VERTICAL STIFFNESS OF PILES AND REDISTRIBUTION DUE TO INTERACTION FOR LINEAR CHANGE OF SOIL MODULUS BY DEPTH

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Abstract:
The paper shows some characteristic problems of determining the axial stiffness of the pile and redistribution in a group. Some advantages and drawbacks of the use of individual design models are indicated. In the redistribution, the zero iteration is considered, which a condition is when the connecting beams or rafts are fully flexible, and the final iteration when the deflections of the piles are equalized. In the zero iteration, all the forces in the piles are of the same intensity, because only the centrically loaded raft is considered, but the deflections of individual piles during interaction are different. Due to the symmetry of the problem, in the zero iteration, only the certain groups have the equalized deflections of piles, but there is no equalized deflection of the groups themselves. In the final iteration, a distribution of forces in which the deflections of the individual piles during interaction are equalized is sought, i.e. a tolerance that is a relative error of 10^{-5} in relation to the mean deflection is set. The soil model with the linear distribution with depth is considered. The effect of the distance of piles on the interaction is studied.

Key words*: Vertical stiffness, Vertical interaction, Redistribution during Pile-Soil-Pile interaction, distance influences of interaction.*

1. Introduction

A large number of researchers dealt with determining the vertical stiffnes and pile-soil-pile interactions, but only some are listed in this paper, without an in-detail review of results. Those are: Poulos and Davis [1], Milović and Đogo [2], [3], Scot [4], Pender [5], Fleming [6],[7], Gazetas, Prakash, Seed and Reese, Van Impe, Maymond, Vesić [8], Coop [9], Mosher[10], Folić,[11], [12] et al. The interaction problems are analyzed in the papers, by introduction various models of behavior of the groups of piles. i.e. for the different properties of soil and variation of individual parameters, including the experimental results and in situ testing. These papers are presented and analyzed in the paper [12].

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In this paper are analyzed soil modulus with the linear distribution of soil modulus by depth and the methodology for calculation of the values of vertical stiffness of the piles. In the process, elastic dependence of the force-displacement curve and redistribution of interaction for the considered soil model are used.

2. Soil modulus and distribution used in the analysis of vertical stiffness and pile-soil-pile interaction

Usually, three idealized models of variation of soil stiffness by depth are used, and those are: linear, parabolic and constant value. Often, the trapezoid distribution is used as a model, which, for the linear-elastic behavior, can be obtained by the superposition of constant and linear cases. For the stiffness of the static pile, special expressions and the model are used.

The interaction of nine piles in the group of a symmetrical arrangement 3x3 (figure 1) is considered.

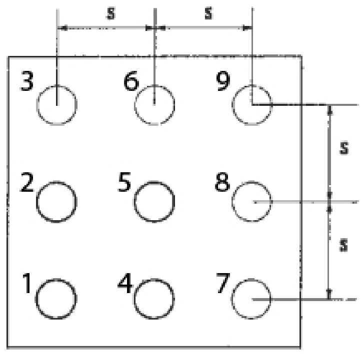

Figure 1. Arrangement of piles in the 3x3 group, and redistribution of forces

Force on the corner pile (1,3,7 and 9) is 117 kN, on mid-span (2, 4, 6 i 8) 91.8 kN, and on the center (5) 64.8kN. By Pender, the total force is 1000 kN, while the authors of this paper set on each pile the force having intensity of 100 kN, because it is more convenient to monitor the redistribution for the total force of 900 kN.

2.1. Problem formulation and numerical treatment of the deterministic problem

For the linear variation of soil modulus by depth the equations for the linear variation of soil modulus profile are given by Budhu and Davies, cited after (Scot R. F., 1981, [4], and Pender M. [5]).

