

## A NUMERICAL PROBABILISTIC APPROACH FOR POUNDING EFFECTS ON THE SEISMIC RESPONSE OF ADJACENT RC STRUCTURES IN DUAL SYSTEMS STRENGTHENED BY TENSION-TIES

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**Abstract:** *Dual systems of reinforced concrete (RC) moment frames with concrete shear walls have been used as common lateral load resisting systems in earthquake prone countries. In order to overcome damages caused by seismic actions in the above systems, a strengthening by cable-like members (tension-only tie-elements) can be used. In the present study, a numerical probabilistic treatment for the pounding problem concerning the seismic interaction between adjacent structures strengthened by cable-ties in such dual RC systems is presented when the input parameters are uncertain. This problem concerns the elastoplastic-fracturing unilateral contact between neighbouring structures during earthquakes and is considered as an inequality problem of dynamic structural contact mechanics. The Monte Carlo method is used for treating the uncertainty concerning input parameters. The purpose here is to estimate numerically and to control actively the influence of the cable-ties on the seismic response of the adjacent structures. Finally, in a practical case of two structures in a dual system, the effectiveness of the proposed methodology is shown*

**Key words:** *Seismic Pounding, Dual RC systems, Strengthening by cables-ties.*

### 1. Introduction

As well-known, see e.g. [1], the so-called “mixed-system” or “dual system” of frames and shear walls are often used in reinforced concrete (RC) building structures. In such systems, the case of the seismic pounding between adjacent frames and shear walls can become a crucial

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problem, especially when the adjacent structures have non-equal story heights. It is reminded that pounding concerns the seismic interaction between adjacent structures, e.g. neighboring buildings in city centers constructed in contact when the so-called “continuous” building system is allowed to be applied. On the common contact interface, during an earthquake excitation, appear at each time-moment either compressive stresses or relative removal displacements (separating gaps) only, see for details [1-9].

In general, lessons learnt from strong earthquakes have shown that pounding can cause significant strength degradation and damages on existing adjacent structures. To overcome strength degradation effects, various repairing and strengthening procedures can be used for the seismic upgrading of existing buildings [1]. Among them, cable-like members (ties) can be used as a first strengthening and repairing procedure [10]. These cable-members can undertake tension, but buckle and become slack and structurally ineffective when subjected to a sufficient compressive force. So, in the mathematical problem formulation, the constitutive relations for cable-members are also inequality conditions. These requirements result to inequality conditions in the mathematical problem formulation [10,11].

On the other hand, for the numerical analysis of such systems of old existing reinforced concrete (RC) structures, many uncertainties for input parameters must be taking into account [6,12]. These uncertainties mainly concern the holding properties of the old materials that had been used for the building of such structures, e.g. the remaining strength of the concrete and steel, as well as the cracking effects etc. Therefore, an appropriate estimation of the input parameters and use of probabilistic methods must be performed [12-16].

In the present study, the seismic problem of old colliding structures in dual RC systems is analyzed in a numerical stochastic way. Emphasis is given to the uncertainty concerning the input parameters. For this purpose, the input-parameters are considered as interval parameters with known upper and lower bounds, characterized in Civil Structural Engineering as uncertain-but-bounded parameters [13,14]. The herein numerical stochastic approach is based on Monte Carlo simulation methods, see e.g. [17-19]. Finally, an application is presented for a simple typical example of pounding concerning a dual RC system of an industrial RC frame adjacent to a RC shear wall.

## 2. Method of analysis

The methodology presented recently in [6] for the analysis concerning the seismic pounding of existing adjacent RC framed structures is followed herewith. Briefly, the probabilistic approach may be obtained through Monte Carlo simulations. As well-known [17-19], Monte Carlo simulation is simply a repeated process of generating deterministic solutions to a given problem. Each solution corresponds to a set of deterministic input values of the underlying random variables. A statistical analysis of the so obtained simulated solutions is then performed. Thus the computational methodology consists of solving first the deterministic problem for each set of the random input variables and finally realizing a statistical analysis.

