Eigenvalue Sensitivity Analysis in Structural Dynamics

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Structural dynamic modification implies the incorporation, into an existing model, of new information gained either from experimental testing or some other source, which questions or improves the accuracy of the model. The sensitivity approach is based on the prior selection of updating parameters (design variables) in the initial FE model. This paper deals with analysis of the dynamic behavior of shaft of electromotor. Two cases are done. The second example problem is dynamic analysis of 12-node cantilever beam. Distribution of potential and kinetic energy in every finite element is used for analysis. In this study it is shown that structural dynamic modification is important in structural reanalysis.

Keywords: *structural dynamics modification, eigenvalues, potential and kinetic energy.*

1. INTRODUCTION

Structural design for optimal dynamic behavior is an important problem, especially for structures whose operational performance and integrity strongly depends on the structural dynamic characteristics. Some of the important application areas of this technology are inegrated controls-structures design, flutter control and buckling load modification. An excellent review of the field can be found in a paper by Grandhi [1].

The dynamic response of a structural system is primarily governed by the natural frequencies and mode shapes. Hence, formal modification techniques can be used to achieve the desired dynamic behavior by changing the design variables to manipulate the natural frequencies and mode shapes. The design variables depend on the type of modification problem. In the design of structural components, such as stiffened panels and cylinders, the design parameters represent the spacing of the stiffeners, the size and shape of the stiffeners, and the thickness of the skin. If the skin and/or stiffeners are made of layered composites, the orientation of the fibers and their proportion can become additional variables. The sizes of the elements are design variables of a structural system of fixed configuration (frames, trusses, wings, fuselages, etc). The thickness of plates, cross-sectional areas of bars, areas, moments of inertia, and torsional constants of beams represent sizes of the elements.

It is becoming widely accepted that sensitivity analysis can be a valuable tool in structural reanalysis where (enough of) the modal properties are known, either through theoretical or experimental analysis. In the modal analysis literature there have been two primary applications. In the first case sensitivity data are used solely as a qualitative indicator of the location and approximate scale of design changes to achieve a desired change in structural properties. The consequences of candidate design changes would then be evaluated using exact methods. The second strategy uses the design sensitivities directly to predict the effect of proposed structural changes. The use of sensitivities in this fashion relies on the Matrix Taylor Series expansion, with the usual implications of convergence and truncation errors. Use only of first order design sensitivities assumes implicitly that the second (and higher) order derivatives are negligible. The use of these second order sensitivities as suitable criteria for the acceptability of first order sensitivities for predictive analysis can be interested in some detail. Sensitivity analysis may be applied to candidate design modifications distributed across a number of degrees of freedom of the structure but is limited in scale.

Modal design sensitivities are the derivatives of the eigensystem of a dynamic system with respect to those variables which are available for modification by the designer. A typical modification would be the change in diameter of a circular section. This would affect both the mass of the section, proportional to the square of the diameter, and its stiffness, which depends on the second moment of area of the section. A change in length would have a mass effect directly proportional to length, but a stiffness change depending on the cube of length. Changing material would similarly affect mass, stiffness and damping. Shape sensitivity analysis of physical systems under dynamic loads may be important from different points of view (i) to understand and model the system's behavior better with respect to shape, (ii) to optimize the physical shapes of the desired systems responses in a prescribed time interval, or (iii) to identify shapes by utilizing the system's measured response in time. There are a lot of papers which deal with structural dynamics modification [6-35].

The general perturbation procedure followed in major papers is diagrammatically shown in Fig. 1.

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2. PROBLEM STATEMENT. DERIVATION. THE PARAMETERS OF DYNAMIC MODIFICATION

Specific structural analyses provide closer determination of structural behaviour [2]. The parameters of modification (distribution of membrane and bending stresses [29], distribution of deformation energy, kinetic and potential energy within the element of the structures) provide very efficient identification of structural behaviour. They define necessary modification of structure providing better behaviour of structure in service life. The problem of modification, mathematicaly, comprises of minimization of objective function $F_i(v_j)$ (weight, deformation energy, stress level, eigenvalues,...) of design variables v_j (nodal coordinates, area of cross section, depth...) with constraints $g(v_j)$ (constraints of stress level, displacements, length, area, volumen, frequency [28-31],...). In general, considered functions are nonlinear. The main goal of modification represents analysis of sensitivity of objective function. Sensitivity analysis has been briefly described in the next paragraph [3-5].