For this case the Young's modulus of soil and stiffness is:

$$E_S = mD \;;\; K = \frac{E_P}{mD} \tag{1}$$

The axial stiffness of the pile for the linear variation of the soil modulus by depth is:

$$K_V = 1.8 \cdot E_{SL} D \cdot \mathsf{L}^{0.55} \cdot \Re^{-b} \tag{2}$$

$$\Re = k = E_P / E_{SL} \tag{3}$$

E_{SL} - is the soil modulus at the pile tip (base) $E_{SL} = E_S(z=mL)$

Where m is the range increase of the Young's modulus with depth. Budhu and Davies give values m for different densities of sand. This is appropriate for the static pile load, but not for the dynamics excitation of piles bounded in loose saturated sands. Other coefficients, equations and theory can be seen in [5], [6], or [10].

2.2. Variation of vertical stiffness of the pile with the increase of the depth of placement

The analysis provided and presented diagrams of variation of the vertical stiffness of the pile with the increase of the depth of placement, for the soil with linear increase of the modulus by depth.

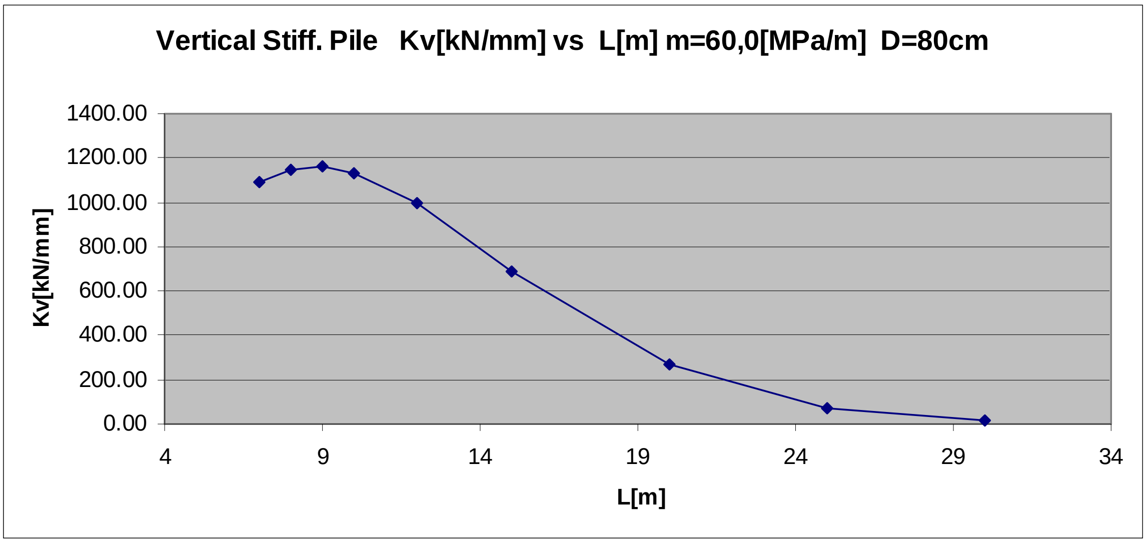

Figure 2. Variable vertical stiff. of pile vs depth piles, in soil modulus with linear variation stiffness *m*=60,0 [MPa/m], diameter *D*=80cm. Depth of pile from 8 to 30m.

