Error estimates for Gaussian quadrature of analytic functions for various weight functions

Davorka Jandrlić[1], Miodrag Spalević[1] and Aleksandar Pejčev[1]

[1]*Department of Mathematics, Faculty of Mechanical engineering, University of Belgrade, Serbia*

djandrlic@mas.bg.ac.rs

ABSTRACT

We studied the error bound of Gaussian quadrature for analytic functions. The basic idea is to express the remainder of Gaussian quadrature as a contour integral, then the error bound is reduced to find the maximum of the kernel function:

$$K_n(z;\omega) = \frac{\varrho_n(z;\omega)}{\pi_n(z)}, \quad \varrho_n(z;\omega) = \int_{-1}^{1} \frac{\pi_n(t)}{z-t}\,dt, \quad z \in \mathbb{C}\backslash[-1,1]. \tag{1}$$

The integral representation of the error term leads directly to the error bound

$$|R_n(f)| \le \frac{l(\Gamma)}{2\pi}\left(\max_{z\in\Gamma}|K_n(z)|\right)\left(\max_{z\in\Gamma}|f(z)|\right), \tag{2}$$

where $l(\Gamma)$ is the length of the choosen contour Γ.

A common choice for the contour Γ is one of the confocal ellipses with foci at the points ∓ 1, also known as the Bernstein ellipses, and the sum of semi-axes $\rho > 1$,

$$\mathcal{E}_\rho = \left\{z \in \mathbb{C}\colon\ z = \frac{1}{2}(u + u^{-1}), u = \rho e^{i\theta},\ 0 \le \theta < 2\pi\right\}.$$

For such Γ we studied the estimates (2) when w is one of the four generalized Chebyshev weight functions.

Key words: Gauss quadrature formulae, Orthogonal polynomials, remainder term for analytic function, error bound.

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