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STATIC LOAD INFLUENCE IN THE PILE CAP AND ALONG THE PILES

Summary: *This paper presents the methodology for determining changes in lateral displacements, inclination and bending moments and shear forces in piles for static loading. The coefficients of flexibility and stiffness for piles embedded in different types of soil are shown. Values were determined for the mentioned deformations and forces, which were presented and analyzed. The effects of pile depth for horizontal loading were considered. For the vertical load, the impacts of the depth of the pile according to Vesić were analyzed, such as the friction distribution of along the pile shaft and the bearing capacity ratio between the base and the shaft. The results include piles with a hinged connection with the caps and for piles restrained in the caps. The Winkler model was used for the p-w curve connection for nonlinear analysis. The relationship between the stiffness matrix and the flexibility of the pile caps is shown. The friction on the pile casing was also analyzed according to the recommendations of several authors.*
Keywords: *piles**,** caps, flexibility and stiffness of piles, effects along piles, friction along shaft, static load.*

UTICAJI U NAGLAVICI I DUŽ ŠIPA ZA STATIČKO OPTEREĆENJE

Rezime: *U ovom radu prikazana je metodologija određivanja promene bočnih pomeranja, nagiba i momenata savijanja i smičućih sila u šipovima, za statičko opterećenje. Prikazani su koeficijenti fleksibilnosti i krutosti za šipove ugrađene u različite vrste tla. Određene su vrednisti za pomenute deformacije i sile koje su prezentirane i analizirane. Razmatrani su uticaji po dubini šipa za horizontalno opterećenje. Za vertikalno opterećenje analizirani su uticaji po dubini šipa prema Vesiću, kao što je raspodela trenjem po omotaču šipa i odnos nosivosti bazom i omotačem. Rezultati obuhvataju šipove sa zglobnom vezim sa naglavnicom i za šipove uklještene u naglavnicu. Za vezu p-w krive za nelinearnu analizu korišćen je Winklerov model. Prikazana je veza matrice krutosti i flaksibilnosti naglavnica šipova. Analizirano je i trenje po omotaču šipova po preporukama više autora.*
Ključne reči: *šip, naglavnice, fleksibilnost i krutosti šipova, uticaji duž šipa, trenje po omotaču, statičko opterećenje.*

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1. INTRODUCTION

Individual, and particularly grouped piles are connected with blocks, beams or caps. For this reason, the analysis of the state stress and strain along the piles and in the caps under a specific load are important. The parameters for consideration of the behavior are deformations *w* and skin friction. In the process, it is necessary to assess the pile-soil interaction, and in the pile group the interaction of piles, among themselves and with the soil. The solutions for the behavior of the individual piles under the static load were provided by: (Vesić 1977, [3]), (Paulos i Dejvis 1980, [13] i [14]), (Reese and Van Impe, 2001 in [16]), (Scot, 1981 in [17]), Maksimović [5], (Pender 1993, in [12]) provide the recapitulation of coefficients and theories of different authors.

As for the vertical interaction, there are several load-deformation models (*p-w* curves), of different authors, which are used for nonlinear analysis of load transfer from the pile onto the soil, and the review was provided by Mosher [7]. Nonlinear behavior to horizontal load and effects of changes of input soil parameters to the pile response were provided by Meyer and Reese [6].

Poulos and Davis, 1974, [14], listed equations and diagrams of solutions of the transfer of axial load of compressible and non-compressible pile in a limited thickness soil, for different values of Poisson's number and the ratio of the layer thickness and pile length. Also the solution for the standing pile and grouped piles was provided.

Milović and Đogo, in [8], [9] and [10] provided the review of theories of different distributions of the soil modulus by depth of pile, and in relation coefficients of stiffness and flexibility in functions where displacement diagrams u, w, and pile rotation ϕ, are determined, as well as the effects along the depth (u, w, ϕ, moment $M(z)$ and transversal force $S(z)$). Also, the solution for the individual pile and piles in the group was provided.

Among the first, the distribution of soil resistance for the horizontal direction, for the failure state in the conditions of the border equilibrium of the long, medium and short piles for cohesive and non-cohesive soil was provided by Broms [1], [2].

In this paper, soil models used in static and dynamic analysis of piles are discussed. The coefficients of flexibility and stiffness for piles embedded in different types of soil are shown. The coefficients for the static load and influences in the pile cap for hinged connection and restraining of piles in the cap are shown. The effects of pile depth for horizontal and vertical loading were considered.

