

INFLUENCE OF SPECIFIC ELECTRICAL RESISTANCE ON THE HEATING OF METAL PLATES EXPOSED TO HIGH-FREQUENCY ELECTROMAGNETIC WAVES

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Temperature distribution in the metallic plate influenced by high-frequency electromagnetic wave depends on thermal and electrical properties of the plate material. The goal of the paper is to present the significance of the temperature dependence of the electrical resistivity and its influence on the plate heating. A mathematical model of the problem was established and an analytical closed-form solution to the problem was presented. The active power of the electromagnetic wave was calculated by Poynting vector. Temperature field was obtained by integral-transform technique. Numerical examples were presented for three different materials (copper, aluminum, and steel). Calculated results show the large influence of temperature dependent electrical resistance on heating of the metallic plate.

Key words: electromagnetic wave, magnetic field strength, temperature, electrical resistance, numerical simulation

1. Introduction

The aim of the paper is to present the important influence of temperature dependent specific electrical resistance on temperature field of the plate made of electrically conductive materials, as copper, aluminum, and steel. If structures made of conductive materials are exposed to high-frequency electromagnetic waves, such as constructions in nuclear plants and aerospace constructions, and also in motors, generators, and inductors, the formation of conduction currents occurs due to the change in the electromagnetic field. That is why appropriate electrical characteristics of the material such as electrical conductivity or electrical resistance, which largely depend on temperature, must be taken into consideration. Depending on the strength of the electromagnetic wave, or on the electric and magnetic field intensities and wave frequency, significant changes in temperature may occur. In the mathematical modeling of various problems where relatively small temperature changes are expected, of the order of 100 K, the thermal, elastic, and electrical characteristics of the material can be considered constant. As a result, the values of the physical properties of the material that correspond to the average temperature of the problem can be used in the calculation. For larger temperature changes, the temperature dependence of these properties must be considered. To show the importance of the change in the physical properties of the material with respect to temperature, and to obtain the closed-form solution, the paper considers the heating of a thin plate of electrically conductive material under the influence of a plane

electromagnetic wave. Boundary conditions represent the plate thermally insulated on both sides and with constant temperature at the edges. In that case, small-sized plate would cool in a short period of time, so in this paper, calculations are presented for larger plates.

As electromagnetic radiation is both wave and heat, and heat is characterized by temperature as the ratio between energy and entropy, in Ref. [1] was presented the calculation of entropy and temperature of a single-mode electromagnetic radiation from the wave properties. The influence of the exerting magnetic field, at the surface of the magnetized cone, on its heat transfer properties was discussed in Ref. [2]. Corresponding coupled equations were numerically simulated. Mathematical model for the temperature field of a steel plate, influenced by high frequency induction heater, was set up and solved in Ref. [3]. Also, appropriate comparison of temperature history of numerical simulations and experimental tests had been discussed. Ref. [4] presents the closed-analytical solution of the temperature field for the problem with the moving heat sources. Distribution of the eddy-current power was obtained by use of complex analysis. Depending on the boundary conditions, thermal stresses appear in thermally loaded structures. That is why analysis of this type of problems must include both thermal and mechanical aspects. Modeling of thermal stresses in low alloy steels was provided in Ref. [5]. Also, heat capacity, heat conductivity and elastic properties for hypo-eutectoid steel were discussed. Boundary value problems of 2-D half space with different types of heating under gravity effects were presented in Ref. [6]. Numerically obtained results were presented with comparisons in the absence and the presence of the influence of the gravity and magnetic fields. Thermal switching of asymmetric transmission of linearly polarized terahertz waves was demonstrated in Ref. [7] numerically and experimentally. Physical mechanism based on simulated surface current distribution was also explained. The problem of electromagnetic wave absorption at extremely high temperatures for some special materials as the binary SiC was investigated in Ref. [8].

1.1. Description of the problem

A homogeneous, isotropic plate with dimensions $a \times b$ and small thickness h is placed perpendicular to the direction of propagation of the plane electromagnetic wave (axis x). The plate material is linear magnetic which is also a good conductor (Figure 1).

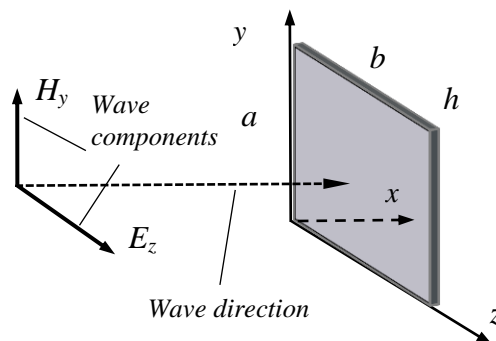


Figure 1 Position of the plate with respect to the direction of the electromagnetic wave

The analysis of electromagnetic waves is performed using Maxwell's equations (Ref. [9]). In the case of the high plate conductivity (copper, aluminum, steel) dielectric current can be neglected in

comparison with the conducting current. So, a system of Maxwell's equations for homogeneous, isotropic, and linear magnetic plate material has the following equations form (Ref. [10])

$$\begin{aligned} \text{rot } \vec{H} &= \frac{\vec{E}}{\rho_t}, \quad \text{div } \vec{E} = 0, \\ \text{rot } \vec{E} &= -\mu \frac{\partial \vec{H}}{\partial t}, \quad \text{div } \vec{H} = 0, \end{aligned} \quad (1)$$

in which \vec{E} and \vec{H} are vectors of electric and magnetic fields, ρ_t is specific electric resistance and μ is magnetic permeability of the material. Using by the method of Fourier analysis, electromagnetic wave with complex time changing field can be represented as a sum of simple plane waves. With a simple plane wave only normal components of the electric and the magnetic field depend on each other (Refs. [9,10]). For the clarity of presentation, only one plane wave with H_y and E_z components which vary in time t as $\exp(j\omega t)$, where ω is angular frequency, will be observed in the paper.

