Vertical interaction of grouped 3x3 piles

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ABSTRACT

This paper presents the calculation method of the vertical interaction of grouped piles for very a rigid cap and a very flexible cap (raft/beam). The expressions for the calculation of axial stiffness of piles along depth for 4 soil models are listed: constant, linear, parabolic variation of soil modulus for the floating pile and two-layered model for the end bearing pile. A program for the analysis of the interaction in the form of a flowchart has been made and presented. Results of numerical analyses of redistribution between the piles in the group, of 3x3, due to the vertical interaction for the soil of constant stiffness by depth with two modules of elasticity of soil of 25 MPa and 50 MPa have been presented. For linear and parabolic distribution soil modulus by depth, the impact of varying the mutual distance of the piles 3*D*, 3.5*D*, 4*D*, 5*D* and 10*D*, for a RC pile length (7 to 12m) and having diameter of 60 cm has been studied. Forces and settlements for the zero and final iteration are presented in tables, as well as the bending moment for the zero iteration, determined using a difference method. Some possible development directions of such models have been indicated.

KEYWORDS

Vertical interaction factor; Piles group; Soil models; Axial stiffness.

1. INTRODUCTION

In the past decades extensive research work has been carried out and considerable effort has been devoted to the procedures end methods for the evaluation of the settlement of piled foundations, involving very complex models of soil and effects of interaction soil - foundation - structure (Milović and Đogo, 2009a).

The study of vertical interaction of grouped piles is necessary for determining the distribution of axial forces for each individual pile, and it is determined depending on the type soil, which is expressed: via the type (method) of varying of the soil elasticity modulus by depth, stiffness of the cap beam, mutual distance, their length and diameter etc. A large number of researchers dealt with these topics, and only some of them are listed here: (Scot, 1981), (Fleming et al., 1998), (Poulos and Davis, 1980), (Poulos, 1989; 2001; 2011), (Pender, 1983), (Reese and Van Impe, 2001), (Milović and Đogo, 2009b), (Mosher and Dawkins, 2000), (Russo, 1998), (Sander and Baleshwar, 2010), (Azizkandi and Fakher, 2014) and recently Deb and Pal, 2019), (Celik, 2019), etc.

This paper presents the program for the analysis of vertical interaction in the form of a flowchart. Results of numerical analyses of redistribution between the piles in the group due to the vertical interaction for the soil of constant stiffness by depth with two modules of elasticity of soil of 25 MPa and 50 MPa have been presented. The impact of varying the mutual distance of the piles 3*D*, 3.5*D*, 4*D*, 5*D* and 10*D*, for a RC pile 12m long and having diameter of 60 cm has been studied.

2. SOIL MODELS FOR VERTICAL INTERACTION OF PILES

In the case of vertical interaction, and of horizontal, too, usually 3 models of soil for floating piles and the model of two-layer soil for end bearing piles are used. In accordance with these functions of varying the soil modulus by depths, the axial stiffness of piles is determined. Axial stiffness of piles for constant, linear and parabolic soil are given according to Gazetas, and for the end bearing pile in a manner set by Randolf and Wroth, adapted by Kulhawy and Carter, cited after (Pender, 1983), (Folić B. and Folić R., 2018) and (Folić, 2017).

The soil presented as having the constant stiffness by depth is adequate for over consolidated clays. The linear change of stiffness is characteristic for soft clays, normal consolidated clays and sand for higher strain levels. Parabolic change of modulus by depth is characteristic for sand with low strain levels (Nguyan et al., 2013).

2.1. Constant modulus of soil

For a constant modulus of soil, the equations for this soil model profile are given in the form:

$$K = \frac{E_P}{E_S} \tag{1}$$

Where: E_p – is Young's modulus of pile, and E_s – the soil modulus, usually given at the corresponding design depth of the soil model.

The axial stiffness of a pile is:

$$\mathrm{K_v} = 1.9 \cdot \mathrm{E_S D} \cdot \mathcal{L}^{0.67} \cdot \mathrm{k}^{-b} \tag{2}$$

$$\mathrm{b} = \mathcal{L} / \mathrm{k}\,,\ \mathcal{L} = \Im = \mathrm{L/D} \tag{3}$$

2.2. Linear variation of soil modulus by depth

For the linear variation of soil modulus by depth the equations for the linear variation of soil modulus profile are given by Budhu and Davies, cited after (Scot, 1981). For this case the Young's modulus of soil and stiffness is:

$$E_S = mD\ ;\ k = \frac{E_P}{mD} \tag{4}$$

The axial stiffness of the pile for the linear variation of the soil modulus by depth is:

$$K_v = 1.8 \cdot E_{SL} D \cdot \mathcal{L}^{0.55} \cdot \Re^{-b} \tag{5}$$

$$\Re = k = E_P / E_{SL} \tag{6}$$

E_{SL} - is the soil modulus at the pile tip (base) $E_{SL} = E_S(z{=}mL)$

Where m is rang increase of Young's modulus with depth. Budhu and Davies give values m for different densities of sand. This is appropriate for the static pile load, but not for the dynamics excitation of piles bounded in loose saturated sands. Other coefficients, equations and theory can be seen in (Pender, 1983), (Fleming et al. 1998), or (Folić, 2017).

2.3. Parabolic variation of soil modulus by depth

For parabolic variation of soil modulus by depth, the axial stiffness of the pile:

$$K_v = 1.9 \cdot E_{SL} D \cdot \mathcal{L}^{0.60} \cdot \mathfrak{R}^{-b} \tag{7}$$

$$E_{SL} = E_{SD}\sqrt{\mathcal{L}} \tag{8}$$

The end bearing pile is considered as a two-layer soil, where E_B is the soil modulus at the pile base, and E_{SD} at the depth of one diameter of the pile. The stiffness of the end bearing pile is determined according to the following formula (Pender, 1983) and (Folić, 2017):

$$K_V \; end\ bearing = \left(\frac{E_S D}{1+\nu_S}\right) \cdot \frac{\Omega + \dfrac{\mathcal{L}\Xi}{\zeta}}{1 + \dfrac{4\Omega \mathcal{L}\Xi}{\pi K(1+\nu_S)}} \tag{9}$$

Where:

$$\Omega = \xi(1+\nu_S)/(1-\nu_B^{\,2}) \tag{9a}$$

$$\xi = E_B / E_{SD} \tag{9b}$$

$$\Xi = \mathrm{tgh}(T)/T \tag{9c}$$

$$T = 2\mathcal{L} \cdot [\zeta(1+\nu_S)K]^{-0.5} \tag{9d}$$

$$\zeta = \ln[5(1-\nu_S)\mathcal{L}] \tag{9e}$$

3. PROGRAM FOR THE CALCULATION OF VERTICAL INTERACTION OF GROUPED PILES

Research of the vertical interaction of piles is a continuation of those already published in (Folić, 2017) and (Folić, et al., 2016). A program (in Basic) was written to examine the vertical interaction of grouped piles. The program also includes a procedure for calculating the equivalent modulus of a hollow pile, because steel piles are often used, while prestressed concrete piles can also be of tubular cross-section. It is assumed that all piles have the same properties (material, length and diameter). The zero iteration also assumes an even distribution of forces, but the user has the ability to modify it by specifying three forces: in the inner (centre) pile N_5, the pile on the edge of the mid-side N_2, and the force in the corner pile N_1 (for the 3x3 arrangement).