2. SOIL MODELS

The usual soil models used in the static and dynamic analysis of the structure-soil interaction are:

1. model with the constant soil stiffness by depth,
2. model with the linear increase of soil stiffness by depth,
3. model with the parabolic increase of soil stiffness by depth.

For these three soil models, there are solutions in the closed analytical form.

The model for layered soil is also widely used, with a sudden change in stiffness between the layers, whereby even individual layers can vary in stiffness, although the model with constant stiffness in the layer is most often used. If necessary, when discretizing the model, some complex function of soil stiffness variation can be represented as a series of layers with constant stiffness.

Figure 1. Typical stiffness profiles for foundation strata a) Constant Stiffness (Typical of overconsolidation clay) b) Parabolic stiffness (Typical of sand), c) Linearly increasing stiffness (Typical of soft clay)

3. IMPACTS ALONG THE PILE LENGTH

The solution for the horizontally loaded pile in the Winkler medium is expressed using the following parameters:

Where: E_p - Young's modulus of pile material,

I_p –moment of inertia of pile cross-section.

If it is known that in the linear change of the modulus of soil the fifth root and the similar stiffness ration as for the one in the expression (1) are used.

Parameter λ has dimensions L^{-1} and depends on the characteristics of pile and soil. When the pile is very stiff in relation to the surrounding soil, λ is low and the load on the pile cap causes deformations at a greater distance from the pile cap. For the relatively elastic pile, in relation to the soil, λ is high and the load has only a local impact on the stress and strain. According to[12], Hetenyi, 1946, proposed the following classification for the relative stiffness of the beam, based on the λL parameter (L being the length of the beam):

I Short beam $\lambda \mathrm{L} \leq \pi/4$

II Medium beam $\pi/4 \leq \lambda \mathrm{L} \leq \pi$

III Long beam $\pi \leq \lambda \mathrm{L}$

Prof. D. Milović investigated these recommendations and proposed appropriate corrections. For short piles, bending deformations can be neglected because they are very small compared to the deformations of the surrounding soil. If the pile is stiff, the displacement is of a simple form, and accordingly the reactive pressure is easily estimated. For long piles, it is considered that if the load acts on one end, it is not transferred to the other end. For piles of medium length, a much wider analysis is required because the load applied at one end is transferred to the other end. Of interest is the solution for a semi-infinite beam on a Winkler support. Hetenji and (Lee and Harrison, 1970) gave a solution for various cases of finite beams, cited in [insert number].

In order to determine impacts by the length of the pile (Hateny, 1948) the horizontal load is replaced to the moment and horizontal force. Trigonometric and exponential functions in the solutions of both cases are very similar (figure 1), and only constants are different.

The equations listed below employ the following relations: inclination $\phi=du/dz$, moment $M/EI=d\phi/dz$, and shear force $S=dM/dz$. For the positive displacement of the pile head, the inclination is always negative on the pile cap, because the displacement decreases with the increase of the value *z*. For consistency reasons, it is adopted that the pile cap rotation is positive. This is achieved by defining that the pile cross-section rotation is equal to the negative value of the inclination, i.e. $\Theta=-\phi$. For the horizontal load, *H,* applied on the pile cap, varying the displacements and other values along the pile shaft was defined by:

$$u(z)=\frac{2\lambda H}{k}(\cos\lambda x)e^{-\lambda x}=\frac{2\lambda H}{k}D \tag{1}$$

$$\theta(z)=\frac{2\lambda^2 H}{k}(\cos\lambda x+\sin\lambda x)e^{-\lambda x}=\frac{2\lambda^2 H}{k}A \tag{2}$$

$$M(z)=\frac{H}{\lambda}(\sin\lambda x)\cdot e^{-\lambda x}=\frac{H}{\lambda}B \tag{3}$$

$$S(z)=-H(\cos\lambda x-\sin\lambda x)e^{-\lambda x}=-H\cdot C \tag{4}$$

u(z) - lateral pile displacement,
θ(z) - pile shaft rotation (minus inclination),
M(z) - moment of bending, and
S(z) - shear force.

