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## A COMPUTATIONAL ANALYSIS OF OLD RC STRUCTURES STRENGTHENED BY TIES UNDER REMOVAL OF MEMBERS

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### ABSTRACT

In existing old reinforced concrete (RC) buildings, some structural element-members, e.g. columns, are sometimes removed due to various reasons. These reasons can concern changes of serviceability and requirements, or environmental effects, e.g. corrosion, earthquakes, which cause strength degradation etc. In such cases, a modification of the structural response and a redistribution of internal actions can result to a requirement for strengthening the remaining structure. The present study deals with such a case, which concerns the computational analysis of framed RC structures under the removal of some columns and the so-induced requirement of a strengthening by cable-ties (tension only elements). These cable-ties can undertake tension but buckle and become slack and structurally ineffective when subjected to a sufficiently large compressive force. Thus the problem governing conditions take an equality as well as an inequality form and the problem becomes a high non-linear one. The problem of structures containing as above cable-like members belongs to the so-called Inequality Problems of Mechanics, as their governing conditions are of both, equality and inequality type. A realistic numerical treatment of such problems can be obtained by mathematical programming methods (optimization algorithms). Here, and taking into account all the non-linearities concerning the cable and the RC elements behavior, a computational approach based on an incremental problem formulation is appropriately applied. Finally, an application is presented for a simple typical example of an industrial RC frame strengthened by bracing ties after the removal of some ground floor columns.

**Keywords:** *Old RC structures; computational structural mechanics; removal of columns; strengthening by ties.*

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## 1. INTRODUCTION

Existing old reinforced concrete (RC) structures are subjected to various environmental actions, e.g. corrosion, earthquakes etc., which can often cause significant damages. A main such defect is the strength degradation, resulting into a reduction of the loads bearing capacity of some structural elements. For some of such degraded elements is sometimes obligatory to be removed, and so a further reduction of the whole structure capacity is caused, which can lead to progressive collapse.

To overcome such defects, strengthening of the remaining structure is suggested, mainly after structural assessment. Among the available strengthening methods [1-4], cable-like members (tension-only tie-elements) can be used as a first strengthening and repairing procedure [5]. Cables can undertake tension but buckle and become slack and structurally ineffective when subjected to a sufficiently large compressive force. Thus the governing conditions take an equality as well as an inequality form and the problem becomes a high non-linear one. So, the problem of structures containing as above cable-like members belongs to the so-called Inequality Problems of Mechanics, as their governing conditions are of both, equality and inequality type [6-10]. A realistic numerical treatment of such problems can be obtained by mathematical programming methods (optimization algorithms).

The aim of this paper is to deal with a numerical approach for the analysis of existing old industrial framed RC buildings, which are strengthened by cable elements in order to overcome capacity reduction after the obligatory removal of some degraded structural elements. The computational approach is based on an incremental problem formulation. Finally, an application is presented for a simple typical example of an industrial RC frame strengthened by bracing ties after the removal of some ground floor columns.

## 2. METHOD OF ANALYSIS

A reinforced concrete (RC) framed structure containing cable-like members is considered. For the general analysis of such a structure, see details as described in [5]. Generally, a double discretization is applied: in space by finite elements and in time by a direct time-integration method. The RC structure is discretized to frame elements with generally non-linear behavior. For the cables, pin-jointed bar elements with unilateral behavior are used. The rigorous mathematical investigation of the problem can be obtained by using the variational or hemivariational inequality concept, see Panagiotopoulos [9,10]. So, the behavior of the cables and the generally non-linear behavior of RC elements, including loosening, elastoplastic or/and elastoplastic-softening-fracturing and unloading - reloading effects, can be expressed mathematically by the subdifferential relation:

$$s_i(d_i) \in \hat{\partial} S_i(d_i) \quad (1)$$

Here, for the example case of a typical  $i$ -th cable element,  $s_i$  and  $d_i$  are the (tensile) force and the deformation (elongation), respectively,  $\hat{\partial}$  is the generalized gradient and  $S_i$  is the superpotential function [9,10].

