SIMULATION OF THE CRACK PROPAGATION THROUGH A PLANAR PLATE WITH THE MIDDLE POSITIONED CYLINDRICAL HOLE

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Abstract
A prediction of crack propagation is subject of research of many scientists. Understanding crack occurrence and propagation could save lives (i.e. in airplane industry), or at least reduce the costs occurred by unexpected failures and stoppages. In this paper it is provided a brief theoretical background of the crack propagation regarding to stress intensity factor, and simulation of crack propagation through a planar plate made of steel with the middle positioned cylindrical hole, by means of Abaqus software. In addition, it is performed calculation of stress intensity factors with MORFEO, software that could be implemented onto Abaqus.

Keywords: *Crack Propagation, Abaqus, Morfeo, Stress Intensity Factor*

1. INTRODUCTION

If taken on macro-level polycrystalline materials could be assumed as homogenous. But if it is observed in micro-level, they are certainly not. Due to this inhomogeneity, and under applied load, there occur micro cracks between the grains of crystals. They are usually initiated on the surface of the specimen, and represent the stress concentration factors, which locally increase the stress value. When this value achieves the critical value, the crack begins to propagate. Usually, only one of the micro cracks continues to propagate, until the final fracture of the specimen occurs.

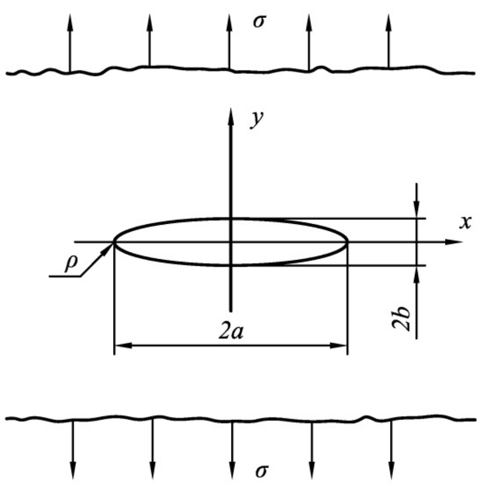
Prediction of the crack propagation is very important in sense of economical and safety reasons as well. By predicting the crack propagation, it is possible to evaluate the remaining time before the final fracture occurs, and plan the replacement of the part.

Stress intensity factor analysis of interface cracks using X-FEM was presented by Nagashima et al [1]. It was 2D bi-material interface crack problem. Shi et al. [2] developed a 3D finite element method for the analysis of fatigue crack growth based on the extended finite element method (X-FEM). The crack morphology was described by level set methods, which is convenient modelling of growing cracks without remeshing. Numerical modelling crack propagation under Mode II fracture in plain concretes containing siliceous fly-ash additive was presented by Golewski et al [3]. Xue et al. [4] did numerical modelling crack propagation of sheet metal forming based on stress state parameters using XFEM method, applying ABAQUS program and its user subroutine UVARM. It is found that the stress state parameters can well reflect the micro-crack propagation, which is useful for the prediction of service life and it plays an important role in the damage tolerance design field. Zhuang and Cheng [5] performed Development of X-FEM methodology and study on mixed-mode crack propagation. The computational results show that the fracture mode and the crack growth path can be strongly affected by the interface of bi-materials and the loading asymmetry.

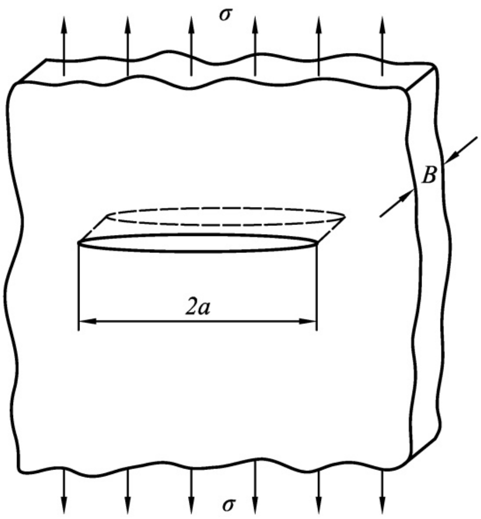
2. THEORETICAL BACKGROUND

Alan Arnold Griffith was the first to explain the phenomena of local stress increase due to crack occurrence. He made assumption that crack could be considered as elliptic hole (

Figure 1).



a)



b)