The following holds for the moment, M, applied on the pile cap:

$$u(z)=\frac{2\lambda^2 M}{k}(\cos\lambda z-\sin\lambda z)e^{-\lambda z} \tag{5}$$

$$\theta(z)=\frac{4\lambda^3 M}{k}(\cos\lambda z)e^{-\lambda z} \tag{6}$$

$$M(z)=M(\cos\lambda z+\sin\lambda z)e^{-\lambda z} \tag{7}$$

$$S(z)=-2\lambda M(\sin\lambda z)e^{-\lambda z} \tag{8}$$

Functions λz occurring in expressions (1) to (4) are the same as those in expressions (5) to (8) which are jointly presented in figure 2.

Maximum moment point is found by using the transformation of expressions (2) to (8) which is provided with the transcendent equation:

$$\lambda z = \tan^{-1}\left(\frac{1}{1 + 2\lambda \frac{M}{H}} \right) \tag{9}$$

By including *z* from this expression into the previous equations the maximum moment is obtained.

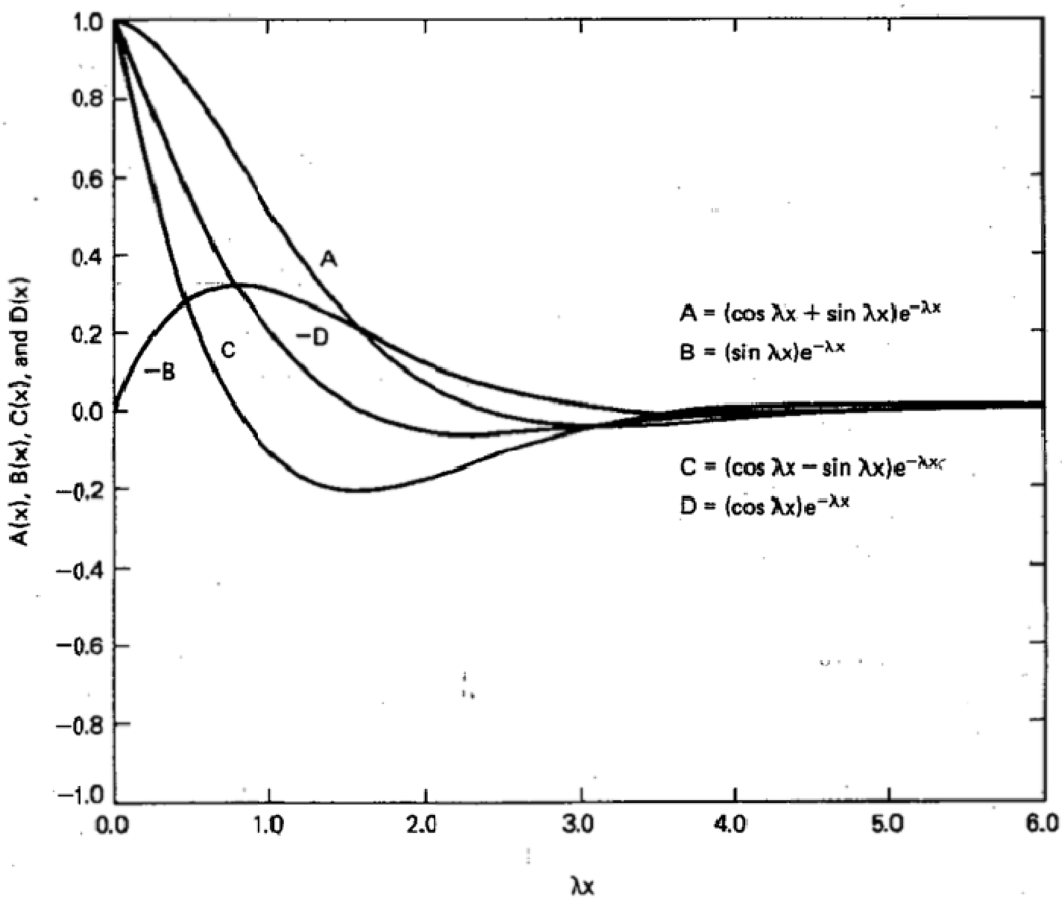

Figure 2 Functions of variation of displacement, inclination, moment and shear forces in the semi-infinite beam on the Winker base

λx	x(m)	u(z)	Θ(z)	M(z)	S(z)
0,00	0,00	0,0585	0,0068	0	-10000
0,23	2,00	0,0450	0,0065	15682,6	-5862,5
0,47	4,00	0,0327	0,0058	24143,7	-2763,3
0,70	6,00	0,0221	0,0048	27349,2	-583,1
0,94	8,00	0,0136	0,0038	26985,7	832,8
1,17	10,00	0,0071	0,0028	24418,8	1647,2
1,40	12,00	0,0024	0,0019	20694,5	2014,4

