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Predicting fatigue crack growth in complex geometries using numerical methods

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Abstract

Fatigue cracks growth in three different wing spar geometries is simulated using xFEM, as the most suitable numerical method for this purpose. The goal of this analysis was to find out how crack paths affect number of cycles, i.e. fatigue life, so that the optimal geometry can be defined. The extended finite element method was applied using Morfeo/Crack for Abaqus software. Results provided better understanding and prediction of multiple cracks propagation in complex 3D structures and provided data for the optimal design of wing spar.

Keywords: Fatigue crack growth, xFEM, wing spar, crack path

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1. Introduction

In recent decades, fracture mechanics parameters became essential for the prediction of crack initiation and propagation, i.e. for fatigue life estimation. The stress intensity factor (SIF) is one of the most important parameter which provides data on crack initiation and propagation. In complex geometries, such as wing spar, it is only natural

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to use numerical methods for any kind of analysis regarding crack initiation and/or growth, including fatigue (Khalid E. et al. 2018, Petrašinović D. et al. 2012, Božić Ž. et al. 2017, Shin, W. et al. 2021 and Naderi M. and Iyyer N. 2015).

Different numerical techniques, finite element method (FEM), boundary element method (BEM), mesh-free methods and extended finite element method (XFEM), have been developed to simulate the fracture mechanics problems (Sedmak A. 2018 and Grbovic A. and Rašuo B. 2015). In XFEM, crack growth is modelled by adding discontinuous functions into standard finite element approximation, enabling simple application of standard FEM software (Belytschko T. and Black T. 1999 and Moës N. et al. 1999). Sukumar N. et al. 2000, presented an improvement of XFEM, which enabled modelling of the three-dimensional crack and calculation of its SIFs. Jovicic G. et al. 2010, proposed similar improvement.

The XFEM has successfully been used to calculate SIFs for problems involving multiple, interacting cracks, resulting from multiple site damage (MSD). Aldarwish M. et al. 2017, conducted SIFs calculations based on implementation of XFEM in Abaqus for a typical problem with MSD. Fatigue crack growth in welded skin-stringer panel was also analyzed using XFEM (Sghayer A. et al. 2017 and Sghayer A. et al. 2018). Similar approach was used to analyze fatigue crack growth in friction stir welded T joints under three-point bending by Kredegh A. et al. 2017[a] and Kredegh A. et al. 2017[b], and to verify the SIFs solutions calculated by proposed approximation method, based on superposition, (Kastratović G. et al. 2015).

Here, some aspects of numerical simulation of possible crack paths in wing spar under amplitude load are presented, including 3 different wing spar geometries previously analyzed: the existing differential spar, the integral spar with same dimensions and redesigned integral spar, (Khalid E. et al. 2018, Petrašinović D. et al. 2012, Grbovic A. et al. 2019[a] and Grbovic A. et al. 2019[b]). Redesigned integral spar was established by optimization of 3 different cross-sections in respect to fatigue life. Here, special attention was paid to the analysis of cracks growth and their paths along the spar in each case. All computations for crack propagation simulation and fatigue life estimation were carried out by XFEM, using Morfeo/Crack for Abaqus software.

The Paris law model has been employed for the evaluation of the fatigue life for the compact tension specimen (CTS) crack where closed-form solution for SIF exists, as well as for cracks in typical aerospace structure where there are no closed-form solutions.

1.1. Stress intensity factor calculation and fatigue crack growth simulation

Abaqus uses the interaction integral, (Grbovic A. et al. 2019[c]), to perform the stress-intensity factors (SIFS) calculation:

$$I_0 = -\int_V q_{i,j}\left[\sigma_{kl}\varepsilon_{kl}^{aux}\,\delta_{ij} - \sigma_{kj}^{aux}\,u_{k,i} - \sigma_{kj}\,u_{k,i}^{aux}\right]dV \Big/ \int_S \delta q_n\,dS \tag{1}$$

where: σ_{ij}, ε_{ij}, u_i are stress, strain and displacement respectively, ${\sigma_{ij}}^{aux}$, ${\varepsilon_{ij}}^{aux}$, ${u_i}^{aux}$ are stress, strain and displacement of the auxiliary field, and q_i is crack extension vector. The interaction integral is associated with the stress intensity factors as follows:

$$I = \frac{2}{E^*}\left(K_1K_1^{aux} + K_2K_2^{aux}\right) + \frac{1}{\mu}K_3K_3^{aux} \tag{2}$$