### 2.1. Numerical Treatment of the Deterministic Problem

For the herein pounding problem formulation, a system of two adjacent structures (A) and (B) is considered for simplicity. Following the methodology presented in Liolios [4-7], the system of the two structures (A) and (B) is discretized by the finite element method. Let  $j_A$  and  $j_B$  be two associated nodes on the interface (joint) of (A) and (B), where unilateral frictional contact can take place during an earthquake. These nodes are considered (see Liolios [4-7]) as connected by two fictive unilateral constraints, normal to interface the first and tangential the second one. The corresponding force-reactions and retirement relative displacements are denoted by  $r_{jN}$ ,  $z_{jN}$  and  $r_{jT}$ ,  $z_{jT}$ , respectively. They satisfy in general nonconvex and

nonmonotone constitutive relations of the following type (1), expressing mathematically the unilateral elastoplastic hardening/softening contact with friction:

$$(1) \quad r_j(d_j) \in \partial R_j(d_j).$$

Here  $\partial$  is the generalized gradient of Clarke,  $d$  the deformation and  $R_j(\cdot)$  is the superpotential function, see Panagiotopoulos [11]. Relation (1) also can simulate environmental effects, that cause a capacity degradation of the interaction interface.

By piecewise linearizing the above relations as in elastoplasticity [3-6], the following linear complementarity conditions are obtained:

$$(2) \quad w_j \geq 0, \quad r_{jN} \leq 0, \quad w_j \cdot r_{jN} = 0.$$

Taking into account the interaction and the second-order geometric effects (P-Delta effects), the incremental dynamic equilibrium conditions under earthquake ground excitation  $u_g(t)$  for the coupled system of the interacting buildings (A) and (B) are [6]:

$$(3) \quad M_A \Delta \ddot{u}_A + C_A \Delta \dot{u}_A + (K_A + G_A) \Delta u_A = -M_A \Delta \ddot{u}_g + B \Delta p,$$

$$(4) \quad M_B \Delta \ddot{u}_B + C_B \Delta \dot{u}_B + (K_B + G_B) \Delta u_B = -M_B \Delta \ddot{u}_g - B \Delta p,$$

$$(5) \quad p = p_N + p_T.$$

Here  $M_L$ ,  $C_L$ ,  $K_L$  are the mass, damping and stiffness matrices, respectively, for structure L (L=A,B);  $u(t)$  is the sought node displacement (relative to ground) vector corresponding to given ground earthquake excitation  $u_g(t)$  and appropriate initial conditions; dots over symbols indicate time derivatives;  $G_A$  and  $G_B$  are the geometric stiffness matrices, by which P-Delta effects are taken into account;  $B$  is a transformation matrix. The pounding stress vector  $p$  is decomposed to the vectors  $p_N$ , of the normal, and  $p_T$  of the tangential interaction forces between frames (A) and (B), satisfying in general the nonconvex and nonmonotone constitutive relations (1).

To above conditions are adjoined the initial conditions. So the problem consists in finding the time-dependent vectors  $\{u_A, u_B, p\}$  which satisfy the rels. (1)-(5) for the given earthquake excitation  $u_g(t)$ .

As mentioned, each solution of the above deterministic problem corresponds to a set of deterministic input values of the underlying random variables. The time-history responses  $\{u_A, u_B\}$ , satisfying (3)-(5) for the system of the adjacent structures (A) and (B), can be also numerically evaluated by means of the structural analysis software Ruaumoko [20]. Ruaumoko provides results which are related to the following critical parameters: local or global structural damage, maximum displacements, interstorey drift ratios, development of plastic hinges and response using the incremental dynamic analysis (IDA) method.

Here the assessment is based on a relevant evaluation of suitable damage indices [10,21]. After Park/Ang [21], the *local* damage index  $DI_L$  is computed by the following relation:

$$(6a) \quad DI_L = \frac{\mu_m}{\mu_u} + \frac{\beta}{F_y d_u} E_T$$

where:  $\mu_m$  is the maximum ductility attained during the load history,  $\mu_u$  the ultimate ductility capacity of the section or element,  $\beta$  a strength degrading parameter,  $F_y$  the yield generalized force of the section or element,  $E_T$  the dissipated hysteretic energy, and  $d_u$  the ultimate generalized deformation.