1,64	14,00	-0,0008	0,0012	16568,4	2069,8
1,87	16,00	-0,0027	0,0007	12548,3	1925,0
2,11	18,00	-0,0036	0,0003	8943,2	1667,4
2,34	20,00	-0,0039	1E-05	5910,3	1361,6
2,57	22,00	-0,0038	-0,0002	3498,7	1052,1
2,81	24,00	-0,0033	-0,0003	1685,3	767,1

Table 1. Impacts along the pile length according to Hetenyi

Impacts in table 1 are presented for k=40000, H=10 kN, Ep=25 GPa, rectangular pile b/d=40cm, Ip= 0,0021 m^4, λ= 0,117. Za 24m, $\lambda \cdot x$ = 2,81. It is assumed that for $\lambda \cdot x$ = between 4 and 6, the functions i.e. tend to zero. For such conditions and pile diameter, the length of 12m would probably be sufficient, but because of monitoring the shape of the impact, it was extended to 24m.

4. CONNECTION BETWEEN THE STIFFNESS MATRIX AND FLEXIBILITY OF PILE CAPS

The stiffness matrix is an inverse flexibility matrix. In one case, the coefficients are the forces caused by unit displacements, and in the other case those are displacements caused by unit forces in the generalized direction of action.

$$\begin{bmatrix} K_{HH} & K_{HM} \\ K_{MH} & K_{MM} \end{bmatrix} = \frac{1}{(f_{uH} f_{\theta M} - f_{uM}{}^2)} \begin{bmatrix} f_{\theta M} & -f_{uM} \\ -f_{\theta H} & f_{uH} \end{bmatrix} \tag{10}$$

K_{HH}, K_{HM}, K_{MH}, K_{MM} are coefficients of the pile cap stiffness, based on the reciprocity theorem $K_{HM}= K_{MH}$. Determinant of the flexibility matrix is:

$$f_{uH} f_{\theta M} - f_{uM}{}^2 = \det[f_{ij}] \text{, za } f_{uM} = f_{\theta H} \tag{11}$$

$$K_{HH} = f_{\theta M} / (f_{uH} f_{\theta M} - f_{uM}{}^2) \tag{12}$$

$$K_{MM} = f_{uH} / (f_{uH} f_{\theta M} - f_{uM}{}^2) \tag{13}$$

$$K_{HM} = -f_{uM} / (f_{uH} f_{\theta M} - f_{uM}{}^2) \tag{14}$$

First tier / column shows the impacts on the pile cap which are generated for the unit displacement applied on the pile cap, with the rotation which equals zero. The second tier are the data of the impacts on the pile cap which are generated for the unit rotation with the displacement which equals zero.

The next case is the free pile cap in which the horizontal force can be applied above or below the ground surface level. In the case when the pile cap rotates, the equivalent horizontal stiffness would be lower than K_{HH}, and the rotational stiffness lower than K_{MM} .

Equivalent stiffnesses can be obtained by extending the impact on the pile cap, in terms of stiffness and displacement:

$$H = K_{HH}u + K_{HM}\theta \tag{15}$$

$$M = K_{HM}u + K_{MM}\theta \tag{16}$$

First, θ from (16) is solved and substituted in (15) with the assumption that e originates from H:

$$\theta = (M - K_{HM}u)/K_{MM} \tag{17}$$

$$H(1 - eK_{HM} / K_{MM}) = u(K_{HH} - K_{HM}{}^{2} / K_{MM}) \tag{18}$$

Thence, the equivalent stiffness of the pile cap is:

$$K_h = \frac{H}{u} = \frac{K_{HH}K_{MM} - K_{HM}{}^{2}}{K_{MM} - eK_{HM}} \quad (kN/mm) \tag{19}$$

Where: K_h is horizontal (condensed) stiffness of the free pile cap (for the known eccentricity, as a ratio of moment and horizontal force).

The equivalent rotation stiffness of the pile cap is obtained in a similar way starting from the use of expression (15) and solving by substituting in expression (16):

$$u = (H - \theta \cdot K_{HM})/K_{HH} \, , \tag{20}$$

$$M = K_{HM}(M/e - \theta \cdot K_{HM})/K_{HH} + K_{MM}\theta \, , \tag{21}$$

$$M(1 - K_{HM} / eK_{HH}) = (-\theta \cdot K_{HM}{}^{2})/K_{HH} + K_{MM}\theta \tag{22}$$

$$K_\theta = \frac{M}{\theta} = \frac{K_{HH}K_{MM} - K_{HM}{}^{2}}{K_{HH} - K_{HM}/e} \, . \tag{23}$$

Vertical load distribution along the pile shaft can be very different, as shown in Vesić [3], Maksimović [5], Nonveiller [11], Paolus and Davies [13] etc.

