepsilon \to 0^+$ denotes a control vector of the just realized control at time t. If the feedback delay can be neglected then:

$$u(t^-) = u(t) \tag{9}$$

yields:

$$u_{i+1}(t) = -\Delta u_{i+1}(t) + u_i(t) + K(t)\, \dot{e}_i(t) \tag{10}$$

or:

$$u_{i+1}(t)\, [I + \Delta] = u_i(t) + K(t)\, \dot{e}_i(t) \tag{11}$$

In ILC a fundamental problem is to guarantee the ILC convergence property, i.e. to guarantee the system is output trajectory converging to the desired one within a prescribed desired accuracy as the number of ILC iterations increases. A sufficient condition for the convergence of a new iterative learning control is given in the following theorem.

Theorem 1:

If an existing non–linear repetitive system satisfied the previously introduced assumptions (P1–P4) with the proposed new iterative learning control and matrices Δ and K exist such that $\|(I + \Delta)^{-1}\|\, \|I - Kg_x B\| \le \rho < 1$ then, when $i \to \infty$ the bounds of the tracking errors $\|u_d(t) - u_i(t)\|$, $\|x_d(t) - x_i(t)\|$, $\|y_d(t) - y_i(t)\|$ converge asymptotically to a residual ball centered at the origin.

Proof:

Let

$$g_i = g_d(x_d(t)) - g(x_i(t)),$$

$$\delta u_i = u_d(t) - u_i(t),$$

$$\delta B_i = B_d(t) - B(x_i(t))$$

$$\dot{x}_i = \dot{x}_d(t) - \dot{x}_i(t)$$

$$f_i = f_d\,(x_d\,(t),\, x_d\,(t-\tau),\, t) - f\,(x_i\,(t),\, x_i\,(t-\tau),\, t) \quad (12)$$

$$g_{x_i} = g_{x_d}\,(x_d(t)) - g_{x_i}\,(x_i(t))$$

Also, the tracking error can be presented as:

$$\dot{e}_i = \delta g_{x_i}\,\dot{x}_d + g_{x_i}\,\delta\dot{x}_i \quad (13)$$

or,

$$\dot{x}_i = \delta f_i + \delta B_i u_d + B_i \delta u_i - \nu_i \quad (14)$$

Taking that the control law is given as:

$$u_{i+1}(t)\,[I+\Delta] = u_i(t) + K(t)\,\dot{e}_i(t) \quad (15)$$

it yields:

$$[I+\Delta]\,\delta u_{i+1} = \delta u_i + \Delta u_d - K\dot{e}_i \Rightarrow$$

$$\Rightarrow u_{i+1} = [I+\Delta]^{-1}\,\delta u_i + [I+\Delta]^{-1}\,\Delta u_d - [I+\Delta]^{-1}\,K\dot{e}_i \quad (16)$$

where is $\Delta = \mathrm{diag}\{\delta_i,\ i = 1, 2, 3 \ldots n\}$ and:

$$[I+\Delta]^{-1} = \mathrm{diag}\left\{\frac{1}{1+\delta_i}\right\},\ [I+\Delta]^{-1}\,\Delta = \mathrm{diag}\left\{\frac{\delta_i}{1+\delta_i}\right\},$$

$$i = 1,2,3 \ldots n \quad (17)$$

$$a_\Delta^{-1} = \sup_{t\in[0,T]} \|(I+\Delta)^{-1}\| \quad a_{\Delta^*} = \sup_{t\in[0,T]} \|(I+\Delta)^{-1}\,\Delta\| \quad (18)$$

also:

$$u_{i+1} = [I+\Delta]^{-1}\,\delta u_i + [I+\Delta]^{-1}\,\Delta u_d$$

$$-[I+\Delta]^{-1}\,\langle K[\delta g_{x_i}\,\dot{x}_d + g_{x_i}\,(\delta f_i + \delta B_i u_d + B_i \delta u_i - \nu_i)]\rangle$$

$$= [I+\Delta]^{-1}\,[I - K g_{x_i} B_i]\delta u_i + [I+\Delta]^{-1}. \quad (19)$$

$$\{(\Delta - K g_{x_i}\,\delta B_i)u_d - K(\delta g_{x_i}\,\dot{x}_d + g_\xi\,\delta f_i - \nu_i)\}$$