The Park/Ang *global* damage index is given by the following relation:

$$(6b) \quad DI_G = \frac{\sum_{i=1}^n DI_{Li} E_i}{\sum_{i=1}^n E_i}$$

where  $DI_{Li}$  is the local damage index and  $E_i$  the energy dissipated at location  $i$ , and  $n$  the number of locations at which the local damage is computed

## 2.2. Numerical Treatment of the Probabilistic Problem

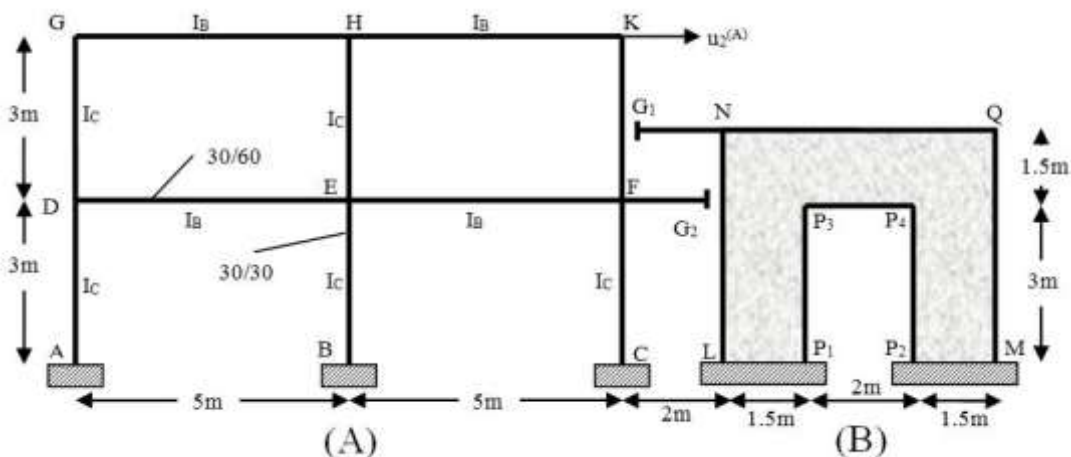
In order to calculate the random characteristics of the response of the considered RC buildings, the Monte Carlo simulation is used [17-19]. As mentioned, the main element of a Monte Carlo simulation procedure is the generation of random numbers from a specified distribution. Systematic and efficient methods for generating such random numbers from several common probability distributions are available. The random variable simulation is implemented herein by using the technique of Latin Hypercube Sampling (LHS) [12].

In more details, a set of values of the basic design input variables can be generated according to their corresponding probability distributions by using statistical sampling techniques. As concerns the uncertain-but-bounded input parameters [14,15] for the stochastic analysis, these are estimated here by using available upper and lower bounds, denoted as  $U_B$  and  $L_B$  respectively. So, the mean values are estimated as  $(U_B + L_B)/2$ .

## 3. Numerical example

### 3.1. Description of the considered RC structural system.

The system shown in Fig. 1 is investigated. It is a 2-D “mixed” system consisting of the two adjacent reinforced concrete (RC) structures, the frame (A) and the shear wall (B)..



**Figure 1.** The initial system of the RC structures (A) and (B), without cable-strengthening and with two possible unilateral contacts on  $G_1$  and  $G_2$ .

The shear wall (B) has an orthogonal opening of 2mx3m. Both structures are of estimated concrete class C20/25, and have been designed according to Greek building codes and to current European seismic codes. The frame beams are of rectangular section 30/60 (width/height, in

cm), with section inertia moment  $I_B$  and have a total vertical distributed load 30 kN/m (each beam). The frame columns, with section inertia moment  $I_C$ , have section dimensions, in cm: 30/30. The thickness of the shear wall (B) is 20cm. The structures are parts of two adjacent buildings, which initially were designed and constructed independently in different time periods. Due to connections shown in Fig. 1, pounding is expected to take place on frame column FK (point  $G_1$ ) and on shear wall part LN (point  $G_2$ ) of structures (A) and (B), respectively. The gaps on  $G_1$  and  $G_2$  are taken initially as zero. The system of the seismically interacting RC structures (A) and (B) has been subjected to various extremal actions (seismic, environmental etc.). So, corrosion and cracking have been taken place, which have caused a strength and stiffness degradation. The effective stiffness of the concrete members are estimated according to [22]. The so resulted reduction for the section inertia moments  $I_C$  and  $I_B$  was estimated to be 20% for the internal column BH and the shear wall (B), 40% for the external columns AG and CK, and 60% for the frame beams.