The arrangement of piles in a rectangular shape is designed, but special cases such as the arrangement in one row or square shape can also be specified.

Through iterations, the program calculates the redistribution of forces on the cap beam or raft, observing the influence of the mutual interaction of all piles in the group. The assumption is that both the slab and the beam are very rigid.

The results of the iteration are written in a special file, and 4 types of soil are planned for (Folić, 2017):

1. Constant Young's modulus.
2. Linear variation of modulus by depth.
3. Parabolic variation of modulus by depth.
4. End bearing pile in two layer soil.

The end bearing pile requires caution, when the soil at the base has considerable stiffness; there is no point in calculating redistribution, because the axial stiffness of the piles becomes dominant.

Types of soil are introduced through the modulus of elasticity of soil, at the half of pile depth and at the base ($\rho = E_S\ (L/2)\ /\ E_S(L)$). Calculating coefficients of mutual action (interaction) of grouped piles (impact coefficients α_{ij}). The function of the variation of modulus by depth was more extensively treated in (Folić, 2017).

3.1. Flowchart for the program of analysis of interaction pile-soil-pile

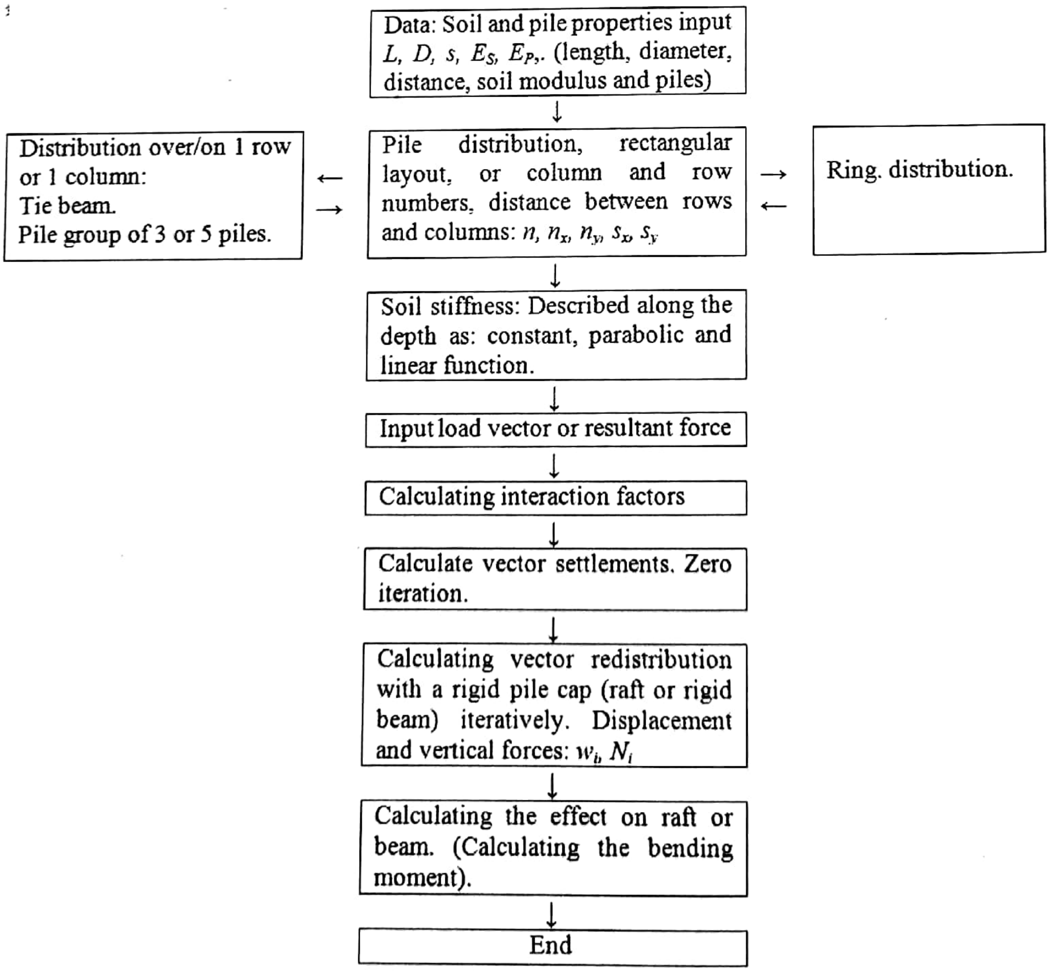

Figure 1. Flowchart

The input parameters of the model are the pile D diameter, and pile length L, Young's modulus of soil at the depth of the D pile diameter is E_{SD}, and the modulus of soil at the depth equal to the length of the pile L is E_{SL}. The type of the soil is determined by the function of the variation of the modulus of soil by depth.

Zero iteration is the initial status of settlement of piles. It is the settlement vector which is obtained when every pile, as free, without a cap) is loaded by the identical force (it is here 100kN, and the total force is 900 kN). This is a settlement of a uniformly loaded group of piles, connected with the totally flexible raft, i.e. $w_i(I_r \rightarrow 0)$; $w_i(I_{tb} \rightarrow 0)$.

The final state is that obtained after a number of iterations, when the deflections of all the piles in the group are equalized, taking into consideration of the piles through the soil. This can be understood as a deflection of the group of piles, evenly loaded, connected with completely rigid cap raft – beam. $w_i(I_r \rightarrow \infty)$; $w_i(I_{tb} \rightarrow \infty)$.

The iterations are obtained by the variation of deflection until the relative difference of averaged deflection and each individual deflection of the pile is lower than 0.0001 ($\Delta w<10^{-4}$). For each iteration in a special file the ordinal number of iteration, force vector and accompanying deflections vector are printed.

4. NUMERICAL ANALYSIS AND DISCUSION OF RESULTS

For analysis we consider group of piles 3x3 linked by a cap with a fixed head (but not restraining rotation). Pile arrangement is shown on figure 2. Pile axis distance is $5D$, in both axes. Soil is the constant modulus E_S=25 MPa and 50 MPa (over consolidated clay). Pile is long (flexible), diameter is D=0.60 m. Pile modulus is E_P=25 GPa and Poisson's ratio v = 0.50.