where: K_i and K_i^{aux} are Mode *i* and auxiliary Mode *i* SIFs, $E^* = E/(1-\nu^2)$, μ shear modulus. Now, typical fatigue crack-growth law formulates the crack-extension increment as function of stress-intensity factor K and stress ratio R:

$$\frac{da}{dN} = f(K,\,R) = C\Delta K^m \tag{3}$$

where $\Delta K = (1-R)\cdot K_{max}$. For mixed-mode fatigue crack growth, an equivalent stress intensity factor range is used:

$$\Delta K_{eqv} = \frac{1}{2}cos\left(\frac{\theta}{2}\right)\left[\Delta K_I(1+cos\theta) - 3\Delta K_{II}(sin\theta)\right] \tag{4}$$

where angle θ (i.e., the direction of propagation) is calculated using the equation:

$$\theta = cos^{-1}\left(\frac{3\left(K_{II}^{max}\right)^2 + K_I^{max}\sqrt{\left(K_I^{max}\right)^2 + 8\left(K_{II}^{max}\right)^2}}{\left(K_I^{max}\right)^2 + 9\left(K_{II}^{max}\right)^2}\right) \tag{5}$$

Walker equation, Forman equation, NASGRO equation and user's own crack-propagation law based on data points can be used, too.

2. Differential wing spar under variable load

First, numerical analysis was carried out and verified, using the experimental data for the first wing spar geometry, which was presented in details in studies by Khalid E. et al. 2018, Petrašinović D. et al. 2012, Grbovic A. et al. 2019[a] and Grbovic A. et al. 2019[b]. Here, we just refer to Fig. 1 to illustrate experimental findings about fatigue crack growth after 58,520 cycles, as shown previously mentioned studies (Khalid E. et al. 2018, Petrašinović D. et al. 2012, Grbovic A. et al. 2019[a] and Grbovic A. et al. 2019[b]). Fig. 2 shows results of numerical simulation by XFEM, as used to simulate the experiment. Results of similar simulation are presented and analyzed in research by Grbovic, A. et al. 2019[c], proving that numerical simulation is in excellent agreement with the experimental results. Here we present results for numerical simulation of alternative crack paths, indicating significantly larger number of cycles (Figs. 3 and 4).

Fig. 1. Cracks on the wing spar in experiment after 58,520 cycles, a) overall appearance; b) first crack; c) second crack.

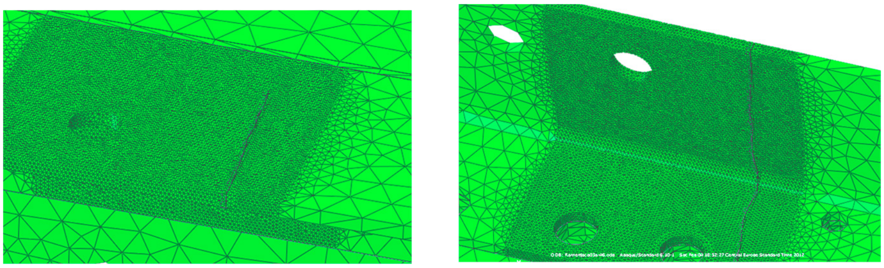

Fig. 2. Cracks on the wing spar in numerical analysis after 58,694 cycles, a) overall appearance; b) first crack; c) second crack.

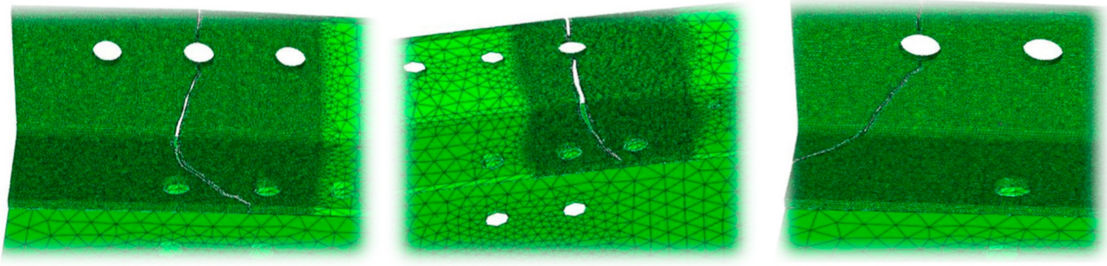

Fig. 3. Cracks on the wing spar in numerical analysis after a) 583,376; b) 6,258,020; c) 68,945,700