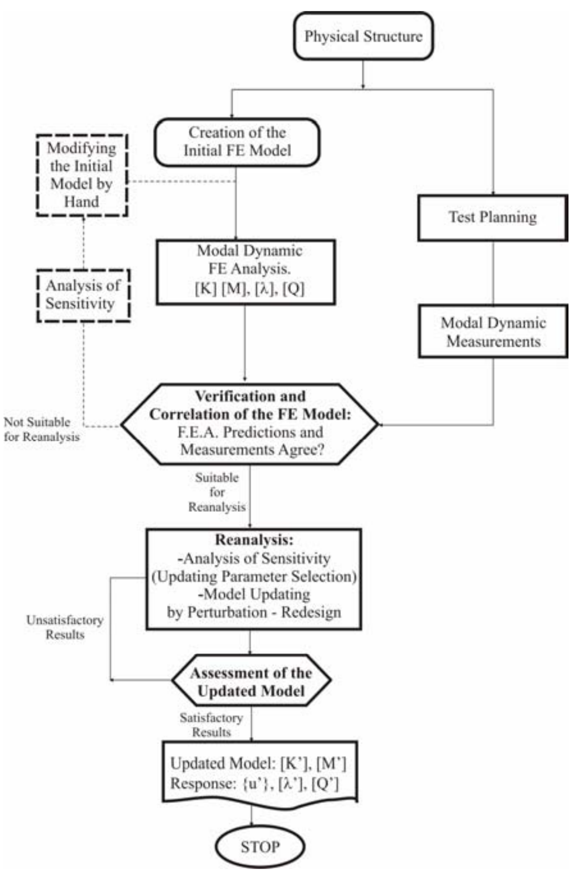

Figure 1. Flowchart of General Perturbation

The matrix form of the equation of undamped motion of an FE model is:

$$\left[M\right]\cdot\left\{\ddot{x}(t)\right\}+\left[K\right]\cdot\left\{x(t)\right\}=\left\{0\right\}. \quad (1)$$

The free-vibration natural frequencies and mode shapes of a linear structural system can be computed by solving the above eigenvalue problem

$$[K]\{Q_i\}=\lambda_i[M]\{Q_i\}, \quad (2)$$

where $[K]$, $[M]$ are the structural stiffness and mass matrix, respectively. The system matrices are considered to be a general function of the design variables denoted by $\{V\}=\{v_1,v_2,...,v_j,...,v_p\}$, and λ_i and $\{Q_i\}$ are the eigenvalue and the eigenvector of mode *i*, respectively.

The eigenvalue and eigenvector derivatives can be calculated by performing partial differentiation of the equation (2) to an updating structural parameter v_j:

$$\frac{\partial\left[K\right]}{\partial p_j}\cdot\left\{\underline{Q}_i\right\}+\left[K\right]\cdot\frac{\partial\left\{\underline{Q}_i\right\}}{\partial p_j}-\frac{\partial\lambda_i}{\partial p_j}\cdot\left[M\right]\cdot\left\{\underline{Q}_i\right\}-$$
$$-\lambda_i\cdot\frac{\partial\left[M\right]}{\partial p_j}\cdot\left\{\underline{Q}_i\right\}-\lambda_i\cdot\left[M\right]\cdot\frac{\partial\left\{\underline{Q}_i\right\}}{\partial p_j}=\left\{0\right\}. \quad (3)$$

Left-multiplying with the transpose of the eigenvector gives

$$-\frac{\partial\lambda_i}{\partial v_j}\cdot\left\{Q_i\right\}^T\cdot\left[M\right]\cdot\left\{Q_i\right\}+\left\{Q_i\right\}^T\cdot\left(\frac{\partial\left[K\right]}{\partial v_j}-\lambda_i\cdot\frac{\partial\left[M\right]}{\partial v_j}\right)\cdot\left\{Q_i\right\}+$$
$$+\left(\left\{Q_i\right\}^T\cdot\left[K\right]-\lambda_i\cdot\left\{Q_i\right\}^T\cdot\left[M\right]\right)\cdot\frac{\partial\left\{Q_i\right\}}{\partial v_j}=\left\{0\right\}. \quad (4)$$