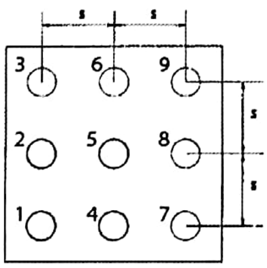

Figure 2. Pile group arrangement is 3x3

Due to the multi-axial symmetry of the pile arrangement, the total number of the unknown deflections is three. The system of equations of deflection is related to the system of impact coefficients α_{ij}. Due to the symmetry, the following relations of equality of forces and deflections are valid:

- Corner piles: $w_1= w_3= w_7= w_9$. (11a)
- Mid-side piles: $w_2= w_4= w_6= w_8$. (11b)
- Corner piles: $N_1= N_3= N_7= N_9$. (12a)
- Mid-side piles: $N_2= N_4= N_6= N_8$. (12b)

Equation of vertical force equilibrium, (due to the symmetry) is in this case:

$$N = 4N_1 + 4N_2 + N_5 \tag{13}$$

Where:

α_{ij} - is the interaction factor between piles i and j, after Randolf and Wroth (1979), (Pender, 1983).
$\alpha_{ij} = \alpha_V$ = is the quotient of displacement caused by unitary vertical action on the adjacent pile and pile displacement due to the unitary action on the pile head.

Broader interpretations of the coefficient, via the relation of settlement of a group of piles are provided in (Folić, 2017), (Scot, 1981), (Pender, 1983), (Milović and Đogo, 2009).

4.1. Analysis for the constant modulus of soil by depth, interaction of 9 piles in the group

Constant modulus of soil by depth of the pile is typical for over-consolidated clays. A pile having diameter of 60 cm, 12 m long in the soil of constant modulus by depth is analyzed for two values: 25 MPa, and 50 MPa. Two border cases of interaction of piles in a group are analyzed. First, consider group without the cap, where each pile is loaded by the equal force of 100 kN. Second case is where the deflections of the piles are equalized due to the interaction with the soil and the connected rigid cap. In the zero iteration, it is assumed that the piles are loaded by the identical forces, and due to the interaction, their settlements are different, and in the final iteration when the settlements are equalized but the forces on the piles are different, due to the interaction with the soil.

In the tables are provided settlements for the zero iteration, where the mean deflection is $w_{mz}=(4w_1+4w_2+w_5)/9$, and for the final iteration is $w_{mf} \approx w_5$, because then all the settlements are almost equal, or they can possibly differ for the preset value (lower than relative difference 10^{-6}), when further iterations and execution of the program is stopped. Tables are usually organized in the groups of three, per example. The first table shows forces in the piles after equalizing the deflection in the final iteration. The second table provides settlements in the zero iteration, and the third compares the settlements for the zero and final iteration in respect to the mean value.

Table 1. Vertical force on pile head N_i [kN] Final iteration. L=12m. Soil modulus E_s=25 [MPa]. ν=0.5

s/D	N_1 [kN]	N_2 [kN]	N_5 [kN]	(N_1-N_5) [kN]	$w_5=w_m$ [mm]
3	116.93	91.96	64.45	52.48	1.714
5	117.34	91.76	63.61	53.73	1.632
4	117.74	91.56	62.78	54.96	1.558
5	118.52	91.18	61.20	57.32	1.425
10	121.84	89.54	54.45	67.40	0.933

$w_m=w_5$; N_1 – (Force on) corner piles; N_2 - mid-side piles;
N_5 - centre pile; s - pile spacing

Table 2. Settlement of pile head [mm]. Zero iteration. L=12m. D=60cm. Soil modulus. E_S=25 [MPa]. ν=0.5

s/D	w_1 [mm]	w_2 [mm]	w_5 [mm]	w_m [mm]	$(w_1-w_5)/w_m$ (%) [%]	$M_x=M_y$ [kNm]	$h_{usvojeno}$ [m]	$s_{min} \approx h$ [m]
3	1.663	1.754	1.859	1.725	-11.332	-274.49	1.8	1.2
3.5	1.579	1.674	1.784	1.644	-12.470	-412.82	2.1	1.5
4	1.502	1.602	1.716	1.570	-13.637	-570.26	2.4	1.8
5	1.366	1.474	1.597	1.440	-16.081	-540.82	3	2
10	0.857	1.001	1.164	0.955	-32.106	-603.28	6	3

$w_m=(4w_1+4w_2+w_5)/9$; w_1- Displacement on corner piles; w_2 - mid-side piles; w_5- centre pile

Table 3. Relative settlement of pile head w_i/w_m [%]. Zero and final iteration. L=12m, D=60cm. Soil modulus const. E_S=25[MPa]. ν=0.5

ν=0.5	Zero iteration				Final. it.	zero-final iteration
s/D	$(w_1-w_m)/w_m$ [%]	$(w_2-w_m)/w_m$ [%]	$(w_5-w_m)/w_m$ [%]	w_{mz} [mm]	w_{mf} [mm]	$(w_{mz}-w_{mf})/w_{mz}$ [%]
3	-3.61	1.68	7.72	1.725	1.714	0.636
3.5	-3.97	1.85	8.50	1.644	1.632	0.717
4	-4.35	2.02	9.29	1.570	1.558	0.803
5	-5.13	2.39	10.95	1.440	1.425	0.988
10	-10.26	4.80	21.85	0.955	0.933	2.325

$w_{mz}=(4w_1+4w_2+w_5)/9$ zero iteration; $w_{mf}=w_5$ final iteration

Table 4. Vertical force on pile head N_i [kN] Final iteration. L=12m, D=60cm. Soil Modulus const. E_S=50 [MPa]. ν=0.5

s/D	N_1 [kN]	N_2 [kN]	N_5 [kN]	(N_1-N_5) [kN]	$w_5=w_m$ [mm]
3	116.93	91.96	64.45	52.48	0.957
3.5	117.34	91.76	63.61	53.73	0.911
4	117.74	91.56	62.78	54.96	0.870
5	118.52	91.18	61.20	57.32	0.796
10	121.84	89.54	54.45	67.40	0.521

Table 5. Settlement of pile head. Zero iteration. L=12m. D=60cm. Soil modulus const. E_S=50 [MPa]. ν=0.5.

s/D	w_1 [mm]	w_2 [mm]	w_5 [mm]	w_m [mm]	$(w_1-w_5)/w_m$ [%]	$M_x=M_y$ [kNm]	h $_{usvojeno}$ [m]	$s_{min}\sim h$ [m]
3	0.929	0.980	1.038	0.963	-11.332	-153.27	1.2	1.8
3.5	0.881	0.935	0.996	0.918	-12.470	-230.51	1.5	2.1
4	0.839	0.895	0.958	0.877	-13.637	-318.42	1.8	2.4
5	0.763	0.823	0.892	0.804	-16.081	-301.98	2	3
10	0.479	0.559	0.650	0.534	-32.106	-336.86	3	6

$w_m=(4w_1+4w_2+w_5)/9$; w_1- Settlement on corner piles; w_2 - mid-side piles; w_5- centre pile

Table 6. Relative settlement of pile head w_i/w_m [%]. Zero and final iteration. L=12m, D=60cm. Soil modulus const. E_S=50[MPa]. ν=0.5