Estimating the norms of (19) with $\|(.)\|$ and using assumptions P1–P4 and the condition of theorem lone may obtain

$$\|\delta u_{i+1}\| \le \|[I+\Delta]^{-1}\,(I - K g_\xi B_i)\|\|\,\delta u_i\| + a_{\Delta^*} a_{ud}$$

$$+ \|\delta x_i\| a_\Delta^{-1}\{a_{ud} a_K a_{gx} k_B + a_K a_{\dot{x}d} k_{gx} + a_K a_{gx} k_f + a_K a_{gx} a_v\} +$$

$$+ a_\Delta^{-1}\, a_K a_{gx} k_{f\tau} \|\delta x_i(t-\tau)\| \quad (20)$$

$$\|\delta u_{i+1}\| \le \rho\|\delta u_i\| + \eta\|\delta x_i\| + \beta\|\delta x_i(t-\tau)\| + \gamma \quad (21)$$

where

$$= a_\Delta^{-1}\,\langle a_K\, a_{\dot{x}d} k_{gx} + a_{gx}(k_f + a_{ud} k_B + a_v)\rangle \quad (22)$$

$$\gamma = a_{\Delta^*} a_{ud}, \quad \beta = a_\Delta^{-1} a_K a_{gx} k_{f\tau}$$

Also,

$$x_i(t) - \delta x_i(0) = \int_0^t \delta\dot{x}_i(\tau)d\tau \quad (23)$$

$$\|\delta x_i(t)\| \le \|\delta x_i(0)\| + \int_0^t \|\delta\dot{x}_i(\tau)\|ds$$

$$\le \|\delta x_i(0)\| + \int_0^t \|\delta f_i + \delta B_i u_d + B_i \delta u_i - \nu\|ds$$

$$\le \|\delta x_i(0)\| + \int_0^t [k_f + a_{ud} k_B]\|\delta x_i\|ds + \int_0^t a_B\|\delta u_i\|ds +$$

$$+ \int_0^t a_v ds + \int_0^t k_{f\tau}\|\delta x_i(t-\tau)\|ds \quad (24)$$

For any function $x_i\,(t) \in R^n$, $t \in [0, T]$, the λ – norm form $\int_0^t \|x_i(s)\|ds$ is

$$\sup_{t\in[0,T]} e^{-\lambda t}\int_0^t \|x_i(s)\|ds = \sup_{t\in[0,T]} e^{-\lambda t}\int_0^t \|x_i(s)\|e^{-\lambda(s-s)}ds \le \quad (25)$$

$$\le \|\delta x_i(t)\|_\lambda \sup_{t\in[0,T]} e^{-\lambda t}\int_0^t e^{\lambda s}ds = \|\delta x_i(t)\|_\lambda \sup_{t\in[0,T]} 1 - \frac{e^{-\lambda t}}{\lambda} \le \|\delta x_i(t)\|_\lambda\, O(\lambda^{-1})$$

where is $O(\lambda^{-1}) = (1 - e^{-\lambda T})/\lambda \le l/\lambda$. Due to fact that $\|x(t-\tau)\| \le \|x(t)\|_\lambda$ by referring to (25), one can find that

$$\sup_{t\in[0,T]} e^{-\lambda t}\int_0^t \|x(s-\tau)\|ds \le \|x(t)\|_\lambda O(\lambda^{-1})e^{-\lambda\tau} \quad (26)$$

New performing the λ–norm operation for (24) and using (25), (26) one obtains

$$\|\delta x_i(t)\|_\lambda \le a_{xo} + a_v T + [k_f + a_{ud} k_B +$$

$$+ k_{f\tau} e^{-\lambda\tau}]\|\delta x_i(t)\|_\lambda O(\lambda^{-1}) + a_B\|\delta u_i\|_\lambda O(\lambda^{-1}) \quad (27)$$

and

$$\|\delta x_i\|_\lambda \le \|\delta u_i\|[a_B/(1 - (k_f + a_{Ud} k_B + k_{f\tau} e^{-\lambda\tau})O(\lambda^{-1}))]$$

$$+ (a_{xo} + a_v T)/(1 - (k_f + a_{Ud} k_B + k_{f\tau} e^{-\lambda\tau})O(\lambda^{-1})) \quad (28)$$

Introducing

$$O_u(\lambda^{-1}) = [a_B/(1 - (k_f + a_{Ud} k_B + k_{f\tau} e^{-\lambda\tau})O(\lambda^{-1}))]$$

$$\varepsilon^* = (a_{xo} + a_v T)/(1 - (k_f + a_{Ud} k_B + k_{f\tau} e^{-\lambda\tau})O(\lambda^{-1})) \quad (29)$$

where it is assumed that a sufficiently large λ is used such that

$$(1 - (k_f + a_{Ud} k_B + k_{f\tau} e^{-\lambda\tau})O(\lambda^{-1})) > 0 \quad (30)$$

or, in condensed form

$$\|\delta x_i\|_\lambda \le \|\delta u_i\|_\lambda O_u(\lambda^{-1}) + \varepsilon^* \quad (31)$$