Because $\left\{Q_i\right\}^T\cdot\left[K\right]-\lambda_i\cdot\left\{Q_i\right\}^T\cdot\left[M\right]=0$ and $\left\{Q_i\right\}^T\cdot\left[M\right]\cdot\left\{Q_i\right\}=1$,

$$\frac{\partial\lambda_i}{\partial v_i}=\left\{Q_i\right\}^T\left(\frac{\partial[K]}{\partial v_j}-\lambda_i\frac{\partial[M]}{\partial v_j}\right)\left\{Q_i\right\}. \quad (5)$$

This is the formula for the eigenvalue sensitivity of the i^{th} mode to the j^{th} design parameter. From this formula, it can be seen that the sensitivity of an eigenvalue to a design parameter can be calculated from the eigenvalue, the corresponding eigenvector, and the sensitivities of the stiffness and mass matrices to the design parameter (variable). Rearranging equation (3) gives:

$$\left([K]-\lambda_i[M]\right)\frac{\partial\left\{Q_i\right\}}{\partial v_j}=$$
$$=\left(\lambda_i\frac{\partial[M]}{\partial v_j}+\frac{\partial\lambda_i}{\partial v_j}[M]-\frac{\partial[K]}{\partial v_j}\right)\left\{Q_i\right\}. \quad (6)$$

This is an equation for the eigenvector sensitivity. It can be seen from Eq. (5) that the computation of the eigenvalue sensitivities involves a simple and straightforward calculation. Equations (2-6) have been derived under the assumption that the baseline eigenvectors have been mass normalized.

Sensitivity analysis of real structures can be complicated task [11-23], and instead of it, analysis of distribution of governing quantities is performed. Distribution of elements of modification represents reanalysis, which can be represented in percentage of quantities within the governing group of elements.

2.1 Distribution potential and kinetic energy within the mode shapes

Left-multiplying of equation (2) with the transpose of the eigenvector gives equation of the balance of potential and kinetic energy of structure.

$$\frac{1}{2}\{Q_i\}^T[K]\{Q_i\} = \frac{1}{2}\lambda_i\{Q_i\}^T[M]\{Q_i\}. \qquad (7)$$

Let $[\Delta K]$ and $[\Delta M]$ be the corresponding perturbation in the stiffness and mass matrices. The perturbed eigenvalue problem (from eq. 2) can be written as

$$\begin{aligned}&([K]+[\Delta K])(\{Q_i\}+\{\Delta Q_i\}) = \\ &= (\lambda_i + \Delta\lambda_i)([M]+[\Delta M])(\{Q_i\}+\{\Delta Q_i\}),\end{aligned} \qquad (8)$$

where $\Delta\lambda_i$ and $\{\Delta Q\}_i$ are the eigenvalue and eigenvector perturbations, respectively. If one assumes that the structural changes are small, changes in frequencies and mode shapes can be also expected to be small. Hence, the second and higher order terms could at first thought be neglected [31]. The first order equation of the perturbed system is:

$$[\Delta K]\{Q_i\} = \lambda_i[\Delta M][M] + \Delta\lambda_i[M]\{Q_i\}. \qquad (9)$$

Left-multiplying with the transpose of the eigenvector equation (9) can be rewritten:

$$\Delta\lambda_i = \frac{\{Q_i\}^T[\Delta K]\{Q_i\} - \lambda_i\{Q_i\}^T[\Delta M]\{Q_i\}}{\{Q_i\}^T[M]\{Q_i\}}, \qquad (10)$$

If modification is performed on e-th finite element, mass and stiffness matrix of this element become:

$$\begin{aligned}&[k]_e^{'} = [k]_e + [\Delta k]_e = [k]_e + \alpha_e[k]_e, \\ &[m]_e^{'} = [m]_e + [\Delta m]_e = [m]_e + \beta_e[m]_e,\end{aligned} \qquad (11)$$

where α_e and β_e are parameters which define modification of *e*-th finite element. In this case, in the perturbation in the stiffness and mass matrices (the matrices of increments the stiffness and mass matrices) all terms are equal to zero, except those which corresponded with e-th finite element, such that numerator of equation (10) for *r*-th mode shape becomes

$$\begin{aligned}&\{Q_r\}^T[\Delta K]\{Q_r\} - \omega_r^2\{Q_r\}^T[\Delta M]\{Q_r\} = \\ &= \alpha_e\{q_r^s\}_e^T[k]_e\{q_r^s\}_e - \beta_e\omega_r^2\{q_r^s\}_e^T[m]_e\{q_r^s\}_e,\end{aligned} \qquad (12)$$

Where are:

ω_r^2 - *r*-th eigenvalue,

$\{Q_r\}$ - *r*-th eigenvector of structure,

$\{q_r^s\}_e$ - governing *r*-th eigenvector, *e*-th finite element with *s* degrees of freedom,

$e_{p,r} = \frac{1}{2}\{q_r^s\}_e^T[k]_e\{q_r^s\}_e$ - potential energy of *e*-th FE for *r*-th main mode shape without structural modification,

$e_{k,r} = \frac{1}{2}\omega_r^2\{q_r^s\}_e^T[m]_e\{q_r^s\}_e$ - kinetic energy of *e*-th FE for *r*-th main mode shape without structural modification.

