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MODIFICATION OF THE DYNAMICS CHARACTERISTICS IN THE STRUCTURAL DYNAMIC REANALYSIS

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Abstract. *Structural dynamic modification (SDM) techniques can be defined as methods by which the dynamic behavior of a structure is improved by predicting the modified behavior brought about by adding modifications like those of lumped masses, rigid links, dampers, beams, etc. or by variations in the configuration parameters of the structure itself. Such methods, especially those with their roots in finite element models, have often been described as reanalysis. Most of the techniques imply a dynamic test at some stage of SDM and currently prefer implementation on a personal computer. The need for SDM arises because of the demands on higher performance capabilities of complex mechanical and structural systems, like machine tools, automobiles, rail vehicles, aerospace systems and high speed rotating systems, which require sound dynamic design, i.e. desired dynamic characteristics like vibration levels, response, resonances, eigenvalues, dynamic stability and mode shapes. Structural dynamic modification implies the incorporation, into an existing model, of new information gained either from experimental testing or some other source, which questions or improves the accuracy of the model. This paper deals with improving of dynamic characteristics of tube collector (protection pipe of conductors of transformer) of the ring cross section. It is shown how change of conditions of support can improve dynamic characteristics of structure. Distribution of potential and kinetic energy in every finite element is used for analysis. In this study it is shown that structural dynamic modification is important in structural reanalysis.*

Key words: *Structural Dynamics Modification, Eigenvalues, Potential and Kinetic Energy*

1. INTRODUCTION

In general, the structural modification problem with frequency constraints is subjected in one of the following ways [1]:

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(i) Maximize the natural frequency or difference between two consecutive frequencies subject to a specified constraint

$$h(v) = g(v) - \tilde{g} = 0 \tag{1}$$

and side constraints on the design variables

$$v_i^l \le v_i \le v_i^u \tag{2}$$

(ii) Minimize structural weight g (v) subject to behavior constraints

$$h_j(v) = \omega_j^2 - \tilde{\omega}_j^2 = 0, \qquad j = 1, 2, ..., k \tag{3}$$

$$h_j(v) = \omega_j^2 - \tilde{\omega}_j^2 \ge 0, \qquad j = k+1, \;\; k+2, ..., m \tag{4}$$

where v_i is the design variable or updating parameter, v_i^l is the lower limit, v_i^u is the upper limit on the design variable, ω_j is the j^{th} natural frequency, $\tilde{\omega}_j$ is the specified value of the j^{th} natural frequency, $g(v)$ is the structural weight, $\tilde{g}$ is the specified weight, n is the number of design variables, and m is the number of design constraints. The design variables depend on the type of optimization problem. In the design of structural components, such as stiffened panels and cylinders, the design parameters represent the spacing of the stiffeners, the size and shape of the stiffeners, and the thickness of the skin. If the skin and/or stiffeners are made of layered composites, the orientation of the fibers and their proportion can become additional variables. The sizes of the elements are design variables of a structural system of fixed configuration (frames, trusses, wings, fuselages, etc). The thickness of plates, cross-sectional areas of bars, areas, moments of inertia, and torsional constants of beams represent sizes of the elements. The parameters may be spatial if the optimization includes configuration. Also, in dynamics problems, the location of nonstructural masses and their magnitudes can be considered as variables. If only frequency constraints are considered in the optimization problem, it is advisable to include nonstructural masses in the structural model representing the fuel, payload, attachments, etc. For performing a model updating procedure, every parameter in an FE model can be considered as a candidate updating parameter. In an FE model for a continuous structure, the number of the independent parameters is equal to the number of degrees of freedom of the model.