For the numerical treatment of practical inequality problems, a piece-wise linearization is usually applied to relation (1), see e.g. [5-8]. So, for the case of cables, the unilateral behavior of the  $i$ -th cable-element ( $i = 1, \dots, N$ ) is expressed by the following relations [5]:

$$e_i = F_{0i} \cdot s_i + e_{i0} - v_i, \quad (2a)$$

$$s_i \geq 0, \quad v_i \geq 0, \quad s_i v_i = 0. \quad (2b)$$

Here  $e_i$ ,  $F_{0i}$ ,  $s_i$ ,  $e_{i0}$  and  $v_i$  denote the strain (elongation), "natural" flexibility constant, stress (tension), initial strain and slackness, respectively. Relations (2b) consist the Linear Complementarity Conditions (LCC) and express that either a non-negative stress (tension) or a non-negative slackness exists on cables at every time-moment. The above considerations lead to formulate the problem and to solve it at every time-moment as a Linear Complementarity Problem (LCP). The numerical treatment of this LCP is obtained by using optimization methods [5-10].

In an alternative approach, the incremental dynamic equilibrium for the assembled structural system with cables is expressed in matrix form by the equation:

$$\underline{M} \cdot \Delta \underline{\ddot{u}}(t) + \underline{C} \cdot \Delta \underline{\dot{u}}(t) + \underline{K}_T \cdot \Delta \underline{u}(t) = \Delta \underline{p}(t) + \underline{T} \cdot \Delta \underline{s}(t) \quad (3)$$

Here  $\underline{u}(t)$  and  $\underline{p}(t)$  are the time dependent displacement and the given load vectors, respectively.  $\underline{C}$  and  $\underline{K}_T(\underline{u})$ , are the damping and the time dependent tangent stiffness matrix, respectively. Dots over symbols denote derivatives with respect to time.  $\underline{T}$  is a transformation matrix. By  $\underline{s}(t)$  is denoted the time dependent cable stress vector with elements satisfying the relations (1)-(2).

The above matrix equation combined with the initial conditions consist the problem formulation, where, for given  $\underline{p}(t)$ , the vectors  $\underline{u}(t)$  and  $\underline{s}(t)$  are to be computed. For the numerical treatment of the above problem, the structural analysis software Ruaumoko [11] is herewith used, as described in [5]. When the static case of the problem is only to be investigated, a Dynamic Relaxation approach [12] is appropriately used.

### 3. NUMERICAL EXAMPLE

The RC plane frame structure of Fig. 1 had been initially constructed with two more internal columns in the ground floor, which are shown as dashed lines and have been removed due to degradation caused by environmental actions. The axial loads, which were initially undertaken by these two columns, are now shown as the two applied vertical concentrated loads of 180 kN and 220 kN.

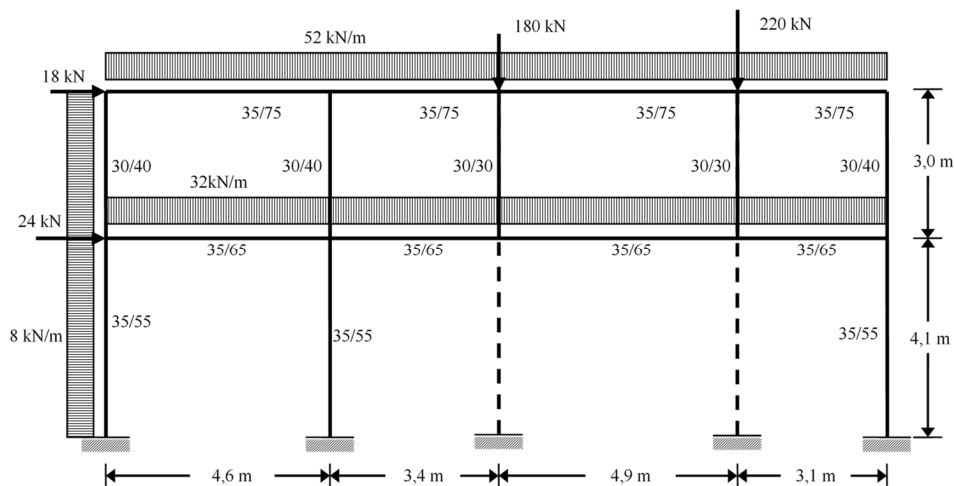


Fig. 1. The initial RC frame F0.