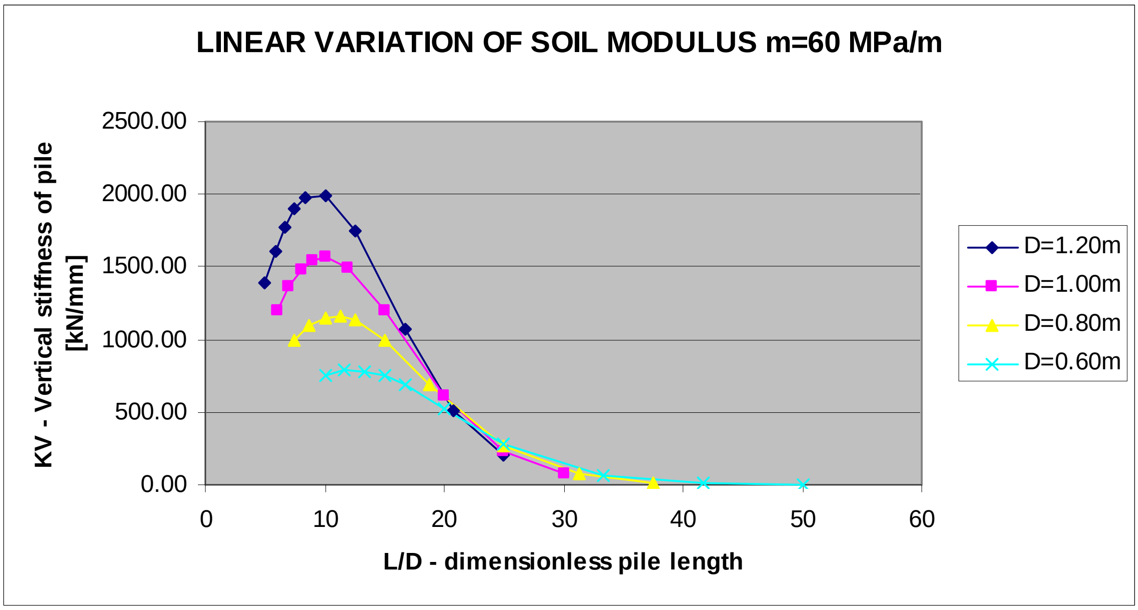

Figure 3. Variable vertical stiff. of pile vs depth piles, in soil modulus with linear variation stiffness *m*=60,0 [MPa/m], diameters *D*=60, 80, 100 i 120 cm. The depth in dimensionless form.

In fig. 2 is presented the diagram of the vertical stiffness for the soil with the linear variation of the modulus of *m*=60,0 [MPa/m], which corresponds to the medium dense sand. The diameter of the pile is *D*=80cm. On the stiffness variation diagram on may observe the mildly convex form of the function for the length of the pile of 8 to 10m. For the length of 10 to 20 m, the stiffness function has a strong linear dependence (R^2=0.9977) and decreases with the depth. Irrespective of the strong linear dependence in this interval, at the depth of about 17m, there is the inflectional point of the vertical stiffness function. Considering the unexpected reduction of vertical stiffness with the depth, the variation for the pile diameters of 60, 80, 100 and 120cm are to be investigated, for *m*=60 [MPa/m], and *m*=1.50 [MPa/m], which corresponds to the medium dense sand and normally consolidated clay, respectively. The following two diagrams (fig. 3 and 4) are given for the depth in the dimensionless form *L/D*, i.e. in relation to the diameter.