2. THE DISTRIBUTION OF THE POTENTIAL AND KINETIC ENERGY WITHIN THE MODE SHAPES

The matrix form of the equation of undamped motion of an FE model is:

$$[M] \cdot \{\ddot{x}(t)\} + [K] \cdot \{x(t)\} = \{0\} \tag{5}$$

The free-vibration natural frequencies and mode shapes of a linear structural system can be computed by solving the above eigenvalue problem

$$[K]\{Q_i\} = \lambda_i [M]\{Q_i\} \tag{6}$$

where $[K]$, $[M]$ are the structural stiffness and mass matrix, respectively. The system matrices are considered to be a general function of the design variables (nodal coordinates, area of cross section, moment of inertia, mass, depth...) denoted by $\{V\} = \{v_1, v_2, ..., v_j, ..., v_p\}$, and λ_i and $\{Q_i\}$ are the eigenvalue and the eigenvector of mode i, respectively.

$$\{Q_i\}^T [K]\{Q_i\} = \lambda_i \{Q_i\}^T [M]\{Q_i\} \tag{7}$$

The perturbed eigenvalue problem (from eq. 6) can be written as

$$([K]+[\Delta K]) \ \ (\{Q_i\}+\{\Delta Q_i\}) = (\lambda_i + \Delta\lambda_i) \ \ ([M]+[\Delta M]) \ \ (\{Q_i\}+\{\Delta Q_i\}) \tag{8}$$

where $\Delta\lambda_i$, $\{\Delta Q\}_i$, $[\Delta K]$ and $[\Delta M]$ are the eigenvalue, eigenvector, the stiffness and mass matrices perturbations, respectively.

The second and higher order terms could be neglected, and after mathematical operations, the perturbed eigenvalue problem (from eq. 8) can be written as

$$\frac{\Delta\omega_r^2}{\omega_r^2} = \frac{\{Q_r\}^T [\Delta K]\{Q_r\} - \omega_r^2 \{Q_r\}^T [\Delta M]\{Q_r\}}{\omega_r^2 \{Q_r\}^T [M]\{Q_r\}} = \frac{E_{p,r} - E_{k,r}}{E_{k,r}} \tag{9}$$

because the potential and kinetic energy of the structure for r-th main mode shape, according to [2], can be written in the next form:

$$E_{p,r} = \frac{1}{2}\{Q_r\}^T [K]\{Q_r\}, \qquad E_{k,r} = \frac{1}{2}\lambda_r \{Q_r\}^T [M]\{Q_r\}. \tag{10}$$

Expression (9) is basic equation for reanalysis of structure, because it shows influence of specific finite elements to the eigenvalue. The distribution of energies within of FE provides necessary information for optimization. In other words, for every FE where the difference between potential and kinetic energy is the largest, the structural modification should be performed for the best influence to change governing eigenvalue. The main goal of dynamic optimization is to increase natural frequencies and to increase the difference between them.

3. DEMONSTRATION EXAMPLES

The first example problem is the tube collector, see Fig. 1 that is modeled using 11 beam elements. The influence of the way of supports and increasing of stiffness to the eigenvalues will be considered. The initial geometry of the tube is: D=200mm – the external diameter of the tube, d=184mm - the internal tube's diameter, L=22m – the length of the tube. All other characteristics, necessary for calculation of tube's eigenvalues, are: $I_z = D^4\pi[1-(d/D)^4]/64 = 2228\text{cm}^4$ – the axial moment of inertia of cross section for z axis, $E=72.10^9 N/m^2$ – Young's modulo of the tube's material (aluminum), q=1.74kg/m – the specific weight of the conductors inside the tube, $\rho=2750 kg/m^3$ – mass density. The area of cross section of tube is $A=48.5 cm^2$, the mass of tube is $m = \rho AL = 286 kg$. This relatively simple model is used to verify the implementation of described method using MatLab 7.