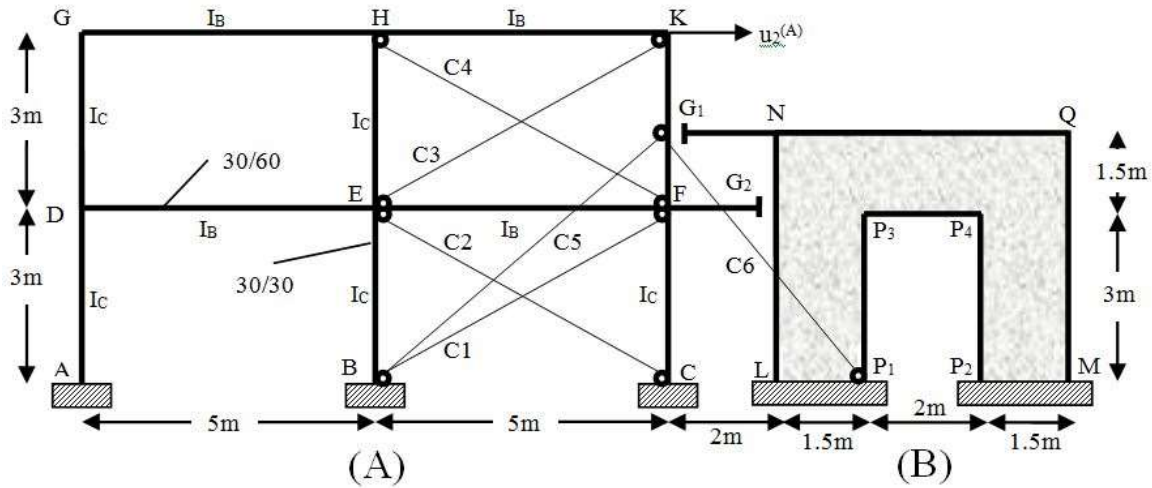
As concerns the discretization in space by using finite elements, for the RC frame (A) the usual 2-D frame elements are used. For the shear RC wall (B), use is made of the displacement-compatible plane stress model proposed and applied in [23]. This model is a quadrilateral plane stress one with 8 nodes totally. Of them, the 4 nodes are the corner ones and the 4 others on the side middles. Each node has three degrees of freedom. So, the displacement vector of each node  $i$  has two translational components,  $u_{ix}$  and  $u_{iy}$ , and one rotational component  $\theta_{iz}$ . This formulation allows the connection of the plane stress elements with the frame elements. Concerning the shear wall (B), 6 square elements with dimensions 1.5m x 1.5m and one orthogonal element with dimensions 2.0m x 1.5m are used.

To overcome the above degradation and rehabilitate seismically the system, various strengthening schemes by cable-elements can be investigated. These schemes are here denoted as SJ, where J is the number of the bracing-cables which are taken into account. So, the frame system of Fig. 1 is denoted as S0 and no strengthening by cable-bracings is considered. In order to upgrade seismically the damaged system S0, here indicatively only one strengthening by cable-bracings is considered. So, the cable-bracing scheme of Fig. 2 is used, denoted as S6R, having the two cable-elements C5 and C6 connect node  $G_1$  with the frame node B and the shear wall node  $P_1$ , respectively.