ν=0.5	Zero iteration				Final Iter.	zero-final
s/D	$(w_1-w_m)/w_m$ [%]	$(w_2-w_m)/w_m$ [%]	$(w_5-w_m)/w_m$ [%]	w_{mz} [mm]	w_{mf} [mm]	$(w_{mz}-w_{mf})/w_{mz}$ [%]
3	-3.61	1.68	7.72	0.963	0.957	0.637
3.5	-3.97	1.85	8.50	0.918	0.911	0.717
4	-4.35	2.02	9.29	0.877	0.870	0.803
5	-5.13	2.39	10.95	0.804	0.796	0.988
10	-10.26	4.80	21.85	0.534	0.521	2.324

4.2. The calculation of the bending moment of the raft

The calculation of the bending moment of the raft is performed using the difference method, i.e. Finite Difference Method (FDM), (Poulos and Davis, 1980), (Chang and Lien, 2019).

$$M_y=-\frac{Eh^3}{12(1-\nu^2)}\cdot\left(\frac{w_2-2w_5+w_2}{s_y^2}+\nu\frac{w_2-2w_5+w_2}{s_x^2}\right)=-\frac{Eh^3}{12(1-\nu)}\cdot\left(\frac{2w_2-2w_5}{s^2}\right)$$

$$M_y = -4936{,}81 \cdot h^3 \cdot \left(\frac{w_2 - w_5}{s^2} \right) [kNm];\ h\ [\mathrm{m}];\ w\ [\mathrm{mm}];\ s\ [\mathrm{m}] \qquad (14)$$

Where: $D=E \cdot h^3/12(1-v^2)$ - raft stiffness; s – pile distance

For the calculation of the cap beam moments, according to the formula (14) the same modulus of elasticity as for the piles are adopted, and that is 25 GPa.

In the table 2, 5 10, 13, 16 and 19 is presented the calculation of the raft bending moment with the approximately adopted thickness depending on the spacing of piles in the group s_{min} (last two columns of those tables).

This moment is obtained when the flexible raft rested on the piles is loaded by the force of 100 kN on each pile (zero iteration), i.e. when the raft of absolute stiffness, around 0.5 s thick (of the pile spacing), is deformed by the vector of the corresponding type. For instance, it is (according to the table 5) for the spacing of the piles 3D (1.8m) and modulus of the soil E_S=50 MPa, and raft thickness of 1.20m, the following deflection vector: corner piles $w_1= w_3= w_7= w_9=$ 0.929 mm; mid-side piles $w_2= w_4= w_6= w_8 =$ 0.980 mm, and the central inner pile w_5=1.038mm. However, the permissible bearing capacity of the pile Ø600mm is several times higher than 100 kN, and it amounts to no less than 300 kN or more (Šuklje, 1979), so the bending moment which would load the raft in the actual loading conditions several times higher. The precise answer would require knowing the exact design load and implementation of the adequate model of structure-foundation-soil interaction, and foundation is here composed of the raft and piles. The stiffer the raft, give the lower the force below the mid pile. If punching shear would be calculated, below the mid pile it would be 65 to 55% of the initial force, i.e. 65 to 55% of 1/9 of the resultant force.

In tables 1 and 4, for the final iteration, there is no difference in the force intensities, even the number of iterations for the spacing from 3D to 4D is the same, only for 5D and 10D, and the number of iterations at 50 MPa is lower for 1.

There is a difference of settlements in the zero iteration, tables 2 and 5. In this case, the deflection of the floating pile for the soil with the constant modulus of soil of 50 MPa in comparison to the modulus of soil of 25 MPa, when all other parameters of the pile are the same, decreases for 44.2%. The same holds for the cap beam bending moment, calculated according to the Finite Difference Method.

In tables 3 and 6 are provided relative settlements of the individual pile in relation to the mean value of the group settlement, for the zero iteration. The deflection of the piles decreases with the increase of pile spacing. The lowest is for the spacing of 10D; however the thickness of the raft in this case is no less than around 3m, so this arrangement should be avoided.

Vertical interaction of piles in a group (3x3) is analyzed with determination of forces after redistribution with the rigid cap for the soil modulus of 25 MPa and 50 MPa, for different spacing of the piles. The state for the zero iteration has been examined, when the cap raft is considered as completely flexible, and practically not participation in the interaction, and the status of final iteration, when the settlements are equalized, which corresponds to a very rigid raft or beam. In the zero iteration, all the forces in the piles are equal, but settlements are different, and the final state when the settlements of the piles are equal, but due to the redistribution, the forces are different.

In tables 7 and 8 are presented the results of the analysis of the Poisson's coefficient for the pile embedded in the soil with the constant modulus by depth, having diameter 60 cm and length 12 m.

The variations of forces values N are relatively small (between N_{max}-N_{min} is around 8%) so they are not displayed, and the settlement variations can be viewed from Table 7.

Table 7. Variation forces in pile of function Poasson's coefficient N_i [kN]. Final iteration L=12m. Soil modulus const. E_s=50 [MPa].

ν	N_1 [kN]	N_2 [kN]	N_5 [kN]	(N_1-N_5) [kN]	$w_5=w_m$ [mm]
0,00	114,41	93,19	69,61	44,80	1,011
0,30	115,95	92,44	66,46	49,49	0,948
0,50	117,74	91,56	62,78	54,96	0,870

Table 8. Vertical settlement of a particular pile in a group in the function of the Poisson's coefficient w_i [mm] when the spacing between piles is s=4D. Zero iteration. L=12m. Soil modulus const. E_s=50 [MPa].

s/D	w_1	w_2	W_5	w_m	$(w_1-w_5)/w_m$	$M_x=M_y$	h_{usvoj}	s_{min}~h
ν	[mm]	[mm]	[mm]	[mm]	[%]	[kNm]	[m]	[m]
0.00	0.982	1.025	1.074	1.011	-9.037	-243.84	1.8	2.4
0.30	0.915	0.964	1.019	0.948	-10.978	-277.60	1.8	2.4
0.50	0.839	0.895	0.958	0.877	-13.637	-318.42	1.8	2.4

The mean value of the settlement decreases with the increase of the Poisson's number.

In table 7 and 8 the results of the zero iteration indicate, that with the increase of the Poisson's number, and for the same spacing of the piles, all the settlements of individual piles and their mean deflection are reduced. The difference of the maximum and minimum deflection increases with the increase of the Poisson's number.

4.3. Linear variation of the soil modulus by depth

For a pile having length L=12 m, D=0.60 m the value of the modulus of soil was examined at the pile depth E_{SD}=25 MPa, which corresponds to the linear variation of m=25/0.6=41.67 MPa/m. This is considerably stiffer soil than the constant modulus 25 MPa, because the settlement of the observed pile for the same load is 50% lower than of the soil with a constant modulus. In line with this, in tables 9, 10 and 11 are provided impacts for the pile at depth of 12 m, and in tables 12, 13 and 14 it is the same pile but for the total depth-pile length of 7 m. For the soil modulus at the depth of 6 m, which is 250 MPa, it can be considered to be a resting pile.