5. FRICTION ALONG PILE SHAFT

In general, the piles are divide into end-bearing and floating, depending on the mechanism of dominant transfer of load to the soil: via the base or friction along the shaft.

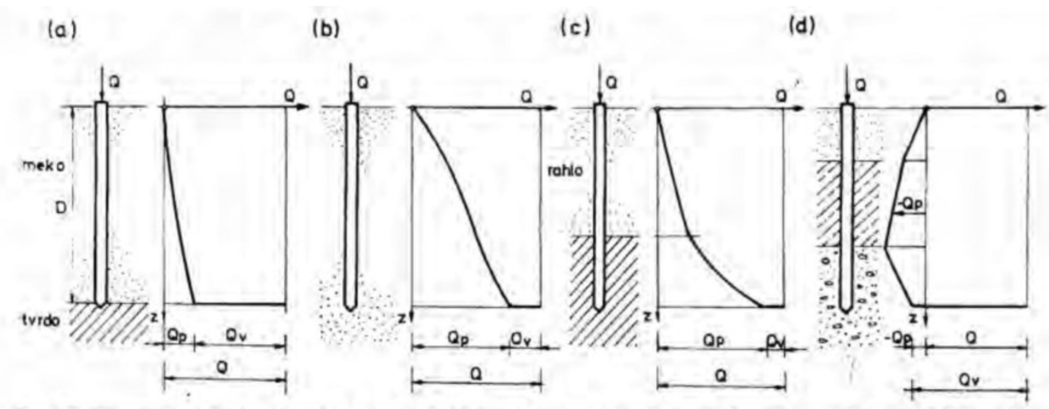

Figure 3. Distribution of pile force to the shaft and the base: (a) pile on a solid ground, (b) pile in homogeneous layer, (c) pile in inhomogeneous soil, (d) negative friction (Nonveiller,1979, [11])

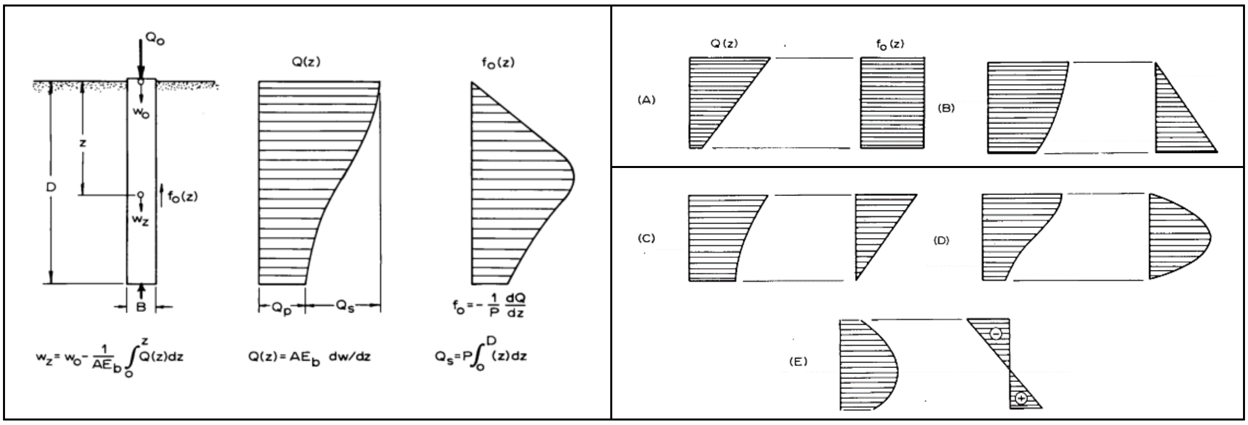

Figure 4. Distribution of vertical force on the pile shaft and on the base: (a) pile on the layer of overconsolidated clay, (b) pile in the soil with linear increase of friction along the shaft, (c) pile in the soil with linear decrease of friction along the shaft, (d) negative friction (Vesić,1977)