Taking the λ–norm of (21) with the substitution of (31) simply yields

$$\|\delta u_{i+1}\| \le [\rho + (\eta + \beta)O_u(\lambda^{-1})]\|\delta u_i\|_\lambda +$$

$$+ (\eta + \beta)\varepsilon^* + \gamma = \rho^\bullet\|\delta u_i\|_\lambda + \varepsilon^\bullet \quad (32)$$

where using a suitable choke of matrices Δ and K i.e. one can find a sufficiently large λ such that $\rho^* < 1$ so that

$$\|\delta u_{i+1}\|_\lambda \le \rho^\bullet \|\delta u_i\|_\lambda + \varepsilon^\bullet \quad (33)$$

Then, according to *Lemma I* it can be concluded that

$$\lim_{i\to\infty} \|\delta u_{i+1}\|_\lambda \le \varepsilon^\bullet / (1-\rho^\bullet) \quad (34)$$

Also,

$$\|\delta x_i\|_\lambda \le \|\delta u_i\|_\lambda \, O_u(\lambda^{-1}) + \varepsilon* \quad (35)$$

now, it can be easily shown that

$$\lim_{i\to\infty} \|\delta x_i\|_\lambda \le O_U(\lambda^{-1})\varepsilon^\bullet / (1-\rho^\bullet) + \varepsilon* \quad (36)$$

or

$$\lim_{i\to\infty} \|\delta e_i\|_\delta \le k_g[O_U(\lambda^{-1})\varepsilon^\bullet / (1-\rho^\bullet) + \varepsilon*] \quad (37)$$

This completes the proof of *Theorem 1*

Example

A simple example of a Siso system

$$\dot{x}(t) = 0.9\,x(t) + 0.1\,x(t) - 0.05 + u(t)$$

$$y(t) = x\,(t)$$

is used to illustrate the previous approach. The given matrices Δ and K are 1x1:0.02 and 0.4 and the output trajectory is $y_d(t) = t$.

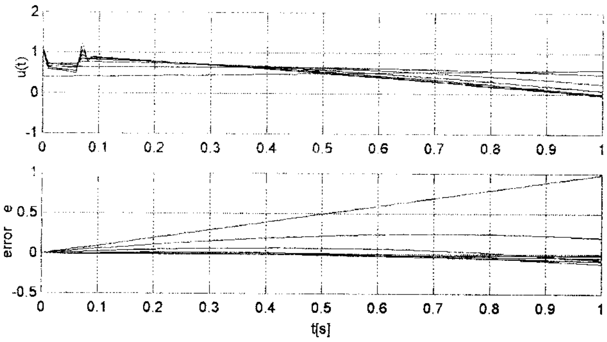

Figure 2. Iterative control and tracking error

CONCLUSION

A new iterative learning algorithm utilizing a biological analog – the principle of self–adaptability for a process plant is considered. It is shown that the new algorithm of ILC improves the speed of convergence and also has better properties of tracking which makes the suggested algorithm more attractive from the viewpoint of practical applications. Also, sufficient conditions for the convergence of ILC are obtained.

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IZVOD

KONTROLA ITERATIVNIM UČENJEM SA NEPOZNATIM VREMENSKIM KAŠNJENJEM

(Naučni rad)

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U ovom radu je prezentovan jedan novi algoritam upravljanja primenom iterativnog učenja (UIU) koji je nazvan upravljanje primenom "prirodnog" iterativnog učenja. UIU je razvijen na bazi biološkog analogona – principa samopodešavanja. Dovoljni uslovi su izvedeni za konvergenciju predloženog algoritma upralvjanja za jednu klasu nelinearnog, vremenski promenljivog sistema sa nepoznatim vremenskim kašnjenjem – jedan metalurški objekat sa nepoznatim kašnjenjem. Novi algoritam ima prednosti koje uključuju upravljanje objektom koji sadrži nepoznato kašnjenje, bolje osobine praćenja kao i povećanje brzine konvergencije željenom rešenju.

Ključne reči: upravljanje objektom • algoritam upravljanja • iterativno učenje • metalurški objekat •
Key words: Object control • Learning algorithm • Iterative learning • Process plant •