Due to removal of the above two columns, and after structural assessment [15] and in order to prevent a progressive collapse, the initial RC frame F0 of Fig. 1 is strengthened by ten (10) steel cables (tension-only tie-elements) as shown in Fig. 2. In the so formulated system, it is wanted to be computed which of the cables are activated and which are not, under the considered static loading of Fig. 1.

Using Ruaumoko software [11], the columns and the beams of the frame are modeled by prismatic frame RC elements. The concrete class is estimated to be C12/15. The effects of cracking on columns and beams are estimated by applying the guidelines of [1-4, 13]. So, the stiffness reduction due to cracking results to effective stiffness with mean values of  $0.60 I_g$  for the external columns,  $0.80 I_g$  for the internal columns and  $0.40 I_g$  for the beams, where  $I_g$  is the gross inertia moment of their cross-section.

Nonlinearity at the two ends of the RC frame structural elements is idealized by using one-component plastic hinge models, following the Takeda hysteresis rule [11]. Concerning the constitutive diagram of plastic hinges, a typical normalized moment- normalized rotation backbone [14] is shown in Fig. 3.

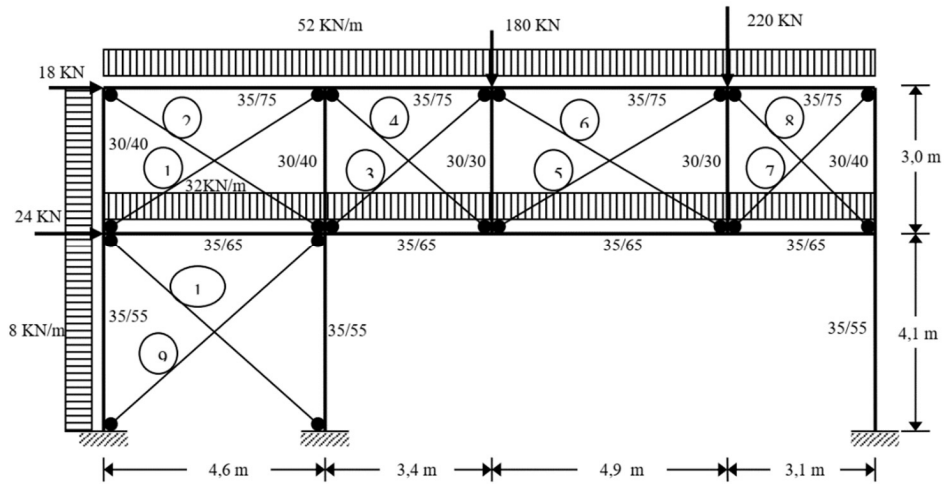


Fig. 2. The RC frame F1 (or F2) strengthened by 10 cables.

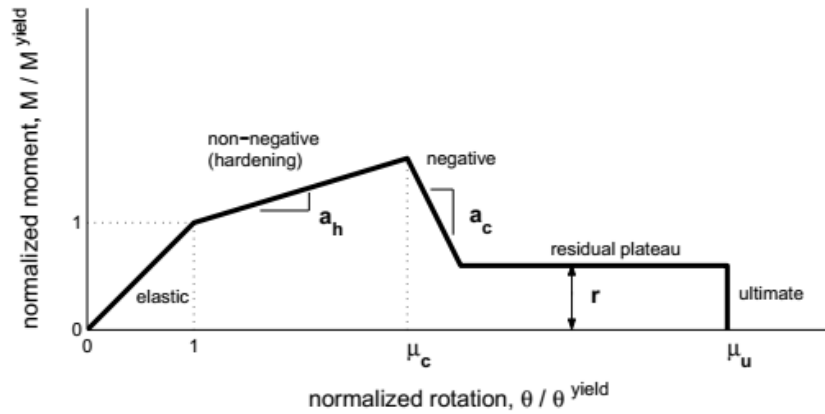


Fig. 3. Constitutive backbone diagramme for normalized moment - normalized rotation in plastic hinges [14].