Potential and kinetic energy of the structure for *r*-th main mode shape, according to eq. (7), can be written in the next form:

$$\begin{aligned}E_{p,r} &= \frac{1}{2}\{Q_r\}^T[K]\{Q_r\}, \\ E_{k,r} &= \frac{1}{2}\lambda_r\{Q_r\}^T[M]\{Q_r\}.\end{aligned} \qquad (13)$$

Now, eq. (10) can be given by expression[1]

$$\begin{aligned}\frac{\Delta\omega_r^2}{\omega_r^2} &= \frac{\{Q_r\}^T[\Delta K]\{Q_r\} - \omega_r^2\{Q_r\}^T[\Delta M]\{Q_r\}}{\omega_r^2\{Q_r\}^T[M]\{Q_r\}} = \\ &= \frac{\alpha_e e_{p,r} - \beta_e e_{k,r}}{E_{k,r}},\end{aligned} \qquad (14)$$

The expression (14) is basic equation for reanalysis of structure, because it shows influence of specific finite elements to the eigenvalue. The distribution of energies within of FE expressed percentage for every main mode shape provides necessary information for modification. In other words, for every FE where the difference between potential and kinetic energy is the largest, the structural modification should be performed for the best influnce to change governing eigenvalue. The main goal of dynamic modification is to increase eigenvalues and to increase the difference between them.

3. DEMONSTRATION EXAMPLE

The first example problem is a shaft of electromotor. The initial geometry of the shaft is given in Figure 2. The cross section of shift of electromotor is stepped. Diameter of shift where coupling is installed is $\phi\ 110\ \text{mm}$. All other characteristics, necessary for calculation of shaft's eigenvalues are: $I_z = D^4\pi/64$ - the axial moment of inertia of cross section for *z* axis, $E = 210\cdot10^9\ \text{N/m}^2$ - Young's moduo of the shaft's material (steel), $M = 1000\ \text{kg}$ – weight of windings of rotor, $\rho = 7800\ \text{kg/m}^3$ - mass density. The area of cross-section of shift is calculated by $A = D^4\pi/4$. This relatively simple model is used to verify the implementation of described method using MatLab 7.

[1] It should be noted that "order" as used above refers to perturbed quantities and does not represent the order in terms of design variables. For example, $[\Delta K]$ may be of up to the third order in the plate thickness, while $[\Delta M]$ is of first order. Hence, it is not clear that higher order terms are always negligible compared with the first order terms.

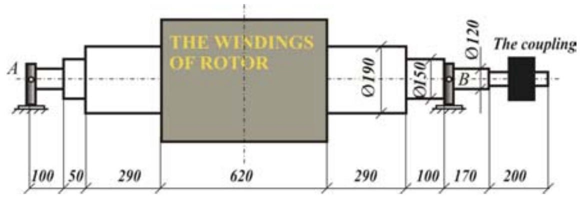

Figure 2. The shaft is supported by a journal bearing at B and a thrust bearing at A.

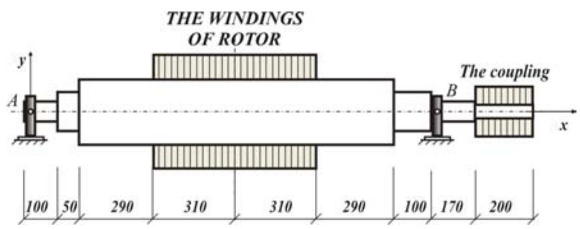

Figure 3. The shaft is modeled using 9 beam elements. The windings of rotor and coupling are presented as the distributed uniform loading

The shaft is modeled using 9 beam elements, see Fig. 2 and 3. There are two degrees of freedom (DOF) at each node corresponding to translation in the *y*-direction and rotation about an axis normal to the *x-y* plane. The shaft is supported by a journal bearing at B and a thrust bearing at A. In that case transversal degrees of freedom at the first and 8-th nodes are constrained to zero, yielding a total of 18 DOF for the model. The influence of the weight of coupling on the free end of shaft and increasing of stiffness to the eigenvalues will be considered.