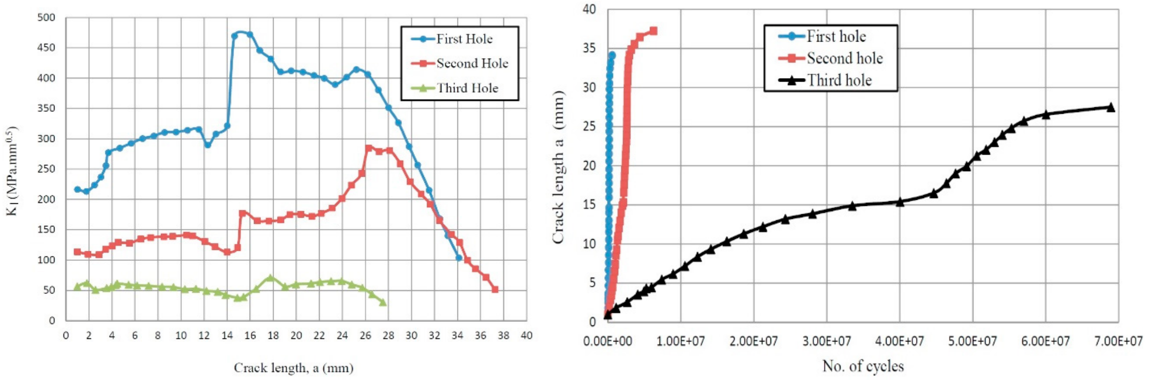

Fig. 4. SIFs (left) and number of cycles (right) for cracks starting from 3 different holes

3. Integral wing spar design improvement

Three different geometries of the integral wing spar were analyzed to define the optimal one, as shown in researches conducted by Grbovic A. et al. 2019[a] and Grbovic A. et al. 2019[b]: the I-section integral spar (case A), U-section spar (case B) and I-section integral spar with intermediate flange (case C), Fig. 5. Design was improved by maximizing the moment of inertia at constant cross-section area, as schematically shown in Fig. 6. Crack paths for 3 different geometries are shown in Fig. 7, providing approximately 345,000 cycles in the case A and C1, 250,000 in the case B. Crack length vs. number of cycles is given in Figure 8, also including two other options of case C, as explained in researches by Grbovic A. et al. 2019[a] and Grbovic A. et al. 2019[b], providing as high as 460,000 cycles in the case C2 and 1,380,000 cycles in the case C2. These 3 geometries (C1, C2 and C3) differ only in dimensions, as defined in Table 1.

