Error estimates for Gaussian quadrature of analytic functions

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ABSTRACT

We studied the error bound of Gaussian quadrature for analytic functions. The basic idea is to express the remainder of Gaussian quadrature as a contour integral, then the error bound is reduced to find the maximum of the kernel function:

$$K_n(z;\omega) = \frac{\varrho_n(z;\omega)}{\pi_n(z)}, \quad \varrho_n(z;\omega) = \int_{-1}^{1} \frac{\pi_n(t)}{z-t}\,dt, \quad z \in \mathbb{C}\backslash[-1,1]. \qquad (1)$$

The integral representation of the error term leads directly to the error bound

$$|R_n(f)| \le \frac{l(\Gamma)}{2\pi}\left(\max_{z\in\Gamma}|K_n(z)|\right)\left(\max_{z\in\Gamma}|f(z)|\right), \qquad (2)$$

where $l(\Gamma)$ represents the length of the chosen contour Γ.

We studied the estimates (2) for various weight functions with respect to this particular Γ.

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