3.1 The First Case

The most important thing, dealing with dynamical improvment of structure, is increasing of the lowest frequencies and increase of intervals between them. Because of that, it is important to examine infuences, such as changes of geometrical characteristics of the shaft or external loads as well (where it is technicaly possible to make a change), to change the values of frequencies.

The first consideration will be taken when coupling as external load doesn't exist on the free end of shafts (see Fig. 4). The natural frequencies of the shaft for this case are given in the first row in Table 1. In Fig. 5 the diagram of distribution of potential and kinetic energy for this case is given. From the diagram it can be concluded that the differences of *Ep* and *Ek* only along the members 3 and 6 are significant, but the dynamic behaviour of the shaft is satisfied.

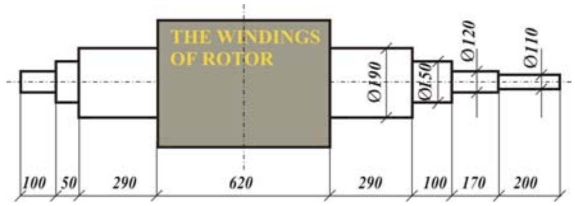

Figure 4. In the first case coupling doesn't exist on the free end of shaft

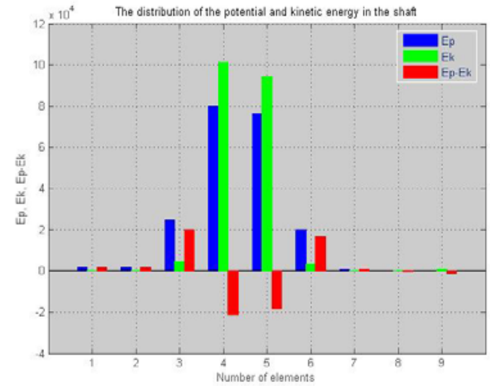

Figure 5. The distribution of the potential and kinetic energy in the case when coupling doesn't exist on the free end of shaft

3.2 The Second Case

The second consideration will be taken when coupling as external load exists on the free end of shafts (see Fig. 6). The weight of the coupling is 600 kg. The natural frequencies of the shaft for this case are given in the second row in Table 1. The distribution of the potential and kinetic energy is shown in Fig. 7. It can be noticed that dynamic behaviour of structure isn't improved. It is expected because of existing external loading (coupling) on the free end of the shaft.

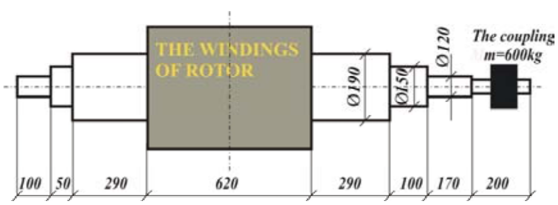

Figure 6. In the second case there is coupling, whose mass is 600 kg, on the free end of shaft

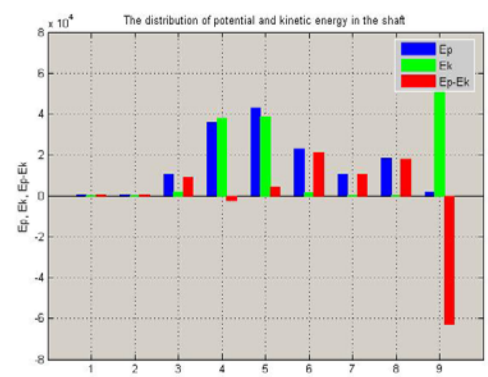

Figure 7. The distribution of the potential and kinetic energy in the case when coupling, whose mass is 600 kg, exists on the free end of shaft

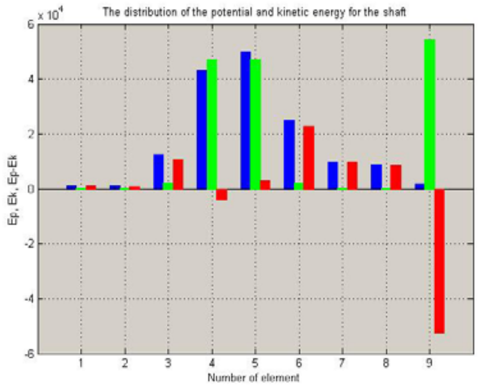

Figure 8. The distribution of the potential and kinetic energy in the case when modification at section 8 is performed. Diameter of the shaft at this section is increased from 0.12 m to 0.14 m.