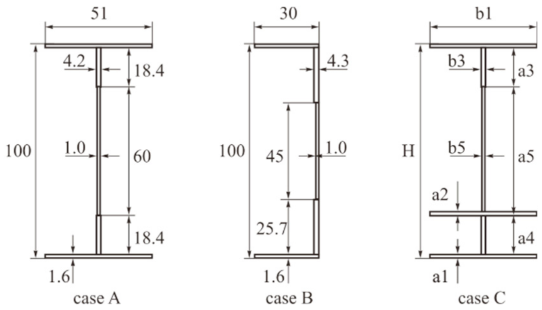

Fig. 5. Three analysed cross-sections of the integral wing spar

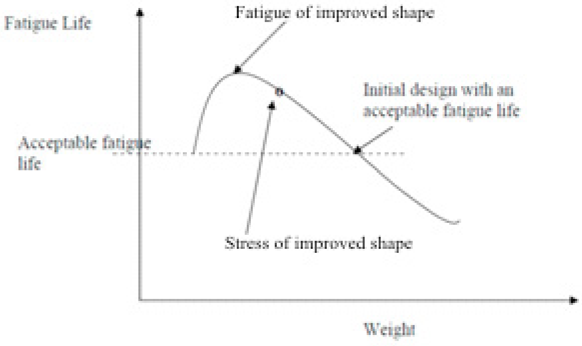

Fig. 6. Fatigue life vs. weight

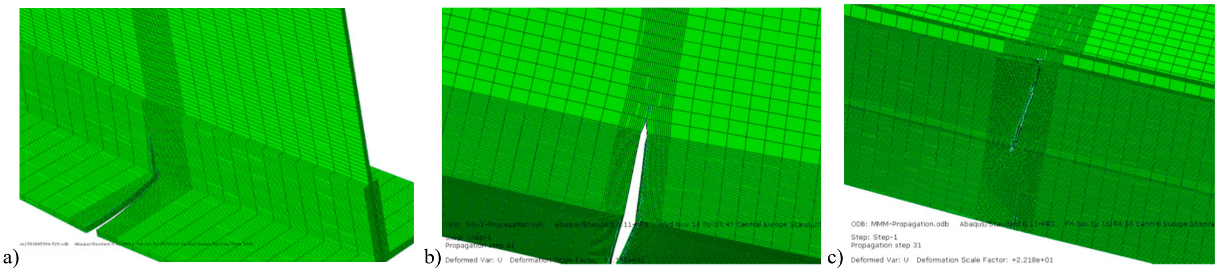

Fig. 7. Crack growth in 3 different geometries of integral spar, a) case A, b) case B, c) case C

Table 1. The overall dimensions for I-section spar with intermediate cap

Case No.	a1 [mm]	b1 [mm]	a2 [mm]	a3 [mm]	b3 [mm]	a4 [mm]	a5 [mm]	b5 [mm]	H [mm]
Case C1	1.6	41	1.6	13.4	4.2	23.4	58.4	1	100
Case C2	1.6	54.7	1.6	5	3	5	85.2	1	100
Case C3	1.6	53.6	1.6	5	3	5	90.2	1	105

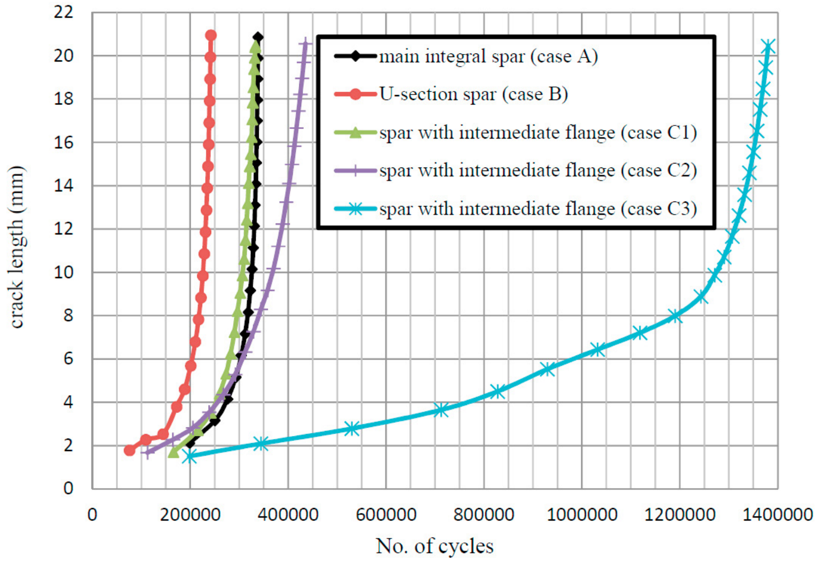

Fig. 8. Crack length vs. Number of cycles for cases A, B and C1-C3

4. Discussion and conclusions

Verified numerical model of the differential wing spar enabled prediction of the most probable crack path. The highest SIFs values and the lowest number of cycles were obtained for the crack emanating from the first hole (Fig. 4) indicating this crack to be the most probable one to occur and propagate. The position of this crack was then used in the model of the integral wing spar for fatigue life estimation.

Thanks to the conducted numerical simulations it was easy to acknowledge that the integral spar can provide scientifically longer fatigue life. Furthermore, analyses of the three different geometries of the integral spar enabled improvement of its design, regarding fatigue life. According to the results presented in Fig. 8 the far longest fatigue life can be achieved by usage of the intermediate flange and minimal increase of the wing spar hi. This can be explained by the fact that the results of the analyses showed similar crack paths for cases A and B, where crack reached deep in the vertical spar wall (web), diminishing drastically the structural integrity of the wing spar. On the other hand, results in case C showed the crack propagation thru bottom flange, while web and intermediate flange remained intact, suggesting that intermediate flange changed the crack path in such a way that even when the bottom flange fails, web and the intermediate flange will remain intact, which consequently leads to increase in fatigue life.

Based on the presented results, one can conclude the following:

- xFEM is suitable method for numerical simulation of fatigue crack growth. It is simple enough and provided good agreement with the experimental results
- geometry of wing spar cross-section has a significant effect on number of cycles. The optimal geometry increases number of cycles approximately 4 times, providing safe life of a wing spar.
- More numerical simulations by xFEM are needed to get more detailed insight into different aspects of this powerful numerical method.