The strengthening cable members have a cross-sectional area  $F_r = 8 \text{ cm}^2$  and are of steel class S1400/1600 with elasticity modulus  $E_s = 210 \text{ GPa}$ . The cable constitutive law concerning the unilateral (slackness), hysteretic, fracturing, unloading-reloading etc. hysteretic behavior, has the diagram depicted in Fig. 4.

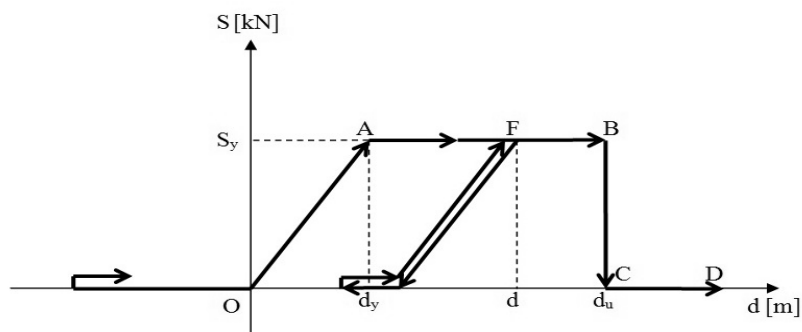


Fig. 4. Constitutive law of the cable-elements.

The application of the proposed numerical procedure by using a pseudo Dynamic Relaxation approach [12] gives the following results for the cable-elements:

- a. The values of the slackness of the no activated cable-elements are:

$$v_1 = 0.848 \cdot 10^{-3} \text{ m}, \quad v_3 = 10.321 \cdot 10^{-3} \text{ m}, \quad v_5 = 1.082 \cdot 10^{-3} \text{ m},$$

$$v_8 = 9.564 \cdot 10^{-3} \text{ m}, \quad v_{10} = 1.652 \cdot 10^{-3} \text{ m}.$$

- b. The elements of the stress vector  $\underline{s}$ , where:  $\underline{s} = [S_1, S_2, \dots, S_{10}]^T$ , are computed to have the following stress-values (in kN) for the non-active cables:

$$S_1 = S_3 = S_5 = S_8 = S_{10} = 0.0,$$

whereas for the active cables, the tension-values are:

$$S_2 = 10.17 \text{ kN}, \quad S_4 = 346.04 \text{ kN}, \quad S_6 = 18.84 \text{ kN},$$

$$S_7 = 342.08 \text{ kN}, \quad S_9 = 25.81 \text{ kN}.$$

Thus, cables 2,4,6,7 and 9 are the only ones which have been activated, appearing non-zero tension. The other cables 1,3,5,8 and 10 cannot contribute to the system resistance under the given loads of Fig. 1.

Obviously, by parametric investigation of the characteristics of the cable-element (sectional area, elasticity modulus etc.), a parametric upgrading investigation of the strengthened structure can be obtained. Such a parametric investigation is shown indicatively in the Table 1. This investigation concerns the response of the initial frame-structure F0 without cables ( $F_r = 0 \text{ cm}^2$ ) and when it is strengthened by cable members having a cross-sectional area either  $F_r = 5 \text{ cm}^2$  (frame F1) or  $F_r = 10 \text{ cm}^2$  (frame F2). By  $U_{y180}$  and  $U_{y220}$  are denoted the node vertical displacements under the single concentrated loads of 180 kN and 220 kN, respectively, (see Fig.1).  $M_1$  and  $M_2$  are the maximum bending moments of the first and the second floor-beams, respectively, between the deleted columns.

Table 1. Comparison of response representative values of the initial frame F0 and the strengthened frames F1 and F2.