Nevertheless, the first natural frequency in the second case is decreased about 9% if compared with the case without coupling. For the sake of increasing values of all frequencies, especially of the lowest, the distribution of the Ep and Ek from diagram 6 is analysed. It can be clearly seen that the largest positive value of the difference between Ep and Ek takes place in the elements 6, 7 and 8 for first mode shape. It means that the change of geometry should take place at this possitions in order to have higher values of natural frequencies. Only modification of element 8 (increasing of diameter of cross section) results of increase of first frequency and decrease of the difference between Ep and Ek in this section, Fig. 8. The natural frequencies of the shaft for this case are given in the third row in Table 1. Because of that it can be concluded that modification of stiffness and mass of structure can't be performed arbitrarily. It depends from distribution of the potential and kinetic energy.

Table 1. Natural frequencies of the shaft for three considered cases for 3 mode shapes

M_s=0. D_8=0.12 m	581.38	363.88	72.10
M_s =600 kg, D_8=0.12 m	387.99	104.19	60.60
M_s =600 kg, D_8=0.14 m	394.57	118.51	62.23

The second example problem is 12-node cantilever beam, see figure 9, that is modeled using 11 rectangular cross-section beam elements. The initial geometry of cantilever beam is defined by: $b \times h = 0.01 \times 0.01\ \text{m}^2$, $L = 1\ \text{m}$. All other characteristics, necessary for calculation of cantilever's eigenvalues are: $E = 2.1 \cdot 10^{11}\ \text{N/m}^2$ - Young's moduo, $\rho = 7800\ \text{kg/m}^3$ - mass density.

The area of cross section of beam is $A = b \times h = 0.01^2\ \text{m}^2 = 10^{-4}\ \text{m}^2$, moment of inertia for z axis is $I_z = bh^3/12 = 0.01^4/12\ \text{m}^4 = 8.333 \cdot 10^{-10}\ \text{m}^4$. The mass of cantilever beam is $m = \rho \cdot A \cdot L = 7800 \cdot 10^{-4} \cdot 1\ \text{kg} = 0.780\ \text{kg}$. There are two degrees of freedom (DOF) at each node corresponding to translation in the *y*-direction and rotation about an axis normal to the *x-y* plane. Both degrees of freedom at the first node are constrained to zero, yielding a total of 22 DOF for the model.

Potential and kinetic energy distribution in every FE of cantilever beam is shovn in Fig. 9a. Based on difference of potential and kinetic energy for every FE it can be concluded that cantilever should be strenghtened from free end to the fixed end.

First approximate modified shape of cantilever is shown in Fig. 10. The profile of cantilever is a cubic parabola, while the width of cantilever is unchanged from the initial one. From Fig. 9a. it can be seen that the smallest difference between potential and kinetic energy along the elements is minimal at the middle of cantiliver. At that point the cross-section shouldn't be changed.

On the other hand, the difference between potential and kinetic energy along the elements is larger if we go to the fixed end of cantiliver and height of cantiliver *h* should be increased. From the middle towards the free end the height should be decreased. Because of that the cubic parabola is chosen as an approximation for the distribution of height of cantiliver. For initial geometric characteristics it means:

$$|y| = -0.04 \cdot (x - 0.5)^3 + 0.005 .$$

Distribution of potential and kinetic energy for this case is given in Fig. 10a. Second approximation of modified shape of cantiliver is shown in Fig. 11. Cantiliver profile is a linear function, and width of cantiliver is again unchanged. Equation of line is determined similarly as in the first case and it is:

$$|y| = -0.01 \cdot x + 0.01 .$$

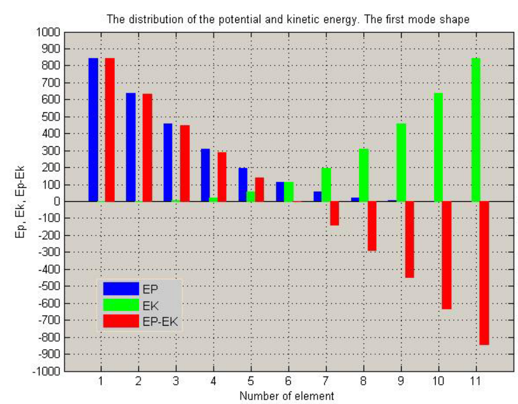

Figure 9a. The distribution of E_p and E_k for initial shape of cantiliver (Fig. 1)