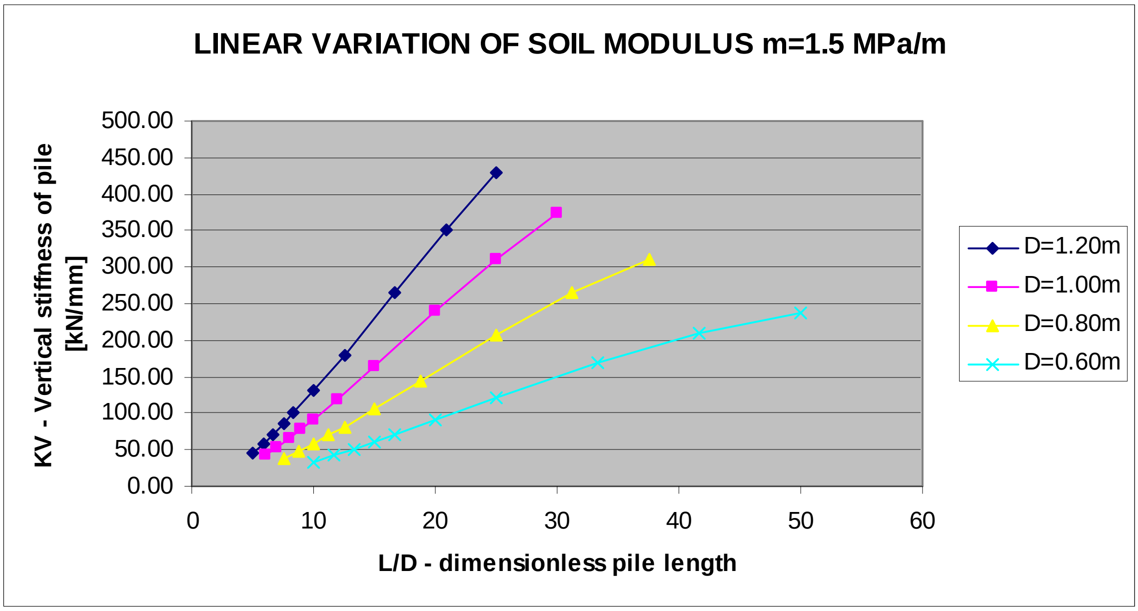

Figure 4. Variable vertical stiff. of pile vs depth piles, in soil modulus with linear variation stiffness *m*=1,50 [MPa/m], diameters *D*=60, 80, 100 i 120 cm. The depth in dimensionless form.

3. Numerical example

For this problem, the centrically loaded group, of the 3x3 pile arrangement, due to the symmetry, the following relations are valid:

(4a) Corner piles: $w_1 = w_3 = w_7 = w_9$.

(4b) Mid-side piles: $w_2 = w_4 = w_6 = w_8$.

(5a) Corner piles: $N_1 = N_3 = N_7 = N_9$.

(5b) Mid-side piles: $N_2 = N_4 = N_6 = N_8$.

Equation of vertical force equilibrium, (due to the symmetry) is in this case:

(6) $N = 4N_1 + 4N_2 + N_5$

Where:

α_{ij} - is the interaction factor between piles *i* and *j*, formula 7 and 8, after Randolf and Wroth (1979), [5], and [11].

$\alpha_{ij} = \alpha_V$ = is the quotient of displacement caused by unitary vertical action on the adjacent pile and pile displacement due to the unitary action on the pile head.

Interaction factors, defined, by:

(7) $$\alpha = \frac{\text{Movement caused by unit action on an adjacent pile}}{\text{Movement of the pile under unit head action}}$$

(8) $$\alpha_V = \frac{1 - \dfrac{s}{(D/\pi + s)} + \pi(1-\nu)\rho L\left(\dfrac{1}{\gamma} - \dfrac{1}{\Gamma}\right)}{1 + \pi(1-\nu)\rho L\ /\ \gamma}$$

(8a) $$\gamma = \ln(2r_m / D)$$

(8b) $$r_m = 2.5(1-\nu)\rho L$$

(8c) $$\Gamma = \ln(2r_m^{\ 2} / Ds)$$

The other formula (9) which is used for the interaction calculation, is simpler and easier to use, but has certain downsides. Firstly, it derived after the analogy f a circular plate, with the stiff disk in the center, so it is adequate only for the constant soil, therefore does not take into consideration the soil modulus variation. Secondly, it does not take into account the ratio of the depth to the diameter, while the procedure according to the formulas 7 and 8 does that. Further, formula 8b, represents the distance to which the interaction between the piles is perceived, and it is not the fixed value.

(9) $$w\left(\frac{r}{a}\right) = w_0\left[1 - \left[\ln\left(\frac{r}{a}\right)/\ln(50)\right]\right]$$

The final state is that obtained after a number of iterations, when the deflections of all the piles in the group are equalized, taking into consideration of the piles through the soil. This can be understood as a deflection of the group of piles, evenly loaded, connected with completely rigid cap raft – beam. $w_i(I_r \rightarrow \infty)$; $w_i(I_{tb} \rightarrow \infty)$.

3.1. Tabular review of the pile interaction results

The table shows the impact of inclusion or exclusion (bold) of negative coefficients of interaction, in the case when the occurrence of negative coefficients is slightly surprising.