Figure 1. Griffith criterion: a) elliptical hole, b) crack

For the above case, highest value of the stress is given by the following formula:

$$\sigma_{\max} = \sigma\left(1+\frac{2a}{b}\right) = \sigma\left[1+2\left(\frac{a}{\rho}\right)^{1/2}\right] \quad (1)$$

There are three load modes, which cause three different movement of one crack surface relative to the other (Figure 2):

1. Mode I – opening, symmetrical opening of crack surfaces. Load type – tension stress σ
2. Mode II – sliding of the crack surfaces at the same plane. Load type – shear stress τ
3. Mode III – tearing of the crack surfaces at the different planes. Load type – shear stress τ

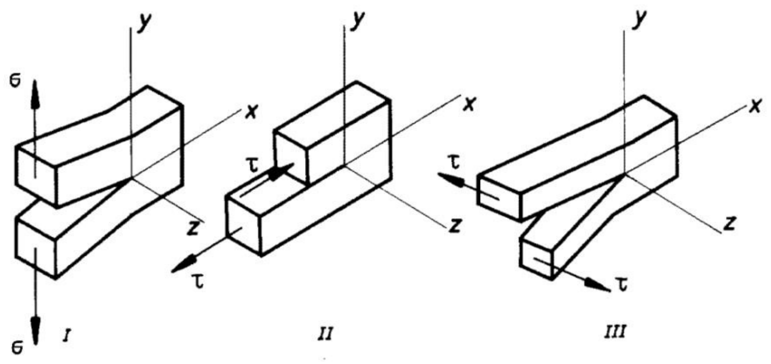


Figure 2. Modes of crack propagation regarding to load direction and relative movement of crack surfaces

2.1. Stress intensity factor

Stress intensity factors are proportional to stresses around the tip of the crack. They define amplitude of the stress singularity. In other words, with the stress intensity factor it is taken into account an influence of stresses, deformations and displacements around the tip of the crack. A unite for the stress intensity factor is MPa $\sqrt{\mathrm{m}}$. The formula for the stress in dependence of the stress intensity factor for the element very close to the tip of the crack is:

$$\lim_{r\to o}\sigma_{ij}^{(I)}=\frac{K_I}{\sqrt{2\pi r}}f_{ij}^{(I)}(\theta),\ \lim_{r\to o}\sigma_{ij}^{(II)}=\frac{K_{II}}{\sqrt{2\pi r}}f_{ij}^{(II)}(\theta),\ \lim_{r\to o}\sigma_{ij}^{(III)}=\frac{K_{III}}{\sqrt{2\pi r}}f_{ij}^{(III)}(\theta) \qquad (2)$$

In the case of the plate of infinite length and width with middle positioned hole stress intensity factor is given by the following formula:

$$K_I=\sigma\sqrt{\pi a} \qquad (3)$$

In case if finite dimensions of the plate, there is coefficient Y added to the formula. This coefficient depends on type and position of the crack, as well as of dimensions of the plate.

$$K_I=Y\sigma\sqrt{\pi a} \qquad (4)$$

2.2. XFEM Method

XFEM Method is developed to improve solutions for problems with local discontinuities (cracks, surface contacts…), which cannot be improved by further mesh refinement. First to introduce this method was Ted Belytschko, back at 1999. It is well known concept of FEM, in which there are standard polynomial functions to describe displacements of the nods. At the XFEM method in addition to polynomial function, there are functions of discontinuity applied to nods which are close to given discontinuity (crack or contact). By means of these functions, it is possible to avoid remeshing in every step of calculation. In other words, only one mesh generation is needed to perform entire simulation of crack propagation