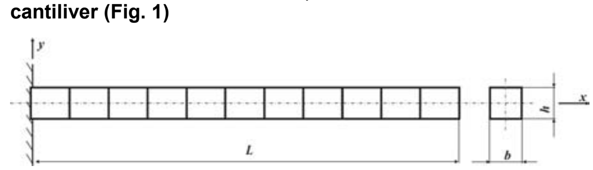

Figure 9. Cantilever beam with height *h* as the design variable. Initial shape

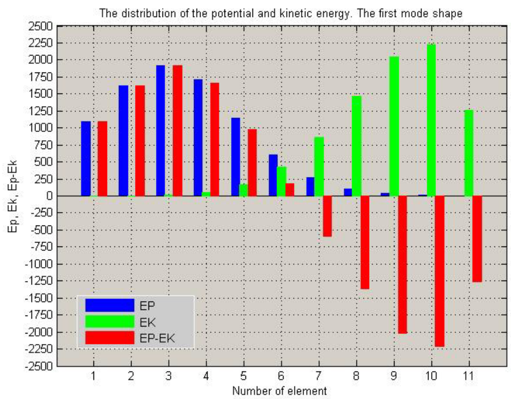

Figure 10a. The distribution of E_p and E_k for modified shape of cantiliver (Fig.10)

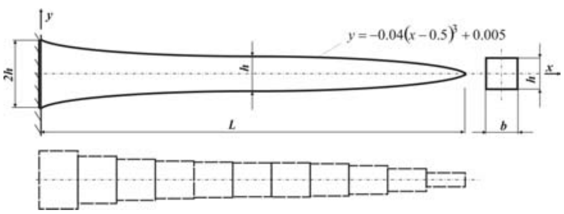

Figure 10. Cantilever beam with cubic parabola profile

Distribution of potential and kinetic energy for this case is given in Fig. 11a. To better understand the dynamic behaviour of cantiliver, the diagram of relative ratio of differencies of potential and kinetic energies is done (Fig. 13), for each of 11 elements and for the first three mode shapes. According to Fig. 13, it can be concluded, that only the changes within the elements where differences of potential and kinetic energies are of the same sign for all three mode shapes should be performed. On the other hand, every increase of first natural frequency will cause decrease of others i.e. it will make a solution worst and vice versa.

According to this analysis, it can be concluded that the cantiliver shape shown in Fig. 10 is approximately the best one for this example for the considered first three modes shape.

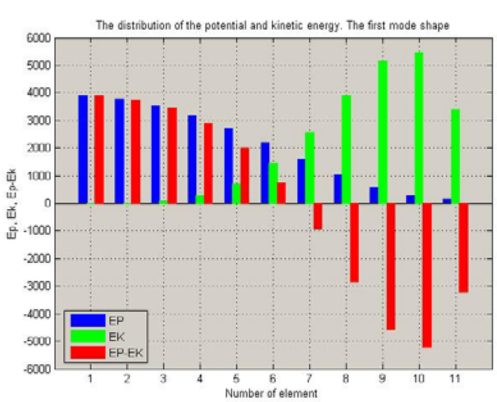

Figure 11a. The distribution of Ep and Ek for modified shape of cantiliver (Fig. 11)

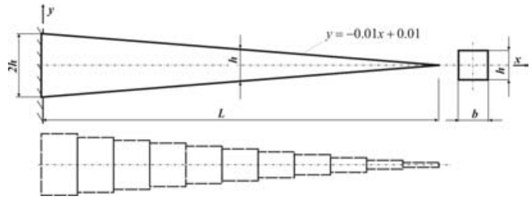

Figure 11. Cantilever beam with the triangular profile

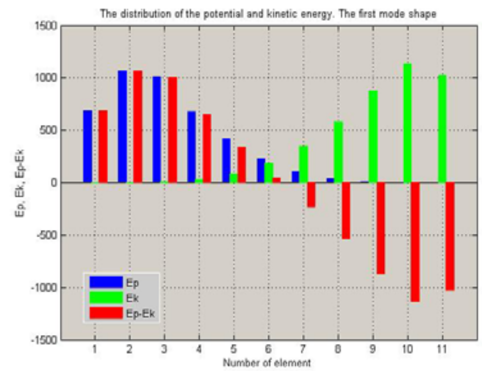

Figure 12a. The distribution of Ep and Ek for modified shape of cantiliver (Fig. 12)