Table 1. Vertical force on pile head N_i [kN] Final iteration.
L=8m. Soil Modul linear m=60 MPa/m, E_s=60 [MPa] ν=0,5

Es=60	Force in pile N_i [kN]		final iteration		[mm]
s/D	N_1	N_2	N_5	$(N_1\text{-}N_5)$	$w_5=w_m$
3	120.75	90.06	56.76	63.98	0.150
3.5	121.54	89.67	55.18	66.36	0.135
	122.37	89.25	53.49	68.88	0.122
4	**120.66**	**90.68**	**54.64**	**66.02**	**0.123**
	124.13	88.37	49.97	74.16	0.097
5	**117.81**	**92.63**	**58.26**	**59.55**	**0.103**

$w_m = w_{5f}$

Table 2. Displac. of pile head [mm]. Zero iterati. *L*=8m. *D*=100cm. Soil Modul lin. *m*=60 [MPa/m] *v*=0,5

Es=60 Displacement of pile head [mm] zero iteration

s/D	W_1	W_2	w_5	w_m	$(w_1-w_5)/w_m$	*Mx=My*	$h_{usvojeno}$	$s_{min}\approx h$
3	0.141	0.157	0.176	0.152	-22.429	-47.77	1.8	1.2
3.5	0.126	0.143	0.163	0.138	-26.395	-72.98	2.1	1.5
4	0.112 **0.114**	0.13 **0.13**	0.151 **0.151**	0.125 **0.125**	-31.040 **-29.547**	-102.54	2.4	1.8
5	0.087 **0.095**	0.107 **0.109**	0.130 **0.130**	0.101 **0.105**	-43.152 **-33.298**	-100.79	3	2

$w_m=(4w_1+4w_2+w_5)/9$; w_1- (Displacement on) corner piles; w_2 - mid-side piles; w_5- centre pile

Based on the results systematized in tables, it can be seen that as soon as at the 4*D* distance, in fact 8*D* (or diagonally 11.31*D*) there occur the negative coefficients of the interaction, which can be a surprise, because, r_m depends on the Poisson coefficient, type of soil and the length of the pile. Possibly, the interaction can exert influence at the distance of r_m increased for D/4. In table 1 and 2 are provided results because of which the negative coefficients of interactions are excluded.

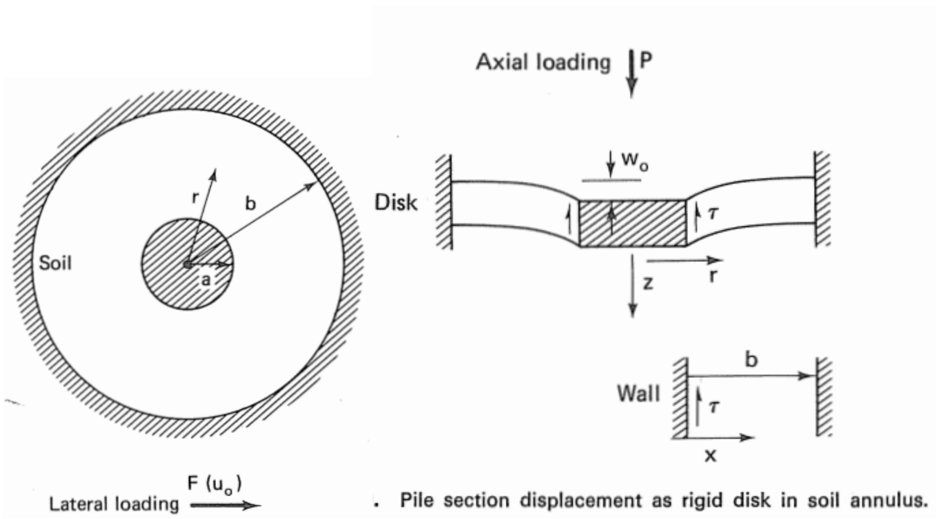

Figure 5 Analogy horizontal layer of soil with circular plate. Pile is rigid disk in centre of plate, after Scot, 1981 [4].

According to Scot [4], the distance between vertically loaded piles, should be around 25d, in order not to have any interaction to take place (Eq. 9).

Table 3 shows effect that change of Poison’s coefficient (Eq. 8a) from 0 to 0.5 has on different border distance of vertical interaction, for piles with diameter 1.00m and lengths of 8 and 12m, when interaction factor is less than 10^{-4}. Here, we consider zero to be any value interaction factor less than 10^{-4}. When Poison’s coefficient decreases from 0.5 to 0, length of the pile remaining the same, the distance border increases almost doubles.