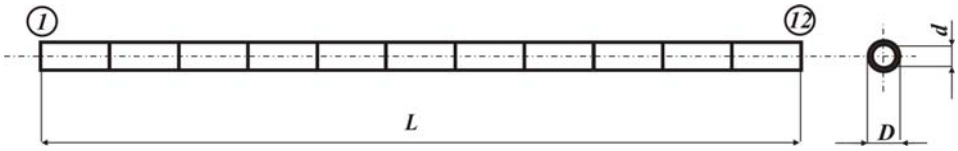

Fig. 1 Tube Collector, Modeled by 11 Beam Elements

The First Case

First consideration will be taken for the tube collector simply supported on both ends (see Fig. 2). In that case transversal degrees of freedom at the first and 12-th nodes are constrained to zero, yielding a total of 22 DOF for the model. There are two degrees of freedom (DOF) at each node corresponding to translation in the y-direction and rotation about an axis normal to the x-y plane. The natural frequencies of the pipe for this case are given in the first column in Table 1. It can be concluded that the first frequency is too low and consequently dynamic behavior of structure is not good enough. The most important thing, dealing with dynamical improvement of structure, is increasing of the lowest frequencies and increasing of the intervals between them. Because of that, it is important to examine influences, such are changes of geometrical characteristics of the tube and boundary conditions as well (where it is technically possible to make a change), to change the values of frequencies. In Fig. 2a the diagram of distribution of potential and kinetic energy for this case is given. From diagram it can be concluded that the differences of *Ep* and *Ek* along the whole span are negligible; then it is inappropriate to change the geometry of the section. Also, the increasing mass of the tube the stiffness will increase as well while frequencies will not be improved, and *vice versa*. Because of that, the change of boundary conditions will be considered firstly.

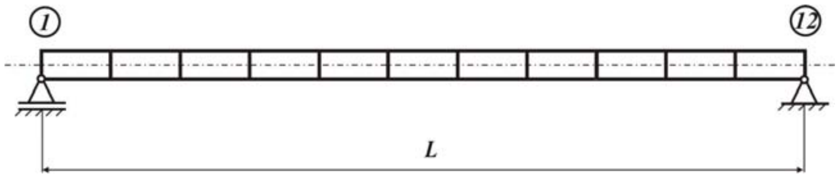

Fig. 2 Tube Collector, Simply Supported at Both Ends

The Second Case

For the sake of increasing values of all frequencies, especially of the lowest, the fixed end is introduced at right end of pipe, while the left end remains hinged (Fig. 3).

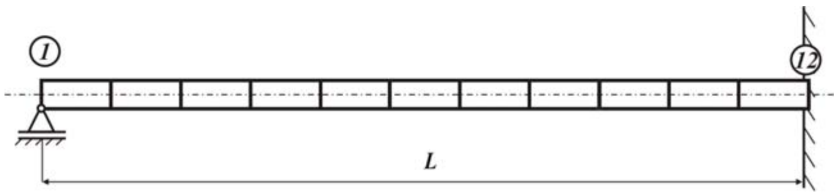

Fig. 3 Tube Collector, Simply Supported at One End and Fixed at the Other

Hence, on the right end of the pipe the rotational degree of freedom is constrained to zero. It can be concluded that natural frequencies are increased if compared with the case of both hinged ends (the natural frequencies of the pipe for this case are given in the second column in Table 1). Hence in this case the unchanging total mass of structure dynamic behavior is considerably improved. If it is impossible to make technically fixed end of the pipe, then changing static system will have similar influence. At the right end two hinged supports are introduced at a very short distance. The system from simple supported beam becomes statically indeterminate, increasing stiffness of the tube (Fig. 4).

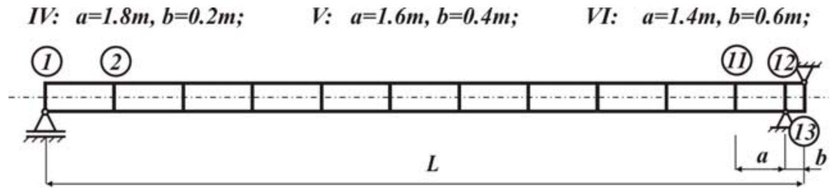

Fig. 4 Tube Collector with Two Hinged Supports at the Right End

We are looking for the distance between supports which will have the same influence as ideally fixed end. For distances of 20cm, 40cm, and 60 cm, frequencies are given in columns IV, V and VI, in Table 1, respectively. It can be seen that the distance of 40 cm leads to the same values of frequencies as ideally fixed end (column II), and that case will be taken into the next considerations. Diagrams of distributions of *Ep* and *Ek* are given in Figures 5, 6, 7 and 8 for the first four mode shapes, respectively. Unlike the diagram in Figure 2a, from Figures 5, 6, 7 and 8 it can be clearly seen that the largest positive value of the difference between *Ep* and *Ek* takes place at the element 11 (the element between nodes 11 and 12, see Fig. 4) for all mode shapes. It means that the change of geometry should take place at this position in order to have higher values of natural frequencies. In columns VII, VIII and X the values of frequencies for some changes of cross sections of elements 11 and 12 are given. Change of element 12 does not give any change, which was already expected from the diagram, while change of element 11 results in increase of frequency less than 10% (Fig. 11, columns IX and XI).