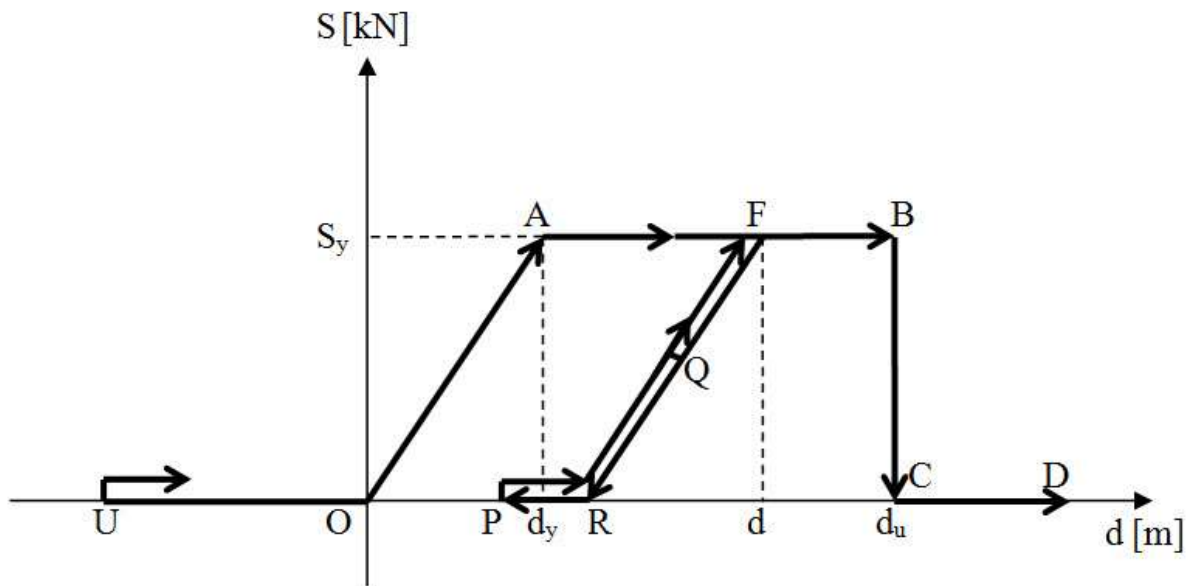
The cable elements have a cross-sectional area  $F_c = 18 \text{ cm}^2$  and they are of steel class S220 with yield strain  $\varepsilon_y = 0.2 \%$ , fracture strain  $\varepsilon_f = 2 \%$  and elasticity modulus  $E_c = 200 \text{ GPa}$ . The cable constitutive law, relevant to the piece-wise linearized form of eq. (2.1), is depicted in Fig. 3. The general unilateral (slackness), hysteretic, fracturing, unloading-reloading etc. behavior of cable-elements is shown. So, segments UO and OA concern the slack-linear elastic behavior, according to which the cables can not undertake compressive stresses (branch UO). Segments AB and DC concern the plastic and fracture behavior, respectively. The cable yield resistance is  $S_y = 438.3 \text{ kN}$ . The yield deformation is  $d_y = 11.7 \text{ mm}$  for cables C1- C4 in frame (A), and  $d_y = 13.5 \text{ mm}$  for cables C5- C6 in Fig. 4. Denoting the ultimate cable elongation by  $d_u$ , the cables ultimate ductility appeared in fig. 3 is  $d_u/d_y = 10$ . Paths FQF or FPRPF concern unloading-reloading cases.

The estimated concrete class is C20/25 and the steel class is S220. According to JCSS (Joint Committee Structural Safety), see [13], concrete strength and elasticity modulus follow Normal distribution and the steel strength follows the Lognormal distribution. So the statistical characteristics of the input random variables concerning the building materials are estimated to be as shown in Table 1. The mean/median values of the random variables correspond to the best estimates employed in the deterministic model according to Greek Building Concrete Code EKOS2000. By COV is denoted the coefficient of variation. The mean/median values of the random variables correspond to the best estimates employed in the deterministic model according to Greek codes. On the contrary, the input variables concerning the steel of the

bracing ties (new material) are considered as deterministic ones.



**Figure 2.** The S6R system with 4 diagonal cables and 2 cable-elements C5 and C6 connecting node G<sub>1</sub> with the shear wall node P<sub>1</sub> and the frame node B.



**Figure 3.** The constitutive law of cable-elements.

### 3.2. Earthquakes Sequence Input

The systems S0 and S6R of Fig. 1 and Fig. 2 are considered to be subjected to the multiple ground seismic excitation Coalinga, shown in Table 1 and presented and discussed in the paper [10]. The complete list of three multiple earthquakes are shown in Table 1 and was downloaded from the strong motion database of the Pacific Earthquake Engineering Research (PEER) Centre.

**Table 1.** Statistical data for the building materials treated as random variables

|                                      | Disribution | mean      | COV |
|--------------------------------------|-------------|-----------|-----|
| Compressive strength of concrete     | Normal      | 20.0 MPa  | 15% |
| Yield strength of steel              | Lognormal   | 191.3 MPa | 10% |
| Initial elasticity modulus, concrete | Normal      | 29.0 GPA  | 8%  |
| Initial elasticity modulus, steel    | Normal      | 200 GPA   | 4%  |

**Table 2.** Multiple earthquakes data

| No | Seismic sequence | Date (Time)        | Magnitude (M <sub>L</sub> ) | Recorded PGA(g) | Normalized PGA(g) |
|----|------------------|--------------------|-----------------------------|-----------------|-------------------|
| 1  | Coalinga         | 1983/07/22 (02:39) | 6.0                         | 0.605           | 0.165             |
|    |                  | 1983/07/25 (22:31) | 5.3                         | 0.733           | 0.200             |
| 2  | Imperial Valley  | 1979/10/15 (23:16) | 6.6                         | 0.221           | 0.200             |
|    |                  | 1979/10/15 (23:19) | 5.2                         | 0.211           | 0.191             |
| 3  | Whittier Narrows | 1987/10/01 (14:42) | 5.9                         | 0.204           | 0.192             |
|    |                  | 1987/10/04 (10:59) | 5.3                         | 0.212           | 0.200             |

### 3.3. Representative probabilistic results

After application of the herein proposed computational probabilistic approach by using 250 Monte Carlo samples, some representative results are shown in Table 3. These results concern the Coalinga case of the seismic sequence of Table 2.