Influence of variation of Poison’s coefficient (Eq. 8a) from 0 to 0.5 on different border distance of vertical interaction is shown in table 3 for piles with diameter 1.00m and length 8 and 12m, for interaction factor is less than 10^{-4}. Here, we consider zero to be any value interaction factor less than 10^{-4}. When Poison’s coefficient decreases from 0.5 to 0, for the same length of the pile, the distance border increases almost doubles.

Table 3. Different limit distances for vertical interaction influence, for 2 neighboring piles. *L*=8 and 12m. *D*=100cm. Soil Modul linear. *m*=60 [MPa/m] *v*=0; 0.1; 0.3; 0,5

	L [m] =	8		L [m] =	12
v=0,5	s/D [1] =	5,243	v=0,5	s/D [1] =	7,73
	r_m =	5		r_m =	7,5
v=0,3	s/D [1] =	7,234	v=0,3	s/D [1] =	10,71
	r_m =	7		r_m =	10,5
v=0,1	s/D [1] =	9,223	v=0,1	s/D [1] =	13,696
	r_m =	9		r_m =	13,5
v=0,0	s/D [1] =	10,216	v=0,0	s/D [1] =	15,189
	r_m =	10,00		r_m =	15,00

Fig. 2 and 3, show variable vertical stiffness of pile vs depth piles for medium dense sand. Decrease of vertical stiffness of floating piles, becomes clear when we keep in mind the Vesić's research, on driven piles shown in fig. 6, and of the research by the group at Imperial College, Lehane et al. 1993, shown in fig. 7. This type of decrease vertical stiffness of pile vs depth, even though it's partially greater than expected, warns us that when working with sand, after installation of the pile up to a certain depth, we can not achieve increase shaft friction. Instead, we must consider increasing the number of short piles in a group.