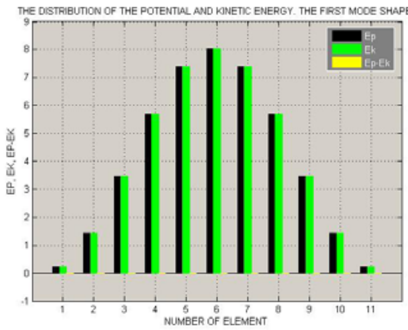

Fig. 2a Distribution of Ep and Ek for the Tube, Simply Supported at Both Ends for the First Mode Shape

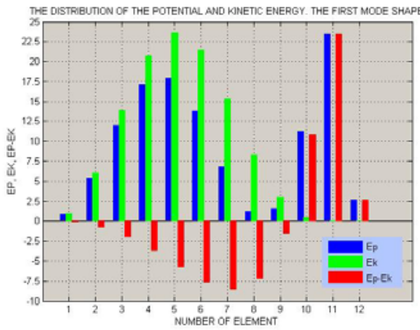

Fig. 5 Distribution of Ep and Ek for the Tube, Simply Supported at One End and Fixed at the Other for the First Mode Shape

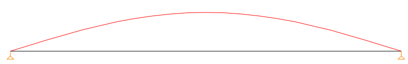

Fig. 2b First Mode Shape for the Tube, Simply Supported at Both Ends, f_{01}=1.06Hz

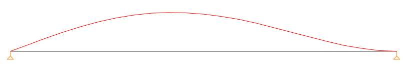

Fig. 5a First Mode Shape for the Tube, Simply Supported at One End and Fixed at the Second, f_{01}=1.70Hz

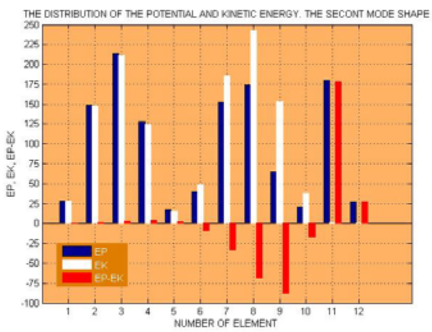

Fig. 6 Distribution of Ep and Ek for the Tube, Simply Supported at One End and Fixed at the Other For the Second Mode Shape

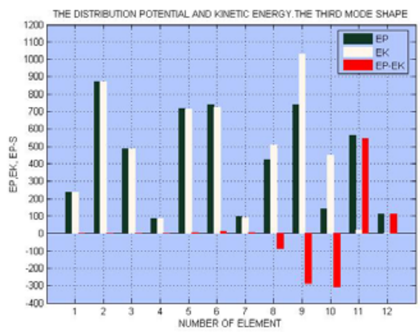

Fig. 7 Distribution of Ep and Ek for the Tube, Simply Supported at One End and Fixed at the Other for the Third Mode Shape

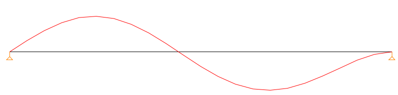

Fig. 6a Second Mode Shape for the Tube, Simply Supported at One End and Fixed at the Other f_{02}=5.51Hz

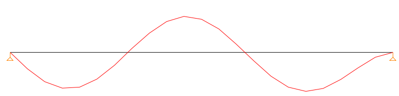

Fig. 7a Third Mode Shape for the Tube, Simply Supported at One End and Fixed at the Other f_{03}=11.49Hz

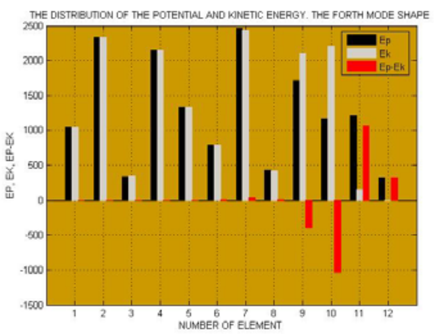

Fig. 8 The distribution of Ep and Ek for the tube, simple supported at one end and fixed at the second, IV mode shape