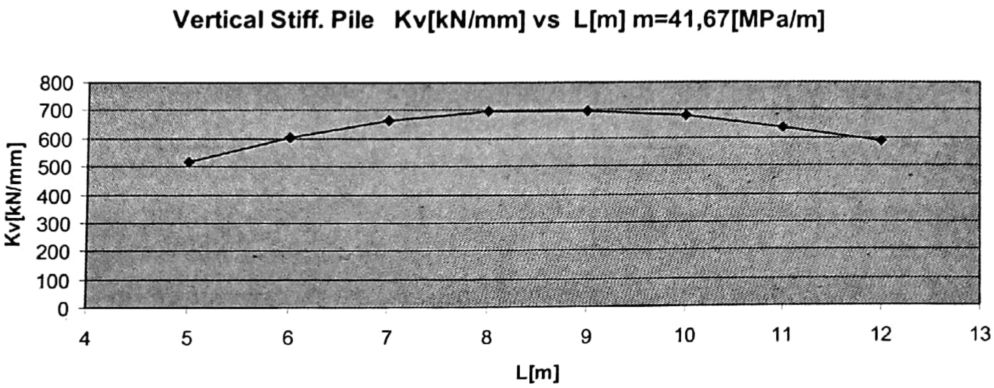

Figure 3. Variable vertical stiffness of pile K_V vs depth piles L, in soil modulus with linear variation stiffness m=41.67 [kN/mm], diameter D=60 cm.

The linear change of soil modulus is characteristic for soft clays, medium compacted and compacted sands. According to (Vesić, 1977), and based on experimental tests, when driving floating piles in medium compacted and loose sands, there are certain "critical depths" after which the bearing capacity of the piles does not increase with lateral (shaft) friction, and it is often irrational to continue driving them beyond that depth.

Based on the dependence shown in Figure 3, the depth of driving by which the maximum stiffness is reached is between 8 and 9 meters. The stiffness of a pile 6 m long is approximately equal to the stiffness at a depth of 12 m, i.e. it is more cost-effective to drive them to the depth of 7 to 9 m. The drop in stiffness can be negatively affected by the friction along the upper part of the pile shaft. This is possible with submerged lightly compacted sands, sensitive clays and muddy-dusty soils.

Table 9. Vertical force on pile head N_i [kN]. Final iteration.
L=12m, D=60cm. Soil modulus linear m_{Es}=41.67 [MPa/m]. $w_m=w_5$

s/D	N_1 [kN]	N_2 [kN]	N_5 [kN]	(N_1-N_5) [kN]	$w_5 = w_m$ [mm]
3	115.51	92.65	67.37	48.13	0.512
3.5	115.77	92.52	66.83	48.95	0.488
4	116.06	92.38	66.24	49.82	0.466
5	116.67	92.08	65.01	51.66	0.427
10	119.54	90.67	59.15	60.40	0.285

Table 10. Displacement of pile head [mm]. Zero iteration. L=12m. Soil modulus linear m_{Es}=41.67 [MPa/m]

s/D	w_1 [mm]	w_2 [mm]	w_5 [mm]	w_m [mm]	$(w_1-w_5)/w_m$ [%]	$M_x=M_y$ [kNm]	$h_{usvojeno}$ [m]	$s_{min}\approx h$ [m]
3	0.497	0.524	0.555	0.515	-11.328	-82.05	1.2	1.8
3.5	0.472	0.500	0.532	0.491	-12.361	-122.34	1.5	2.1
4	0.449	0.479	0.512	0.469	-13.426	-167.89	1.8	2.4
5	0.409	0.441	0.477	0.431	-15.667	-157.78	2	3
10	0.263	0.304	0.351	0.291	-30.241	-172.99	3	6

$w_m=(4w_1+4w_2+w_5)/9$

Table 11. Relative displacement of pile head w_i/w_m [%]. Zero and final iteration. L=12m. D=60cm. Soil modulus linear m_{Es}=41.67 MPa/m, ν=0.5.

m_{Es}=41.67	Zero iteration				Final itera.	Zero-final
s/D	$(w_1-w_m)/w_m$ [%]	$(w_2-w_m)/w_m$ [%]	$(w_5-w_m)/w_m$ [%]	w_{mz} [mm]	w_{mf} [mm]	$(w_{mz}-w_{mf})/w_{mz}$ [%]
3	-3.61	1.68	7.72	0.515	0.512	0.583
3.5	-3.94	1.83	8.42	0.491	0.488	0.647
4	-4.28	1.99	9.15	0.469	0.466	0.716
5	-4.99	2.33	10.67	0.431	0.427	0.867
10	-9.66	4.51	20.58	0.291	0.285	1.960

$w_{mz}=(4w_1+4w_2+w_5)/9$ zero iteration; $w_{mf}=w_5$ final iteration

Table 12. Vertical force on pile head N_i [kN]. Final iteration. L=7m Soil modulus linear m_{Es}=41.67 MPa/m

s/D	N_1 [kN]	N_2 [kN]	N_5 [kN]	(N_1-N_5) [kN]	$w_5 = w_m$ [mm]
3	118.06	91.40	62.17	55.89	0.389
3.5	118.53	91.16	61.22	57.31	0.362
4	119.04	90.91	60.20	58.84	0.338
5	120.09	90.39	58.07	62.02	0.294
10	125.36	87.77	47.46	77.90	0.124

$w_m=w_5$

In table 14, it can be seen that the difference of mean settlements (final and zero iteration) increases over 1% as early as at the spacing of 3.5D, and it amounts to 1.12%, and for 5D the difference is 1.7%, and for 10D 7%, which is not negligible.

Table 13. Displacement of pile head w_i/w_m[%]. Zero iteration.
L=7m. D=60cm. Soil modulus linear m_{Es}=41.67 MPa/m.

s/D	w_1	w_2	w_5	w_m	$(w_1-w_5)/w_m$	$M_x=M_y$	h $_{usvojeno}$	$s_{min}\approx h$
	[mm]	[mm]	[mm]	[mm]	[%]	[kNm]	[m]	[m]
3	0.373	0.402	0.436	0.393	-16.164	-89.06	1.2	1.8
3.5	0.345	0.376	0.412	0.366	-18.226	-134.24	1.5	2.1
4	0.320	0.353	0.390	0.342	-20.472	-186.21	1.8	2.4
5	0.275	0.311	0.351	0.299	-25.623	-178.61	2	3
10	0.098	0.150	0.208	0.133	-81.765	-213.49	3	6

$w_m=(4w_1+4w_2+w_5)/9$

In table 17, it can be seen that the difference of mean deflection of a group of piles for the final and zero iteration is small, and it is 1%, for the spacing of 3 to 4D, for 5D it is 1.3 %, and only for 10D this difference approaches 4%.