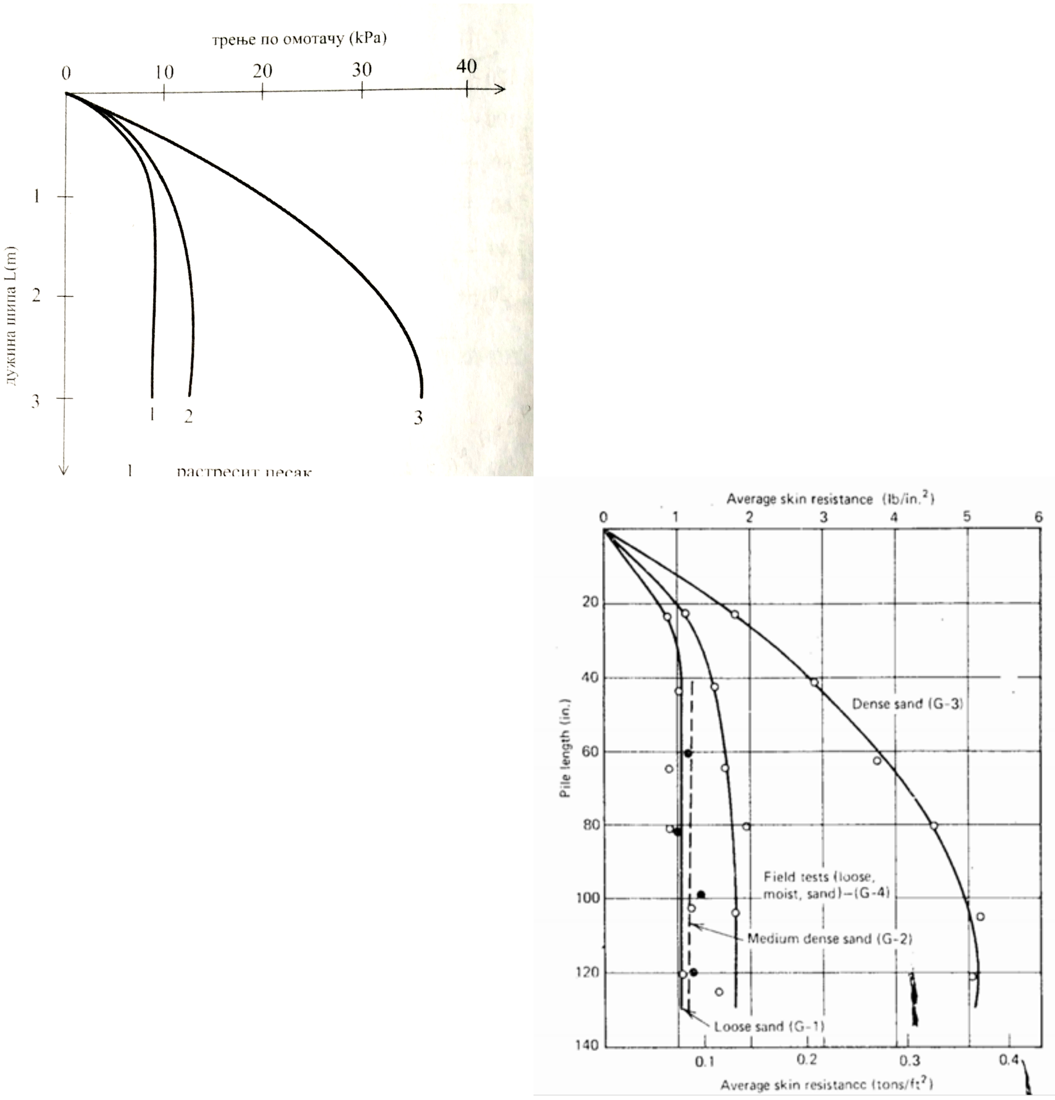

Figure 6. Values of pile friction shaft in function of installation depth, and soil density: 1 loose sand, 2 medium dense sand and 3 dense sand (left after Milović [3], right after Vesić [8])

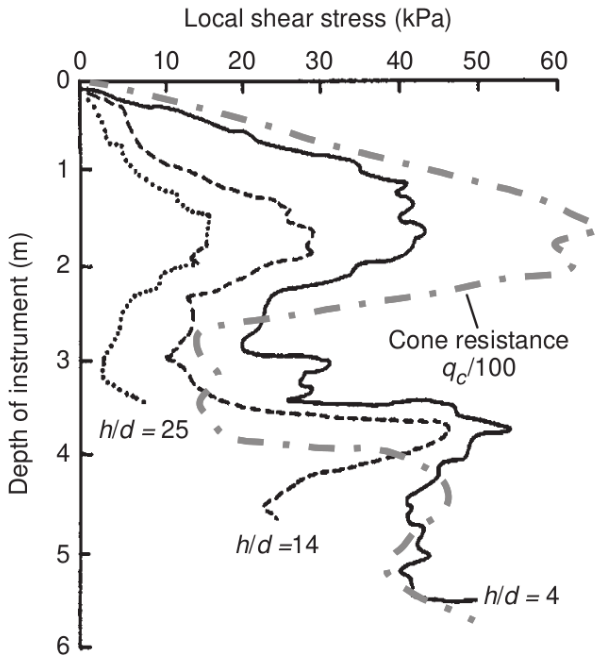

Figure 7. Measured profiles of shaft friction. (Lehane et al., 1993, [7])

The phenomenon of 'friction degradation' is illustrated in Fig. 7 with profiles of shaft friction measured in the three instrument clusters at different distances from the pile, as it is jacked into the ground. The phenomenon of pile group efficiencies in sand, for 4 and 9 piles, is shown in Fig 8. Another example can be seen in [1], [3], [4], [5], [7] and [8].