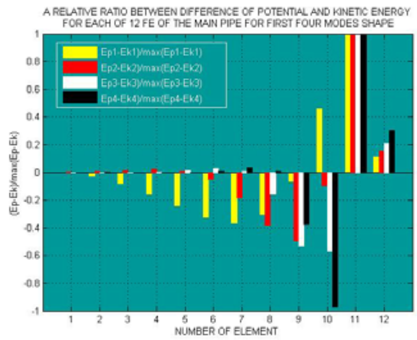

Fig. 9 A relative ratio between difference of potential and kinetic energy for each of 12 FE of beam for first four modes shape

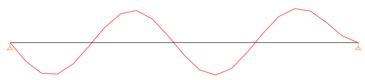

Fig. 8a Fourth Mode Shape for the Tube, Simply Supported at One End and Fixed at the Other, f_{04}=19.67 Hz

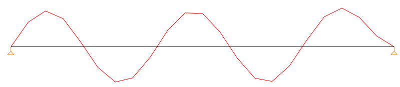

Fig. 10 Fifth Mode Shape for the Tube, Simply Supported at One End and Fixed at the Other, f_{05}=30.07 Hz

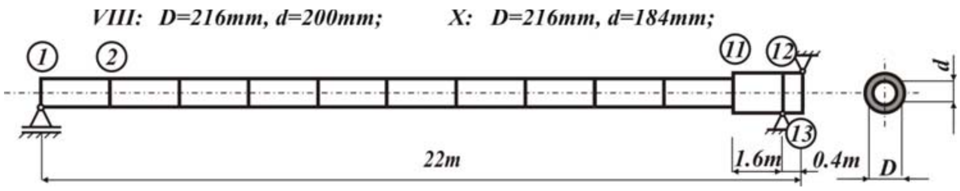

Fig. 11 Tube Collector with Two Hinged Supports at the Right End and Modified Cross Section at One Portion of the Length

CONCLUSIONS

For the sake of improving dynamic characteristics of aluminum tube collector of the ring cross section, the change of boundary conditions and geometry are considered. First it is considered that the tube is simply supported at both ends, and the obtained results for natural frequencies show that dynamic characteristics were not good. Changing boundary conditions only at one end, the lowest frequencies are increased for 60% which considerably improves the solution. Using expression (9), i.e. taking opportunity that at the places where the difference between Ep and Ek is the largest, structural modifications can be done, the analysis is performed along the whole length of the tube. At the place of the largest difference between *Ep* and *Ek* (Fig. 5, 6, 7 and 8) the stiffness of the cross section is increased causing increase of the value of the first natural frequency slightly (Fig. 11 columns VIII, IX, X and XI in Table 1). In this way it is proven that the change of boundary conditions is the most efficient way to increase natural frequencies. The ideal case would be to have both end fixed, but technically it is impossible (for instance because of thermal dilatations) in the case of tube collector of transformer. Only one fixed and the second one hinged supports are allowed, which is considered in this paper.

Table 1 Natural frequencies of the beam for all considered cases for 20 mode shapes, and their relative ratios

