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WAVE PROPAGATION IN PERIODIC TIMOSHENKO BEAMS ON DIFFERENT ELASTIC FOUNDATION TYPES

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Abstract:

Here, the transfer matrix method along with the Bloch theorem are employed to study the frequency spectra in various cases of Timoshenko's beam periodicity, such as structural, material and boundary periodicity. We additionally compare the location and size of band gaps, and the shape of the band structure in general for different types of foundation layers and relevant parameter values.

Key words: elastic wave, Timoshenko beam theory, transfer matrix method, band structure

1. Introduction

Wave propagation in periodic structures is an ongoing area of scientific research. It has been shown that these phononic-like periodic structures exhibit some very interesting and useful phenomena, such as the appearance of band gaps in the frequency spectrum. A periodic Timoshenko beam is examined and its representative cell is taken into consideration. Various types of periodicity are applied: material, geometric and boundary periodicity. We consider one layer in the cell as a uniform beam and we apply Timoshenko's beam theory to describe its transverse motion [1]. Further on, the transfer matrix method and Bloch's theorem are applied in order to study the frequency spectrum [1], [2]. Thus, we state the eigenvalue problem which can be solved by using the so-called "direct" method [3]. By visualizing the frequency band structure and mode shapes, we have achieved a very useful insight into the behavior of such structures, which can be conveniently applied for vibration attenuation without using active control strategies [4]. The transfer matrix method provides any required number of exact results using only one cell, and the order of the global matrix is independent on the number of cells into which the structure is divided, which leads to decrease of computational effort, but suffers from non-physical numerical instability under certain conditions [5].

2. Problem formulation and methodology

When material periodicity is considered, the cell is divided into two layers made out of different materials, while their other aspects remain the same. The geometric periodicity is achieved by varying the cross section of a homogenous beam, thus, a representative cell is consisted of two layers with different cross sections. Finally, three types of boundary periodicity are studied: elastic foundation, a series of suspended masses and elastic foundation with inerters whose reaction force is given as $F = b(\ddot{w} - \ddot{w}_0)$, where b is the coefficient of proportionality and w is transverse displacement. The transfer matrix method is applied to each layer in a cell, which is considered to be one uniform Timoshenko beam, thus relating its transverse displacements, rotation angles, forces and moments, which form a state vector $\mathbf{\Phi}(x)$, where x is the longitudinal dimension of the beam. The transfer matrix for one layer is obtained by relating its left and right boundaries, and the general transfer matrix **T** is formed subsequently. The boundary periodicity is applied by taking into consideration the shear force at the right end of the unit cell, which has been shown in detail in [1]. After applying the Bloch's theorem to relate the adjoining cells, we state and solve the required eigenvalue problem:

$$\mathbf{T}(\Omega)\mathbf{\Phi} = \lambda\mathbf{\Phi}, \ \lambda = e^{i\kappa} \tag{1}$$

where $\mathbf{T}$ is the transfer matrix given as a function of dimensionless frequency Ω, κ is the dimensionless wavenumber, and λ is a Floquet multiplier. The eigenvalue problem (1) is solved by imposing frequency values (the so-called "direct" method [3]), and thus the values of Floquet multipliers are provided. From these results we can determine real and imaginary parts of the wavenumber and evaluate this propagation constant as a function of frequency.

3. Some results and discussion

The representative unit cell for the case of boundary periodicity which is achieved through the application of an elastic layer with inerters is shown in Fig. 1a, while corresponding band structure is given in Fig. 1b, compared with the uniform beam and the case with bare and inerter-based elastic foundation. The results from Fig. 1b show the imaginary (below zero) and real part (above zero) of dispersion curves and demonstrate the existence of many band gaps in such a continuous system. The analogues of optical and acoustic branch of the simple phononic system can be recognized in the higher frequency ranges as well. By introducing the inerter-based foundation, the band gaps and corresponding dispersion branches shift to lower frequency values and slightly change their shapes due to the inertia amplification effect.

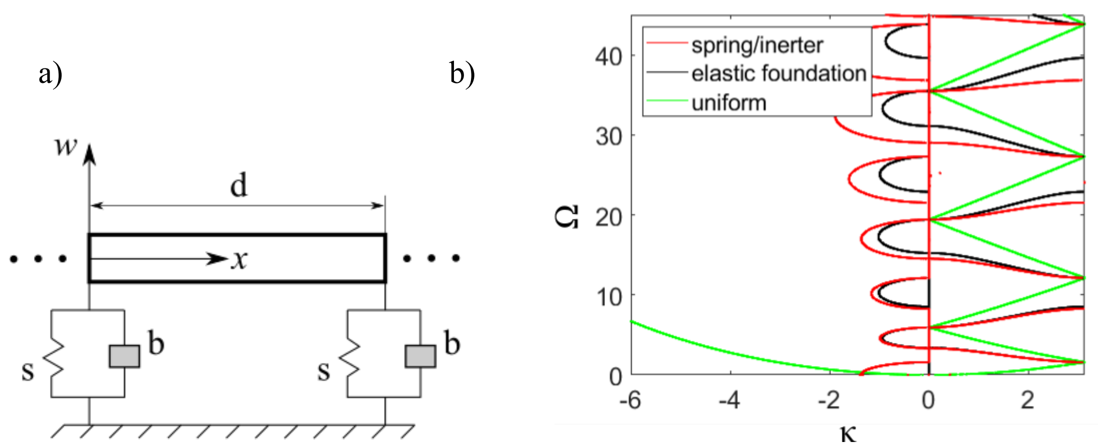

Fig. 1. a) Illustration of a unit cell model and b) Band structures of Timoshenko beam resting on different foundations.

Conclusion

In this work, the analysis of wave propagation in Timoshenko periodic beams resting on different foundation types is performed with the aid of transfer matrix method. The obtained results reveal a significant effect of different periodicity and foundation types on the band structure of the system. The most pronounced effect of the inerter-based elastic foundation reflects in the reduction of the frequency of band gaps and certain dispersion branches, especially those in the higher frequency range.

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References

[1] Liu L., Hussein I.,M., *Wave Motion in Periodic Flexural Beams and Characterization of the Transition Between Bragg Scattering and Local Resonance*, Journal of Applied Mechanics, Vol. 79, 011003, 2012.
[2] Gangli C., Xiaoyun Z., Xingbao L., Xiaoting R., *Transfer Matrix Method for the Free and Forced Vibration Analyses of Multi-step Timoshenko Beams Coupled with Rigid Bodies on Springs*, Applied Mathematical Modelling, Vol. 87, 152-170, 2020.
[3] Hussein I.,M., Leamy M.,J., Ruzzene M., *Dynamics of Phononic Materials and Structures: Historical Origins, Recent Progress, and Future Outlook,* Applied Mechanics Reviews, Vol. 66(4), 040802, 2014.
[4] Santos B.,R., Berres G., Inman J.,D., Gonzales-Bueno G.,C., Bueno D., Douglas, A practical approach to evaluate periodic rods composed of cells with geometric and material periodicity, Journal of Sound and Vibration, Vol. 533, 117646, 2023.
[5] Lee W.,J., Lee Y.,J., Free vibration analysis of functionally graded Bernoulli-Euler beams using an exact transfer matrix expression, International Journal of Mechanical Sciences, Vol. 122, 1-17, 2017.