Figure 4 left show the distribution of vertical load by depth of the pile, on the pile shaft and the pile base. Figure right shows different distributions of the vertical force along the pile shaft. The force variation function derivation *Q(z)* is the function of the variation of friction along the shaft, equation (24), so for the linear variation of force *Q* in figure 4a, there is constant friction by depth. Figure 4b, shows the convex curve of the variation of force *Q*, as a square function, so the friction along the shaft is linearly increasing, while under c the force function *Q* is concave, so the friction along the shaft is linearly decreasing. In figure 4d, there is an inflection point so the friction along the shaft is a second or third order parabolic, with maximum around the middle of the pile length, therefore the force function *Q* is a polynomial of no less than third or higher order. In the right hand part, the label *E* marks the case of negative friction, the force *Q* is the second order parabolic with the maximum around the mid or third of the pile, so in this location the sign of the linear function changes. Negative friction along the shaft is a part of the soil layer which is "suspended" onto the pile, and it reduces is total load bearing capacity along the shaft instead of increasing it. Vesić in [3] talks about ways of applying strain gauges along the pile depth, in order to measure force distribution. Friction along the shaft is:

$$f_0 = -\frac{1}{P} \cdot \frac{dQ}{dZ} \tag{24}$$

Where P is the pile perimeter.

In figure 4 can be seen that Q_p is base bearing, and that Q_s is shaft bearing. Also, the missing parameter is the description of distribution by depth of the pile while the function $Q_s(z)$, because the first function f_0 derivative is friction along the pile shaft. Also the coefficient $k_{sb} = Q_s / Q_p$ should be introduced which shows us the relation of the pile bearing capacity by shaft in comparison with the bearing capacity by friction. Then the reciprocal value is $k_{bs} = 1/ k_{sb} = Q_p / Q_s$, and the relation Q / Q_s and Q_p / Q_s, tells us which is the ratio of bearing capacity by friction along the shaft and by the base and the total vertical load (p - point or base, s -shaft). When $k_{bs} = 1/ k_{sb} = 1$ then the pile bears 50% of the total vertical force with its base. When $k_{bs} > 1$ a pile is prevalently base-bearing.

Vesić in 3 described an elastic approach to the solution of this problem (friction) and transfer function. Table 2 lists the following experiments for the function transfer: Seed & Reese (1955), Reese (1964), Coyle & Reese (1966), and numerical research: Kezdi (1957), Reese et al (1969), Holloway et al (1975) after Vesić [3].

For the elastic approach, Vesić [3] mentioned about 10 theories between 1963 and 1974, of mostly widely known authors, where it is analyzed whether the slip occurs in the base, then slip between the pile and the soil, and one particularly represented author is Poulos.

Autor	Transfer function
Seed & Reese (1955)	experimental
Kezdi (1957)	$f = K_0 \gamma z \tan\phi \{1 - \exp[-kw/(w_c - w)]\}$
Reese (1964)	experimental
Coyle & Reese (1966)	experimental
Reese et al (1969)	$f = K [2 (w/w_c)^{0.5} - w/ w_c]$
Holloway et al (1975)	$f = K \gamma_w (\sigma/p)^n w [1 - (R_f/ \sigma \tan\delta)]^2$

Table 2. Authors and transfer of the friction along the shaft function according to Vesić [3].

The function of distribution of friction along the shaft does not depend only on the variation of soil modulus by depth, but also on the method of embedding of the pile into the soil.

6. CONCLUSION

Often, only the first approximation is used for pile-soil interaction, which is the replacement of pile and soil with springs in the pile head and corresponding coefficients of flexibility or stiffness, while the lower part of the pile is ignored. However, in engineering practice, it is necessary to determine the impact diagrams in the pile and soil by depth. There is a number of theories that deal with this problem, depending on the distribution of soil stiffness by depth, but one minimal set of parameters refers to the relationship between the bearing capacity of the base and the friction of the shaft, as well as the function of the change of friction of the shell with depth, obligatorily for the vertical, and often for the horizontal loading, too.

For horizontal pressure at depth, soil resistance is activated by a mixed mechanism: horizontal pressure and horizontal shaft friction, the shape of the cross-section and the material from which the pile is made also play a significant role. In addition, one should consider that the effects of the pile embedding method are important, as is the function of the distribution of soil modulus by depth. An important element in the analysis is the compressibility of the pile, which tells us whether the pile behaves as very stiff in relation to the soil modulus, when its axial deformation is ignored and when it is not ignored.

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