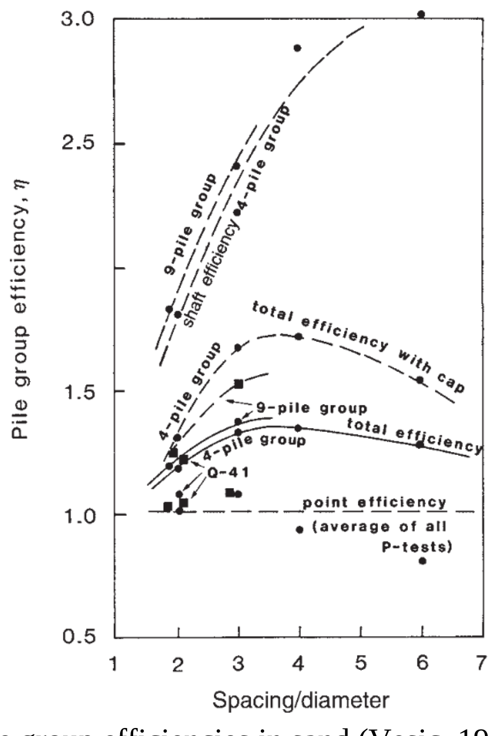

Figure 8. Pile group efficiencies in sand (Vesic, 1969, after [7]).

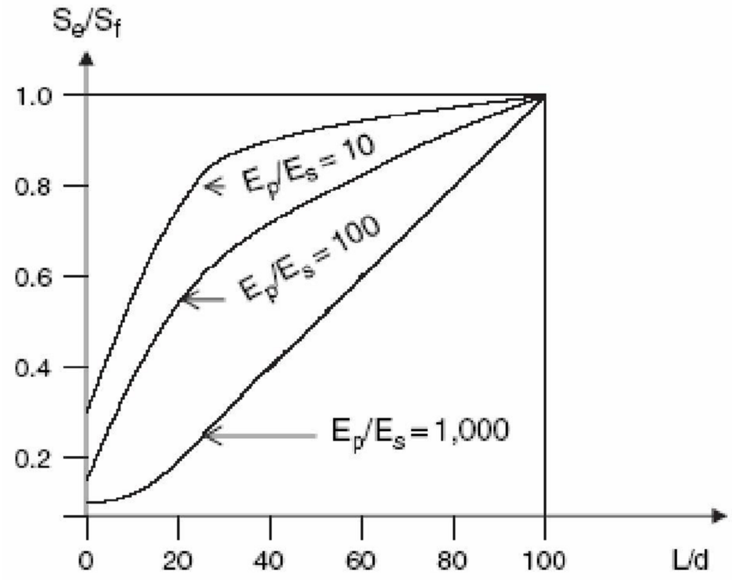

Fig. 7 Pile settlement graph (Poulos, HG, and Davis, E.H 1980, [1])
S_e = Settlement of end bearing pile; S_f=Settlement of a floating pile

Fig. 7 shows the relationship between settlement of the end bearing pile over the settlement of a floating pile (S_e/S_f), and a dimensionless length of pile (L/d), for different coefficients of pile modulus over soil modulus (E_p/E_s). For relatively large value of those coefficients, such as modulus pile/soli 1,000 (for example: E_p=20GPa, E_s=20MPa) for length greater than 20d, dependency function becomes a straight line. Corresponding ratio for S_e/S_f, for 20d shows that settlement of end bearing piles is 5 times smaller than that of floating one, and that by increasing the length up to 100d, these settlements get equalized.

4. Conclusions and final remarks

Based on the numerical research of the arrangement of the piles in the 3x3 group, where the distance between the piles is 4D, there occur the negative coefficients of interaction, because, in fact, the maximum distance between the piles on the side of the raft is 8D. Also, based on the numerical research of the authors of this paper, it is determined that for the constant, linear and parabolic soil, the mean value of the defection of piles, zero and final iteration is between 1 and 12%. On this basis, one can adopt with a relatively small error the first iteration, for which the deflections of all piles are equal to the mean deflection of the zero iteration, to be the final iteration. The zero iteration provides the arrangement of deflections of each individual pile, when each of the piles is loaded by the same force.

A simplified formula of interaction can be applied on the constant modulus of soil, where the mutual effects of the piles depend exclusively on the logarithm of dimensionless distance (expressed as a quotient of distance and pile diameter). Therefore, this procedure can be implemented in the layered soil, if the variation of properties of soil among the layers is minor, fig.5.

Acknowledgements

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