5. Acknowledgements

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References

Khalid E., Grbović, A., Sedmak A., Kastratović G., Petrašinović D., Sedmak S., 2018, Fatigue Life Estimation of Damaged Integral Wing Spar Using XFEM, Technical Gazette 25(6), 1837-1842

Petrašinović D., Rašuo B., Petrašinović N., 2012, Extended finite element method (XFEM) applied to aircraft duralumin spar fatigue life estimation, Technical Gazette 19(3), 557-562

Božić Ž., Schmauder S., Wolf H., 2017, The effect of residual stresses on fatigue crack propagation in welded stiffened panels, Engineering Failure Analysis 84, 346-357.

Shin, W., Kyong-Ho, C. Muzaffer, S., 2021, Fatigue analysis of cruciform welded joint with weld penetration defects, Engineering Failure Analysis 120(16):105111

Naderi M., Iyyer N., 2015, Fatigue life prediction of cracked attachment lugs using XFEM. International Journal of Fatigue, 77, 186-193.

Sedmak A., 2018, Computational fracture mechanics: An overview from early efforts to recent achievements, Fatigue & Fracture of Engineering Materials & Structures, 41, 2438-2474.

Grbovic A., Rašuo B., 2015, Use of modern numerical methods for fatigue life predictions, Chapter 2, Recent Trends in Fatigue Design, Nova Science Publishers, New York.

Belytschko T., Black T., 1999, Elastic crack growth in finite elements with minimal remeshing, International Journal for Numerical Methods in Engineering, 45, 601–620.

Moës N., Dolbow J., and Belytschko T., 1999, A finite element method for crack growth without remeshing, International Journal for Numerical Methods in Engineering 46, 131–150.

Sukumar N., Moës N., Moran B., Belytschko T., 2000, Extended finite element method for three-dimensional crack modelling, International Journal for Numerical Methods in Engineering 48, (11), 1549-1570.

Jovicic G., Zivkovic M., Jovicic N., Milovanovic D., Sedmak A., 2010, Improvement of algorithm for numerical crack modelling, Archives of Civil and Mechanical Engineering 10(3), 19-35.

Aldarwish M., Grbović A., Kastratović G., Sedmak A., Vidanović N., 2017, Numerical Assessment of Stress Intensity Factors at Tips of Multi-Site Cracks in Unstiffened Panel, Structural Integrity and Life 17, 11-14.

Sghayer A., Grbović A., Sedmak A., Dinulović M., Doncheva E., Petrovski B., 2017, Fatigue Life Analysis of the Integral Skin-Stringer Panel Using XFEM, Structural Integrity and Life, 17, 7-10.

Sghayer A., Grbović A., Sedmak A., Dinulović M., Grozdanovic I., Sedmak S., Petrovski B., 2018, Experimental and numerical analysis of fatigue crack growth in integral skin-stringer panels, Technical Gazete, 25(3), 785-791.

Kredegh A., Sedmak A, Grbovic A., Sedmak S., 2017, Stringer effect on fatigue crack propagation in A2024-T351 aluminum alloy welded joint, International Journal of Fatigue 10, 276-282.

Kraedegh A., Li W., Sedmak A., Grbović A., Trišović N., Mitrović R., Kirin S., 2017, Simulation of Fatigue Crack Growth in A2024-T351 T Welded Joint, Structural Integrity and Life 17 (1), 3-7.

Kastratović G., Grbović A., Vidanović N., 2015, Approximate method for stress intensity factors determination in case of multiple site damage, Applied Mathematical modeling 39 (19), 6050-6059.

Grbovic A., Kastratovic G., Sedmak A., Balac I., Popovic M., 2019, Fatigue crack paths in light aircraft wing spars, International Journal of Fatigue 123, 96-104

Grbović, A., Kastratović, G., Sedmak, A., 2019, Design aspects of light aircraft wing spar—Differential and integral approach, Material Design & Processing Communications 1, e105. https://doi.org/10.1002/mdp2.105

Grbovic, A., Sedmak, A., Kastratovic, G., Petrasinovic, D., Videnovic, N., Sghayer, A., 2019, Effect of laser beam welded reinforcement on integral skin panel fatigue life, Engineering Failure Analysis 101(3), 383-393.