Table 14. Relative displacement of pile head w_i/w_m [%]. Zero and final iteration.
L=7m. D=60cm. Soil modulus linear m_{Es}=41.67 MPa/m

m_{Es}=41.67	Zero iteration				Final itera.	zero-final
s/D	$(w_1-w_m)/w_m$	$(w_2-w_m)/w_m$	$(w_5-w_m)/w_m$	w_{mz}	w_{mf}	$(w_{mz}-w_{mf})/w_{mz}$
	[%]	[%]	[%]	[mm]	[mm]	[%]
3	-5.15	2.40	11.01	0.393	0.389	0.968
3.5	-5.81	2.71	12.41	0.366	0.362	1.120
4	-6.54	3.05	13.94	0.342	0.338	1.292
5	-8.19	3.83	17.44	0.299	0.294	1.707
10	-26.21	12.33	55.55	0.133	0.124	6.867

$w_{mz}=(4w_1+4w_2+w_5)/9$ zero iteration; $w_{mf}=w_5$ final iteration

Table 15. Vertical force on pile head N_i [kN] Final iteration.
L=8.5m Soil modulus linear m_{Es}=41.67 MPa/m

s/D	N_1	N_2	N_5	(N_1-N_5)	$w_5=w_m$
	[kN]	[kN]	[kN]	[kN]	[mm]
3	117.02	91.91	64.28	52.74	0.393
3.5	117.40	91.72	63.52	53.87	0.369
4	117.80	91.52	62.70	55.10	0.348
5	118.64	91.11	60.99	57.66	0.310
10	122.79	89.06	52.61	70.18	0.168

$w_m=w_5$

Table 16. Displacement of pile head [mm]. Zero iteration. L=8.5m Soil modulus linear m_{Es}=41.67 MPa/m

s/D	w_1	w_2	w_5	w_m	$(w_1-w_5)/w_m$	$M_x=M_y$	h $_{usvojeno}$	$s_{min}\approx h$
	[mm]	[mm]	[mm]	[mm]	[%]	[kNm]	[m]	[m]
3	0.378	0.404	0.434	0.396	-14.057	-78.10	1.2	1.8
3.5	0.354	0.381	0.412	0.373	-15.624	-117.16	1.5	2.1
4	0.332	0.361	0.393	0.352	-17.291	-161.76	1.8	2.4
5	0.293	0.324	0.359	0.314	-20.968	-153.80	2	3
10	0.145	0.188	0.236	0.174	-51.781	-177.12	3	6

$w_m=(4w_1+4w_2+w_5)/9$

A graph is made according to the tables 16 and 17, where in figure 4 is shown the variation of the deflection of the corner pile w_1, then of the mid-side pile w_2 and of the central pile w_5, depending on the variations of the spacing of piles in the group having a 3x3 arrangement. The dashed line shows the mean deflection of a group of piles, as a final iteration, and it is approximately halfway between the settlements w_1 and w_2.

Table 17. Relative displacement of pile head w_i/w_m [%]. Zero and final iteration
L=8.5m. D=60cm. Soil modulus linear m_{Es}=41.67 MPa/m.

m_{Es}=41.67	Zero iteration				Final itera.	zero-final
s/D	$(w_1-w_m)/w_m$ [%]	$(w_2-w_m)/w_m$ [%]	$(w_5-w_m)/w_m$ [%]	w_{mz} [mm]	w_{mf} [mm]	$(w_{mz}-w_{mf})/w_{mz}$ [%]
3	-4.48	2.08	9.58	0.396	0.393	0.794
3.5	-4.98	2.32	10.64	0.373	0.369	0.902
4	-5.51	2.57	11.78	0.352	0.348	1.021
5	-6.69	3.12	14.28	0.314	0.310	1.297
10	-16.57	7.77	35.21	0.174	0.168	3.910

$w_{mz}=(4w_1+4w_2+w_5)/9$ zero iteration; $w_{mf}=w_5$ final iteration

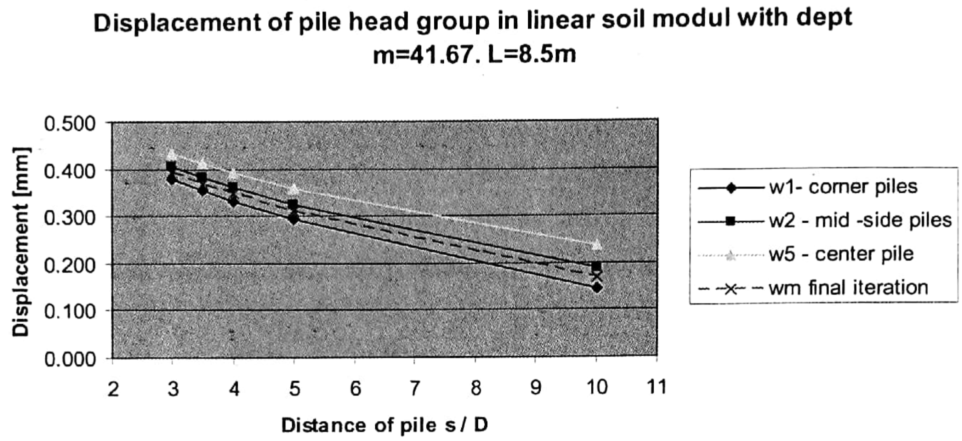

Figure 4. Vertical settlements of pile head distributed in 3x3 group. Settlements w_1, w_2 and w_5 are zero iteration (state). Piles embedded in soil with linear increase modulus by depth. m=41.67 MPa/m, L=8.5m.

For the same load of a group of piles with a diameter of 60 cm, without a cap beam, paradoxically the settlements for piles 8.5 m long are lower compared to piles 12 m long. For soil with linear changing by depth from m=41.67 MPa/m this difference of deflection is 0.12 mm, for the spacing of 3D, 3.5D, 4D, 5D and 10D. This means that driving deeper than 9 m is not cost-effective, and that instead of increasing the depth in this case, some other solutions should be considered. In order to reach a more comprehensive conclusion, the redistribution of forces in the group should be observed, for the case of a very rigid raft (final iteration). The difference between the forces of the piles in the group after the final redistribution for the pile lengths of 8.5 m and 12 m is shown in Table 18 (*Tab15-Tab 9*). The force in the corner is greater for a pile 8.5 m long, but the force in the mid-side pile and inner pile is lower for a pile 8.5 m long. This redistribution affects the calculations of the normalized normal force in the plastic hinges in the heads of the piles, so that in significant structures of founded in such soil in seismically active areas, this should be taken into account.

In Table 18, we note in column 5 that the force difference between the corner N_1 and the inner central pile N_5 is larger for the 8.5 m long pile, or that the force difference is smaller for the 12 m long piles. Therefore by increasing the depth from 8.5 m to 12 m, with this soil a somewhat more even distribution of pile forces in the group when they are connected by a very rigid beam can be achieved.