ω^{*}	ω^{**}	ω^{**}/ω^{*}	ω^{***}	$\boldsymbol{\omega}^{****}$	ω^{*****}	ω^{******}	$\omega^{*******}$	$\omega^{*******}/\boldsymbol{\omega}^{****}$	$\omega^{********}$	$\omega^{********}/\boldsymbol{\omega}^{****}$
I	II	III	IV	**V**	VI	VII	VIII	IX	X	XI
519.56	562.52	1.08	548.82	**550.75**	555.48	551.98	552.38	1.00	551.39	1.00
451.56	491.17	1.09	480.07	**483.31**	489.82	484.69	485.73	1.01	485.61	1.00
387.63	421.90	1.09	413.36	**417.39**	424.18	418.61	420.26	1.01	421.37	1.01
330.38	359.23	1.09	352.77	**356.94**	363.03	357.91	359.85	1.01	361.50	1.01
280.17	304.24	1.09	299.32	**303.20**	308.26	303.93	305.84	1.01	307.25	1.01
236.46	256.54	1.08	252.74	**256.11**	260.16	256.66	258.33	1.01	259.16	1.01
198.49	215.36	1.08	212.33	**215.12**	218.29	215.54	216.87	1.01	217.14	1.01
165.66	180.02	1.09	177.49	**179.66**	182.02	179.96	180.93	1.01	180.82	1.01
142.47	151.81	1.07	149.06	**150.15**	151.37	150.31	150.77	1.00	150.59	1.00
109.77	117.80	1.07	116.32	**118.07**	120.26	118.34	119.09	1.01	118.68	1.01
88.07	95.49	1.08	94.36	**95.75**	97.30	95.96	96.47	1.01	96.28	1.01
69.00	75.35	1.09	74.46	**75.51**	76.66	75.67	76.04	1.01	76.16	1.01
52.50	57.80	1.10	57.11	**57.90**	58.75	58.03	58.32	1.01	58.73	1.01
38.40	42.75	1.11	42.24	**42.81**	43.42	42.90	43.15	1.01	43.78	1.02
26.59	30.07	1.13	29.71	**30.10**	30.52	30.17	30.39	1.01	31.10	1.03
16.99	19.67	1.16	19.43	**19.68**	19.95	19.73	19.93	1.01	20.57	1.05
9.55	11.49	1.20	11.35	**11.50**	11.65	11.52	11.68	1.02	12.15	1.06
4.24	5.51	1.30	5.44	**5.51**	5.58	5.52	5.62	1.02	5.89	1.07
1.06	1.70	1.60	1.68	**1.70**	1.72	1.70	1.74	1.02	**1.84**	1.08

ω^{*} - The natural frequencies with both joint ends of the beam, ω^{**} - The natural frequencies with fixed right end, ω^{***} - The natural frequencies of the beam from Fig. 4, a = 1.8m; b = 0.2m, ω^{****} - The natural frequencies of the beam from Fig. 4, a = 1.6m, b = 0.4m, (taken in the considerations), ω^{*****} - The natural frequencies of the beam from Fig. 4, a = 1.4m; b = 0.6m; ω^{******} - *a* = 1.6m; *b* = 0.4m, *D* = 216mm, d = 200mm, element between nodes 12 and 13 have has been changed, Fig. 4); $\omega^{*******}$ - The natural frekvencies of the beam from Figure 11, a = 1.6m b = 0.4m, D = 216mm, d = 200mm, Iz = 2.831 cm^4, between nodes 11 and 13), $\omega^{********}$ - The natural frekvencies of the beam from Figure 11, a = 1.6m, b = 0.4m, D = 216mm, d = 184mm, Iz = 5058.7 cm^4, between nodes 11 and 13.

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MODIFIKACIJA DINAMIČKIH KARAKTERISTIKA U STRUKTURALNOJ REANALIZI MEHANIČKIH SISTEMA

Nataša Trišović

Tehnika strukturne dinamičke modifikacije (SDM) može se definisati kao skup metoda pomoću kojih se dinamičko ponašanje konstrukcije može popraviti procenom modifikovanog ponašanja dobijenog dodavanjem modifikacija kao na primer koncentrisanih masa, krutih veza, prigušenja, novih elemenata, isl. ili promenom konfiguracionih parametara u samoj strukturi. Takve metode kod kojih je osnov metod konačnih elemenata se često se nazivaju metode reanalize. Potreba za strukturnom dinamičkom modifikacijom se pojavila zbog zahteva za višim performansama složenih mašina i strukturnih sistema, kao što su mašine alatke, automobili, šinska vozila, avioni, i sistemi sa velikim brojem obrtaja, koji zahtevaju zvučno dinamičko projektovanje, odnosno željene dinamičke karakteristike kao što su nivo vibracija/odziv, rezonanca/sopstvene vrednosti, dinamička stabilnost i modalni oblici. Ovaj rad se bavi poboljšanjem dinamičkih karakteristika jedne cevne sabirnice prstenastog poprečnog preseka. Pokazano je kako se promenom graničnih uslova i geometrije mogu popraviti dinamičke karakteristike konstrukcije. U toj analizi se koristi raspodela potencijalne i kinetičke energije u svakom konačnom elementu. Takodje, pokazano je da dinamička modifikacija igra važnu ulogu u reanalizi konstrukcija.

Ključne reči: *dinamička modifikacija konstrukcija, sopstvene vrednosti, potencijalna i kinetička energija*