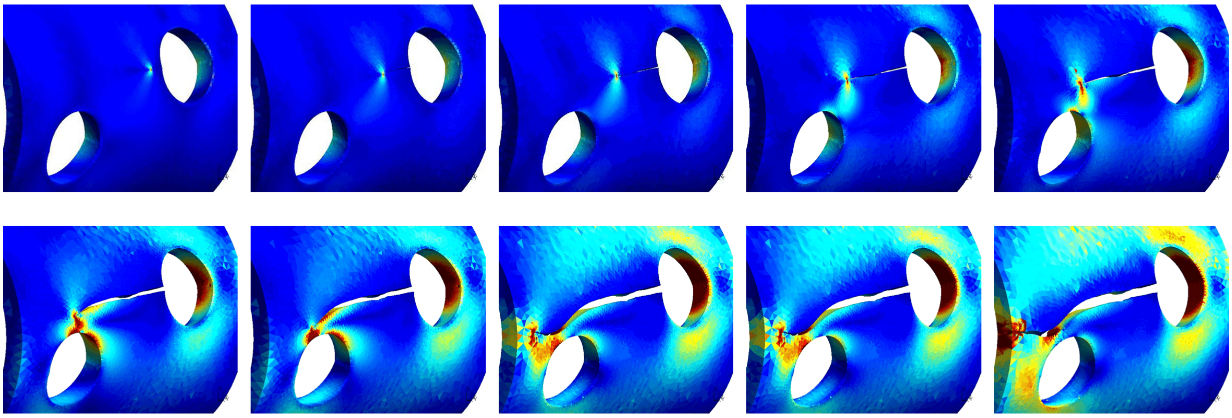
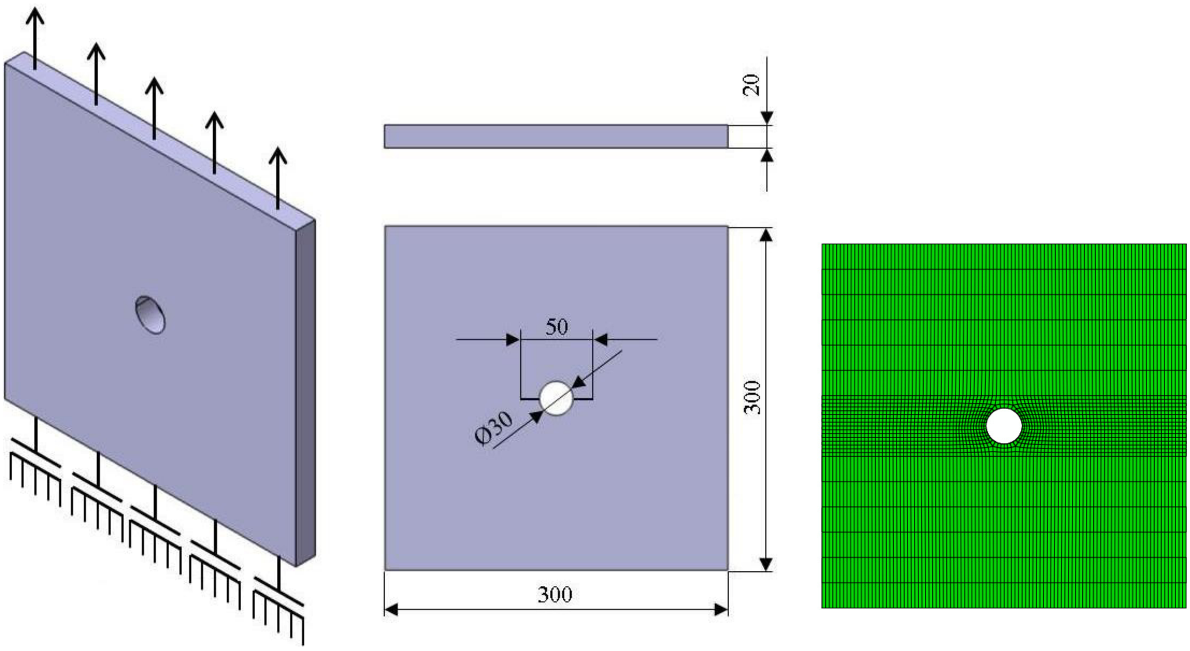


Figure 3 Simulation of the crack propagation by means of XFEM Method [6]

3. CRACK PROPAGATION ANALYSIS BY MEANS OF ABAQUS + MORFEO SOFTWARE

Subject of the simulation was a plate, with dimensions 300x300x20 and middle positioned hole Ø30 (Figure 4a). One end of the plate is cantilevered, while the opposite is subjected to axial load of 10 MPa. At the area of central hole, it is initiated a crack, in direction normal to the direction of load. The length of crack initiation is 10 mm left and right to the hole. The material of the plate is construction steel, which properties are given at the Table 1. It is performed meshing with hexahedral elements. Mesh was much denser in the region of expected crack propagation (Figure 4b) in order to obtain better accuracy. Total number of elements was 11208.



a) b)