In table 19 it can be seen that the deflection difference for the depth of a group of piles of 8.5 m in comparison to 12 m, (when they are embedded in the same soil) is almost constant, and it amounts to around 0.12mm. Although the settlements are almost uniform, the difference of maximum moments varies between 2 and 5% and changes the sign depending on the spacing of piles. The change of sign occurs at the spacing of around 6.5 to 7D. Also, if the difference from w_1, to w_2 and w_5, is observed, it increases for s from 3D to 5D, but for 10D, it changes the direction, i.e. the difference decreases.

Table 18. Difference of vertical force on pile head N_i [kN] Final iteration N_i (L=8.5m)-Ni (L=12m) Soil modulus linear m_{Es}=41.67 MPa/m

s/D	N_1	N_2	N_5	(N_1-N_5)
	[kN]	[kN]	[kN]	[kN]
3	1.51	-0.74	-3.09	4.60
3.5	1.62	-0.80	-3.31	4.93
4	1.74	-0.85	-3.54	5.27
5	1.98	-0.97	-4.02	6.00
10	3.24	-1.61	-6.54	9.78

Table 19. Difference of settlement on pile head w_i [mm]. w_i (L=8.5m)-w_i (L=12m). Zero iter. w_i (L=8.5m)-w_i (L=12m). Soil modulus linear m_{Es}=41.67 MPa/m

s/D	W_1	w_2	w_5	w_m	$(w_1-w_5)/w_m$	$M_x=M_y$	$h_{usvojeno}$	$s_{min}\approx h$
	[mm]	[mm]	[mm]	[mm]	[%]		[m]	[m]
3	-0.119	-0.120	-0.121	-0.120	-2.296	3.95	1.2	1.8
3.5	-0.118	-0.119	-0.120	-0.119	-2.101	5.18	1.5	2.1
4	-0.117	-0.118	-0.119	-0.118	-1.878	6.13	1.8	2.4
5	-0.116	-0.117	-0.118	-0.117	-1.363	3.98	2	3
10	-0.117	-0.116	-0.115	-0.116	2.018	-4.13	3	6

$w_{mz}=(4w_1+4w_2+w_5)/9$ zero iteration; $w_{mf}=w_5$ final iteration

In table 20, it can be seen that the difference of the relative settlements of the corner pile (zero iteration) in comparison to the mean value of the group deflection, is lower for the piles 8.5m long, in comparison to the group 12 m long, for the common spacing of 3D to 5D. For the spacing of 10D, this difference is higher.

Table 20. Relative displacements of pile head [mm]. Zero iteration flexible raft. w_i (L=8.5m)-w_i (L=12m)

s/D	$(w_1-w_m)/w_m$	$(w_2-w_m)/w_m$	$(w_5-w_m)/w_m$	w_{mz}
	[%]	[%]	[%]	[mm]
3	-0.72	0.32	1.58	-0.120
3.5	-0.65	0.29	1.45	-0.119
4	-0.58	0.26	1.30	-0.118
5	-0.41	0.17	0.95	-0.117
10	0.70	-0.36	-1.32	-0.116

$w_{mz}=(4w_1+4w_2+w_5)/9$

The amount of lateral friction per pile shaft depends on the depth, and the type of soil: 1 loose sand, 2 medium compacted sand 3 compacted sand (Vesić, 1977). The friction remains approximately constant at a depth of 1 m for loose sand, at a depth of 2 m, for medium compacted sand and at a depth of 3 m for compacted sand. Although a linear increase in soil modulus by depth is assumed for sands, the occurrence of "critical depth" must be taken into account when determining the depth of compaction, when the friction on the shaft remains constant with depth. The critical depth increases with the compaction of the sand.

It is necessary to precisely define the concept of critical depth of the piles as a "critical depth of constant friction of the pile along the shaft in (1 loose, 2 medium compacted or 3 compacted) sand".

Applying the method of boundary equilibrium to the forms of pile fracture in coherent and incoherent soil, Broms introduced the concept of long, medium and short pile. In the long pile, two plastic hinges appear, in the medium pile - one, and a short pile behaves like a rigid body and there are no plastic hinges in the pile. Broms considers homogeneous models of coherent and incoherent soil, because this does not apply to multilayer soil, when there are large changes in the stiffness of the layers. If the adjacent two layers have a significant difference in stiffness, a plastic hinge will appear in the vicinity of this border.

4.4. Parabolic variation of the soil modulus by depth

Parabolic variation of the soil modulus by pile depth is characteristic for small levels of dilatations in sands. Here, firstly the variation of stiffness of the pile having diameter of 60 cm, for different values of soil modulus at the depth of one diameter of the pipe, in the function of the variation of pile length is analyzed.

By varying the diameter, depth and spacing of the piles in the group, results can be obtained that are completely in accordance with the theory and the selected parameters ( ). To reduce the possibility of error, it is recommended to automate the calculation process, or use verified diagrams, e.g. those given by (Poulos, 2001), (Pender, 1983), (Scot, 1981), and (Vesić, 1977).

For the pile 12 m long, and with the modulus at the depth of 1 diameter E_{SD}=35 MPa it can be noticed that also for the parabolic soil the mean deflection of zero and final iteration for the spacing of piles from 3 to 5D amount to less than 1%, and for 10D, this difference is 2.2%.

Table 21. Vertical force on pile head N_i [kN] Final iteration. L=12m Soil Par.E_{SD}=35 MPa

s/D	N_1	N_2	N_5	(N_1-N_5)	$w_5=w_m$
	[kN]	[kN]	[kN]	[kN]	[mm]
3	116.29	92.27	65.76	50.53	0.543
3.5	116.64	92.10	65.05	51.59	0.517
4	116.99	91.93	64.33	52.66	0.493
5	117.69	91.58	62.90	54.79	0.452
10	120.82	90.05	56.53	64.29	0.298

$w_m=w_5$

Tabela. 22. Displacement of pile head [mm]. Zero iteration. L=12m Soil Par.E_{SD}=35 MPa

s/D	W_1	W_2	w_5	w_m	$(w_1-w_5)/w_m$	$M_x=M_y$	h usvojeno	$s_{min}\approx h$
	[mm]	[mm]	[mm]	[mm]	[%]	[kNm]	[m]	[m]
3	0.526	0.555	0.588	0.546	-11.330	-86.91	1.2	1.8
3.5	0.500	0.530	0.564	0.520	-12.424	-130.22	1.5	2.1
4	0.476	0.507	0.543	0.497	-13.547	-179.39	1.8	2.4
5	0.433	0.467	0.505	0.456	-15.904	-169.47	2	3
10	0.274	0.319	0.370	0.305	-31.300	-187.68	3	6

$w_m=(4w_1+4w_2+w_5)/9$

Table 23. Relativ displacement of pile head [%]. Zero and Final iter. L=12m Soil Par.E_{SD}=35 MPa