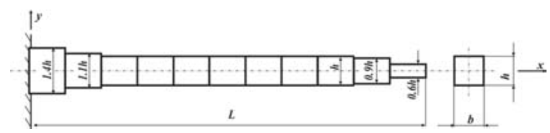

Figure 12. Cantilever beam with stepped cross section

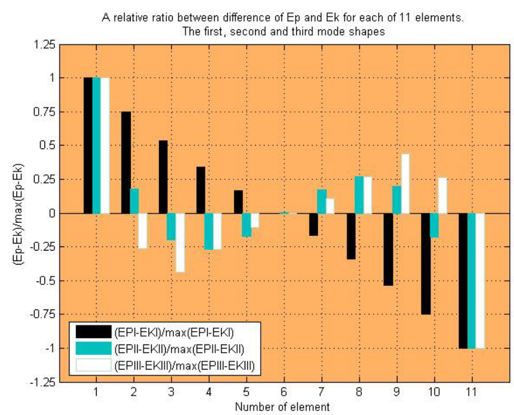

Fig. 13. A relative ratio between difference of potential and kinetic energy for each of 11 FE of cantilever for first three modes shape

Table 2. Natural frequencies [Hz] of cantilever for all considered cases for five mode shapes

	Cantilever beam (fig.9)	Cantilever beam (fig.10)	Cantilever beam (fig.11)	Cantilever beam (fig.12)
1	51.40	92.29	151.49	65.26
2	322.10	453.42	380.44	382.83
3	902.03	1016.79	633.68	1015.25
4	1768.46	1638.42	1114.19	1891.75
5	2926.59	2515.30	1861.50	3002.70

4. CONCLUSION

Two example problems are done in this paper. First consideration is performed for dynamic behavior of shaft of electromotor. The obtained results for natural frequencies for three cases show that dynamic characteristics of considered shaft are satisfied. In the real conditions the mass of coupling doesn't overstep 600 kg. The relative ratio between first frequencies for the first and second cases shows decreasing first natural frequency of 16%. But for the sake of improving dynamic charasteristics of structure, design variables can't be changed arbitrarily. It depends on distribution of the potential and kinetic energy

Second consideration is performed for dynamic behavior of cantilever beam. From this study, and the results shown in Table 2, all modified cantilever shapes are of better dynamic characteristics than the initial one given in the first column of Table 2, or shown in Fig. 9 and Fig. 9a. For the cases shown in column 3 (fig. 11) the increase of the first frequency is the largest one, but the others are decreased, and the difference between two neighboring frequencies is satisfied. From all above considerations, it can be concluded that cantilever of the shape given in Fig. 10 is the best modified one because the first natural frequency is increased enough, but all other natural frequencies preserved almost the same values. In the above explained modification only one constraint is used, with respect to first natural frequency. However, the structure can be modified with the multiple constraints of natural frequencies, which will be done in future investigations.

This paper opens up new possibilities for application of this approach to improve dynamic characteristics of structures, providing increasing of all considered natural frequencies and increasing the difference between them.

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АНАЛИЗА СЕНЗИТИВНОСТИ СОПСТВЕНИХ ВРЕДНОСТИ У ДИНАМИЦИ КОНСТРУКЦИЈА

Наташа Тришовић

Модификација динамичких карактеристика конструкција се дефинише као скуп метода којима се може побољшати динамичко понашање конструкције у експлоатацији. Модификација динамичких карактеристика или реанализа се посебно односи на скуп метода и техника које своје корене и основе имају у примени анализе сензитивности и методе коначних елемената. Анализа сензитивности се заснива на селекцији конструкционих параметара у почетном коначноелементном моделу чијом модификацијом би дошло до поправљања динамичког понашања посматране конструкције. Овај рад се бави анализом динамичког понашања вратила електромотора, као и анализом утицаја облика конзолног носача на вредности основних фреквенција. У основи ове анализе је дистрибуција потенцијалне и кинетичке енергије на главним облицима осциловања у свим елементима посматране конструкције. На основу сензитивности појединих елемената бирају се сегменти за модификацију. На основу овог истраживања показује се важност реанализе у динамици конструкција.