Figure 4 a) Boundary conditions, loads and dimensions, b) Mesh of FE

Table 1. Material properties

Material:	Construction steel
Modulus of elasticity	200 [GPa]
Poisson's ratio	0,33
Maximal principal stress	150 [MPa]

MORFEO is special software developed by GEONX Company, which integrates with ABAQUS as an additional module, and it is able to extract stress concentration factors. To do that, it is necessary to perform "opening" of the crack in ABAQUS, and afterwards to continue with the crack propagation in MORFEO. Input data that should be inserted into MORFEO are number of steps of crack propagation (in this case it was 10 steps) and magnitude of every step (0.5 mm). An output file of MORFEO is consisted of coordinates of each node x, y and z, and values of K_I, K_{II}, K_{III} and K_{ekv} in every single node which belongs to crack front. Since the output file is quiet big to be presented in this paper, there follows a table with average values of K_I, K_{II}, K_{III} and K_{ekv} for every step of crack propagation.

Table 2. Average values of stress intensity factors for every step of crack propagation

Step	K_{ekv} **[MPa√m]**	K_I **[MPa√m]**	K_{II} **[MPa√m]**	K_{III} **[MPa√m]**
1	105,2461071	105,279679	1,28901464	-0,00223466
2	108,6275	108,574214	-0,71594982	0,002269525
3	107,5497857	107,502214	-0,07436818	-0,00236342
4	108,3113214	108,292607	-0,13395814	-0,007464
5	108,9173571	108,991857	0,48802629	0,003186372
6	108,8736429	108,905286	-0,23319675	0,013746009
7	109,4125357	109,352857	0,40155	-0,02416928
8	110,6215333	110,631267	-0,4388207	-0,0003283
9	111,7147333	111,776033	0,25279895	0,007164186
10	112,4803214	112,365286	0,01584482	0,003702531

It was expected for K_I factor to be much higher than other two coefficients K_{II} and K_{III} because of nature of load, which tends to open the crack (Mode I). If we compare the results from

Table 2, it is obvious that assumption is fulfilled. K_I is for two orders of magnitude higher than K_{II} and K_{III}. Also K_{II} and K_{III} are very close to 0, which makes us conclude that their value is just the numerical error. That is also in accordance with Mode I theory.

On the other hand, stress intensity factor should increase for every next step of crack propagation. There follows a diagram which confirms this assumption, too.

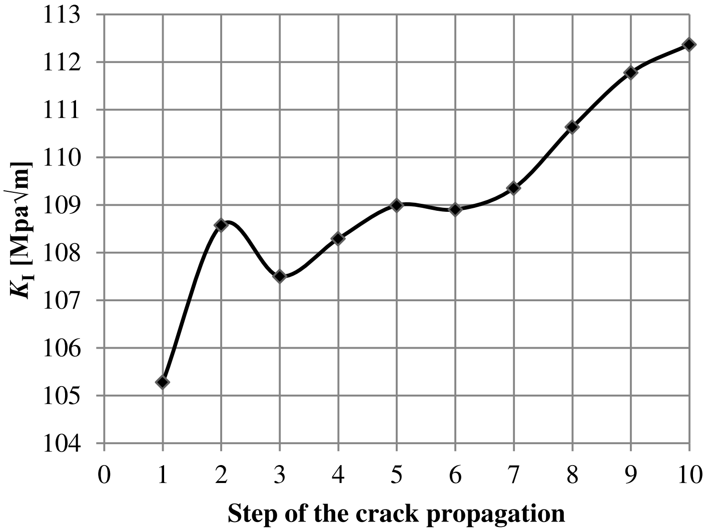


Figure 5. Trend of K_I factor for 10 steps of crack propagation

4. CONCLUSIONS

In this paper it was given a short theoretical background for the crack propagation theory, with special accent on explanation of term of stress intensity factor. Afterwards it was performed modelling of a plate with cylindrical hole in the middle with crack initiations positioned normal to the load direction. By means of ABAQUS software, it was performed complete opening of the crack. Then, it was performed partial opening of the crack by means of MORFEO, and it was obtained values of all stress concentration factors K_I, K_{II} and K_{III}.

In accordance with the theory, regarding to the nature of load, it was obtained the highest values for K_I factor. The other two factors are close to the numerical error.

It is observed the increase of the K_I in every next step of crack propagation. Since the remaining surface of the plate which carries the load is decreasing as the crack propagates, it is also expected behaviour.

5. ACKNOWLEDGEMENT

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