35 MPa	Zero iteration				Final itera.	Zero-final
s/D	$(w_1-w_m)/w_m$	$(w_2-w_m)/w_m$	$(w_5-w_m)/w_m$	w_{mz}	w_{mf}	$(w_{mz}-w_{mf})/w_{mz}$
	[%]	[%]	[%]	[mm]	[mm]	[%]
3	-3.61	1.68	7.72	0.546	0.543	0.613
3.5	-3.96	1.84	8.47	0.520	0.517	0.686
4	-4.32	2.01	9.23	0.497	0.493	0.764
5	-5.07	2.36	10.83	0.456	0.452	0.934
10	-10.00	4.67	21.30	0.305	0.298	2.161

$w_{mz}=(4w_1+4w_2+w_5)/9$ zero iteration; $w_{mf}=w_5$ final iteration

For soils with a linear variation in depth, there is a small difference between the mean deflection of the pile group for final and zero iteration. This can be used to approximate the calculation of the final iteration, or to change the number of iterations, but only for evenly loaded piles in the group. It is not recommended to use this method of redistribution in a large group of piles (Poulos, 2011), when the results are not completely reliable, namely in the method itself the distance to which the direct influence of one pile on another is detected is limited. (Scot, 1981) for the large groups, this distance stated the value of $a/b=r/b=50$, i.e. $b=25$ D. After (Scot, 1981) the deflection of an individual pile can be calculated.

5. CONCLUDING REMARKS

For further research of the cap raft, a model of a thin or thick plate or a STM model (Strut and Tie Method) can be used. When changing the load intensity, the redistribution of the piles in relation to the plate of final stiffness is considered. The behaviour of the plate in that case should be viewed backwards, i.e. from the final iteration to zero. For such a plate, it is possible to redistribute the forces in the piles, which is in one of the calculated iterations. For the calculation of the struts, it is necessary to adopt the dimensions of the plate and the corresponding dimensions of the struts and "wedges" (nodes), and for the ties, it is necessary to adopt the reinforcement and its arrangement (Folić, et al. 2018) . In this case, the forces and settlements as values in certain iteration are multiplied by the intensity of the actually calculated forces as long as it is possible to apply the principle of superposition to the settlements and forces in their linear dependence. Nonlinear effects or fracture states of certain parts of the structure necessitate a special calculation model. This is true when the piles are loaded vertically, but when horizontal interaction is examined, the problem is further complicated. These models can give us more precise safety coefficients, and enable us to better understand the shape of the fracture state of the structure-foundation-soil system, i.e. the structure-piles-soil structure.

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REFERENCES

Azizkandi, S., A.R., Fakher, A., 2014. *A simple algorithm for analyzing a pile raft by considering stress distribution. Civil Engineerig infrastuctures journal.*

Celik, F., 2019. *An analytical approch for piled-raft-foundation design based on equvivalent pier and analysis by using 2D finite element method. Arabian joaurnal of geosciences.*

Chang, D.W., Lien, H.W., 2019. *Developing a three dimensional finit difference analysis for pile raft foundation settlements under vertical loads.* 4th *Bolivian inter. conf. on deep found.*

Deb, P., Pal, K.P., 2019. *Numerical analysis of piled raft foundation under combined vertical and lateral loading.* Ocean Engineering.

Fleming, W.G.K., Weltman, A.J., Randolph, M.F., Elson, W.K., 1998. *Piling engineering.* 2nd edition. E&FN SPON. Routledge. London.

Folić, B., Folić, R., 2018. *Comparative nonlinear analysis of a RC 2D frame soil-pile interaction. Building Materials and Structures 61.* Beograd.

Folić R., Folić B., Miličić I., 2018. Strut – and – Tie Model for analysis of some RC foundation design. Conferenc proceedings 6th International Scientific and Expert Conference. Contemporary achievements in civil Engineering. UNS. Faculty of Civil Engineering Subotica.

Folić, B., Lađinović, Đ., Brujić, Z., Ćosić, M., 2016. *Pile-soli-pile Interaction in designing the foundation of RC structures.* SGI Srbije, in 5th International conference *Earthquake engineering and engineering seismology*, pp. 379 - 386, 29. - 30. Jun, 2016. Sremski Karlovci.

Jayarajan, P., Kouzer, K.M., 2015. *Analysis of pile raft foundations*, pp. 51-57.

Milović, D., Đogo, M. 2009. *Analiza fundiranja na ploči sa šipovima. Materijali i konstrukcije* 52, Br. 3-4, 2009, 3-20. Beograd.

Milović, D., Đogo. M., 2009. *Problemi interakcije tlo-temelj-konstrukcija.* Srpska akademija nauka i umetnosti ogranak u Novom Sadu. Novi Sad 2009.

Mosher, R., Dawkins, W., 2000. *Theoretical Manual for Pile Foundations.* U.S. Army Corps of Engineers, Report ERDC/ITL TR-00-5, Washington, USA.

Nguyan, D. D. C., Jo, S.B., Kim D.S., 2013. *Design method of piled raft foundation under vertical load considering interaction effects.* Computer and geotechics.

Pender, M.J., 1983. *Aseismic pile foundation design analysis. Bulletin of the New Zealand NS of EE,* Vol. 26, No.1, March, 49-160.

Poulos H.G., Davis. E. H., 1980. Pile fundation analysis and design. John Wiley. New York.

Poulos, H.G., 1989. *Pile behaviour-theory and application. Geotechnique.*

Poulos, H.G., 2001. *Methodology of analysis of piled raft foundation. International society of soil mechanic and geotechnical engineering.* A Report prep.on beh. TC18.

Poulos, H.G., 2011. *The design of high-rise building foundation. Geotec Hanoi.*

Randolph, M. F., Reul, O., 2019. *Practical approaches for design of pile groups and piled rafts.* 4th Bolivian interantionl conference on deep foundation.

Reese, L., Van Impe, W., 2001. *Single pile and pile groups under lateral loading.* Balkema, Rotterdam.

Russo, G., 1998. *Numerical analysis of piled rafts. International journal for numerical and analytical methods in geomechanics,* 22, pp. 477-493.

Sandep, R., Baleshwar, S., 2010. *Effect of piles on reasponse of raft foundatin. Indian Geotechnical Conference.* Bombay.

Scot, R. F., 1981. *Foundation Analysis.* Prentence Hall, Englewood Cliffs, NY.

Šuklje, L., 1979. *Objašnjenje pravilnika o tehničkim normativima za projektovanje i izvođenje radova kod temeljenja građevinskih objekta. Časopis Izgradnja.* Beograd.

Vesić, A. S., 1977. *Design of Pile Foundations.* National cooperative highway research program 42. Synthesis of highway practice. Transportation research board. National research council. Washington, D.C.

Фолић, Б., 2017. *Сеизмичка анализа бетонских конструкција фундираних на шиповима.* Докторска дисетација. Универзитет у Новом Саду. Факултет Техничких Наука.