Frame	Cables cross-section	$S_4$ [kN]	$S_7$ [kN]	$U_{y180}$ [cm]	$U_{y220}$ [cm]	$M_1$ [kNm]	$M_2$ [kNm]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
F0	$F_r = 0 \text{ cm}^2$	0.00	0.00	-2.46	-2.59	307.91	402.64
F1	$F_r = 5 \text{ cm}^2$	272.11	268.65	-1.67	-1.70	218.22	290.98
F2	$F_r = 10 \text{ cm}^2$	401.30	412.89	-1.26	-1.27	174.59	236.37

#### 4. CONCLUDING REMARKS

The proposed computational approach can be effectively used for the numerical investigation concerning the inelastic behaviour of existing RC framed-structures, which are subjected to removal of some degraded structural elements and strengthened by cable elements. By parametric investigation of the characteristics of the cable-elements (sectional area, elasticity modulus etc.), the required upgrading of the remaining structure can be obtained.

## REFERENCES

- [1] Penelis, G., Penelis, Gr. (2014). *Concrete Buildings in Seismic Regions*. CRC Press, Taylor and Francis Group Ltd, London.
- [2] Penelis, G., Kappos, A. (2010). *Earthquake Resistant Concrete Structures*. CRC Press, London.
- [3] Fardis, M.N. (2009). *Seismic design, assessment and retrofitting of concrete buildings: based on EN-Eurocode 8*. Springer, Berlin.
- [4] Dritsos, S.E. (2017). *Repair and strengthening of reinforced concrete structures*, (in greek). University of Patras, Greece.
- [5] Liolios, A., Liolios, Ast., and Hatzigeorgiou, G., (2013). *A Numerical Approach for Estimating the Effects of Multiple Earthquakes to Seismic Response of Structures Strengthened by Cable-Elements*. Journal of Theoretical and Applied Mechanics, Vol.43, No. 3, pp. 21-32, 2013.
- [6] Nitsiotas, G., (1971). *Die Berechnung statisch unbestimmter Tragwerke mit einseitigen Bindungen*, Ingenieur-Archiv, vol. 41, pp. 46-60.
- [7] Maier, G., (1968). *A Quadratic Programming Approach for Certain Classes of Non-linear Structural Problems*. Meccanica, Vol. 3, pp. 121-130,.
- [8] Cottle, R.W. (1979). *Fundamentals of Quadratic Programming and Linear Complementarity*, in M.Z. Cohn, D.E. Grierson and G. Maier (Eds.), *Engineering Plasticity by Mathematical Programming*, Pergamon Press, Oxford.
- [9] Panagiotopoulos, P. (1993). *Hemivariational Inequalities. Applications in Mechanics and Engineering*. Springer Verlag, Berlin.
- [10] Mistakidis, E.S. and Stavroulakis, G.E. (, 1998). *Nonconvex optimization in mechanics. Smooth and nonsmooth algorithms, heuristic and engineering applications*. Kluwer, London.
- [11] Carr, A.J. (2008). *RUAUMOKO - Inelastic Dynamic Analysis Program*. Dep. of Civil Engineering, University of Canterbury, Christchurch, New Zealand.
- [12] Papadrakakis, M. (1981). *A method for the automatic evaluation of the dynamic relaxation parameters*. Computer Methods in Applied Mechanics and Engineering, 25(1), 35-48.
- [13] Pauley T. and Priestley, M.J.N., (1992). *Seismic design of reinforced concrete and masonry buildings*. Wiley, New York.
- [14] Vamvatsikos, D., & Fragiadakis, M. (2010). *Incremental dynamic analysis for estimating seismic performance sensitivity and uncertainty*. Earthquake Engineering & Structural Dynamics, vol. 39(2), pp. 141-163.
- [15] Strauss, A., Hoffmann, S., Wendner, R., & Bergmeister, K. (2009). *Structural assessment and reliability analysis for existing engineering structures, applications for real structures*. Structure and Infrastructure Engineering, 2009, vol. 5(4), pp. 277-286.