In column (2) of the Table 4, the Event E<sub>1</sub> corresponds to Coalinga seismic event of 0.605g PGA, and Event E<sub>2</sub> to 0.733g PGA, ( $g=9.81\text{m/sec}^2$ ). The sequence of events E<sub>1</sub> and E<sub>2</sub> is denoted as Event (E<sub>1</sub>+ E<sub>2</sub>). The coefficient of variation COV concerns the Event (E<sub>1</sub>+ E<sub>2</sub>). In table columns (3)-(7) the mean values of the shown quantities and the COV concerning the Event (E<sub>1</sub>+ E<sub>2</sub>) are given. So, in table column (3) the Global Damage Indices DI<sub>G</sub> and in table column (4) the Local Damage Index DI<sub>L</sub> for the bending behavior of the element FK in frame (A) are given. Next, the maximum compressive impact-contact forces on the pounding regions G1 and G2 are given in the table columns (5) and (6), respectively. Finally, in the table column (7), the maximum horizontal top displacement  $u_{top} = u_2^{(A)}$  of the second frame floor is given.

As the table values show, multiple earthquakes generally increase, in an accumulative way, the response quantities, e.g. critical displacements and damage indices. On the other hand, the strengthening of the frame (A) by tie bracings (system S6R of Fig. 2) improves the response behaviour against seismic sequences. So, the mean values of the maximum horizontal top displacement  $u_{top} = u_2^{(A)}$  of the second frame floor in S6R are smaller in comparison to ones of S0. These values can be further reduced by a parametric investigation of the cable-ties characteristics, e.g. by increasing their cross-sectional area  $F_r$  or investigating alternate cable-strengthening schemes.

**Table 3.** Mean values of representative response quantities for the systems S0 and S6R

| SYSTEM | EVENTS                         | DI <sub>G</sub> | DI <sub>L</sub> | IMPACT-G <sub>1</sub> [kN] | IMPACT-G <sub>2</sub> [kN] | $u_{top}$ [mm] |
|--------|--------------------------------|-----------------|-----------------|----------------------------|----------------------------|----------------|
| (1)    | (2)                            | (3)             | (4)             | (5)                        | (6)                        | (7)            |
| S0     | E <sub>1</sub>                 | 0.208           | 0.235           | -118.8                     | -42.4                      | -37.4          |
|        | E <sub>2</sub>                 | 0.290           | 0.268           | -265.4                     | -58.2                      | -50.3          |
|        | E <sub>1</sub> +E <sub>2</sub> | 0.398           | 0.374           | -369.3                     | -82.4                      | <b>-74.2</b>   |
|        | COV                            | 27.2%           | 30.4%           | 28.8%                      | 27.7%                      | 32.8%          |
| S6R    | E <sub>1</sub>                 | 0.012           | 0.084           | -285.0                     | -316.3                     | -12.7          |
|        | E <sub>2</sub>                 | 0.045           | 0.088           | -296.0                     | -333.1                     | -16.7          |
|        | E <sub>1</sub> +E <sub>2</sub> | 0.053           | 0.091           | -307.0                     | -336.3                     | <b>-17.9</b>   |
|        | COV                            | 20.4%           | 25.8%           | 23.7%                      | 26.7%                      | 28.8%          |

#### 4. Concluding remarks

A probabilistic numerical approach for the inelastic seismic behaviour of adjacent existing RC structures in dual systems, strengthened by cable elements, has been presented. Input parameters uncertainty is taking into account in the herein presented numerical approach. The pounding effects and the unilateral behaviour of cable-elements are strictly taken into account. In a numerical example, concerning two adjacent RC structures in a dual system, one frame and one shear wall, under multiple earthquakes, the applicability of the methodology has been proven. As the results show, the optimal strengthening version of the cable-bracings can be decided by computing necessary damage indices. Generally it is concluded that pounding has significant effects on the earthquake response of adjacent structure. Hence, cable strengthening can be effectively used for the seismic upgrading of existing adjacent RC structures.

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