If H_0 is the strength of the magnetic field on the contact surface of the wave and the plate ($x=0$), using the symbolic-complex method given in details in Ref. [11], the active power P absorbed by the conducting plate per unit of its surface can be determined by the Poynting vector method and is as follows

$$P = \frac{H_0^2}{2} \sqrt{\pi f \mu \rho_t}, \quad (2)$$

in which f is the frequency of the observed wave. The amount of heat absorbed by the metal plate depends on the strength of the magnetic field, the frequency of the waves and the specific electrical resistance. The temperature T has the greatest influence on the change in the specific electrical resistance ρ_t because, with the increase in temperature, the speed of the thermal movement of electrons also increases, and thus the frequency of collisions. The corresponding diagram is shown in Figure 2 (Ref. [9]). At temperature between 10 K and 100 K, the specific electrical resistance is proportional to T^5 , while for temperatures from 100 K to the melting point, it can be described by an almost linear function, and can be represented by the following equation (Ref. [12])

$$\rho_t = \rho_0 [1 + \alpha_e (T - T_0)] \quad (3)$$

in which ρ_0 is the specific electrical resistance at temperature T_0 and α_e is the thermal coefficient of resistance.

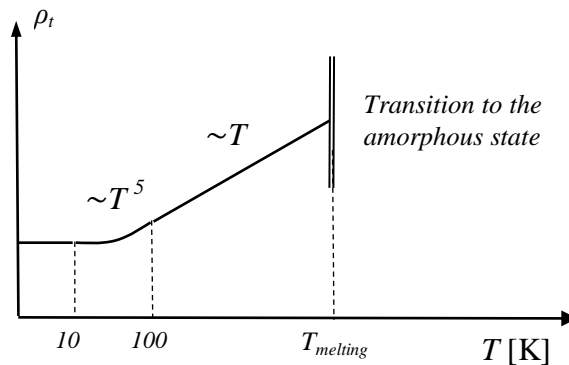


Figure 2 Specific electrical resistance with respect to temperature

In the reference [12] the thermal conductivity coefficient was studied for several types of copper wires and the effect of temperature on superconductors was discussed. Different publications studied temperature dependence of the electrical resistance and conductivity have presented in Ref. [13] as well as the history of discovering the superconductivity.

Three representative groups of electromagnetic materials were selected: aluminum as paramagnetic material, copper as diamagnetic and steel as ferromagnetic material. Magnetic permeability of vacuum is $\mu_0=4\cdot\pi\cdot 10^{-7} \text{ Hm}^{-1}$ and relative magnetic permeability is defined as the ratio $\mu_r=\mu/\mu_0$. The relative magnetic permeability of paramagnetics is slightly higher than 1, and of diamagnetics slightly lower than 1. The value of relative magnetic permeability of ferromagnetics ranges from about 100 for carbon steels, about 4000 for steels used in electrical engineering, and about 5000 for pure iron. Since the thermal behavior of the electrically conductive plate is considered in the paper, the material characteristics corresponding to the given problem must also be introduced into the calculation, such as: thermal conductivity λ , specific heat at constant deformation C_e , material density ρ , coefficient of thermal expansion α_t , coefficient of thermal diffusion $\kappa=\lambda/(\rho C_e)$. Thermal conductivity depends on temperature, as shown schematically in Figure 3 (Ref. [14]).

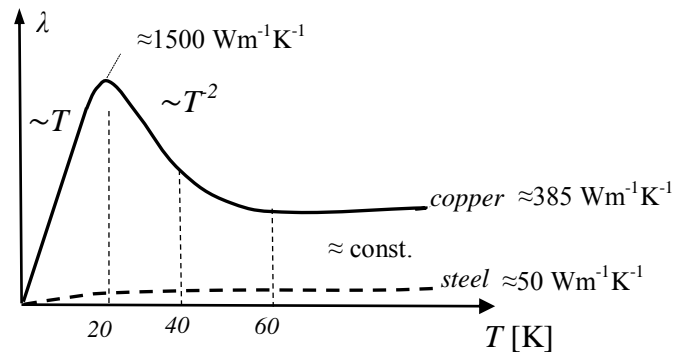


Figure 3 Thermal conductivity coefficient with respect to temperature

For temperatures above 100 K, the coefficient λ can be assumed to have a constant value for most materials. Common values of material properties treated in the paper are shown in Table 1.

Table 1 Common values of material properties

Material	$\rho_0(20^\circ\text{C})$ [Ωm]	α_e [K^{-1}]	μ_r	C_e [$\text{Jkg}^{-1}\text{K}^{-1}$]	λ [$\text{Wm}^{-1}\text{K}^{-1}$]	α_t [K^{-1}]	ρ [kg/m^3]	κ [m^2/s]
Al	$2.8 \cdot 10^{-8}$	$\sim 4.2 \cdot 10^{-3}$	~ 1	910	205	$2.4 \cdot 10^{-5}$	$2.7 \cdot 10^3$	$8.34 \cdot 10^{-5}$
Cu	$1.68 \cdot 10^{-8}$	$\sim 3.9 \cdot 10^{-3}$	~ 1	390	385	$1.7 \cdot 10^{-5}$	$8.96 \cdot 10^3$	$1.1 \cdot 10^{-4}$
Carbon steel	$1.43 \cdot 10^{-7}$	$\sim 5.7 \cdot 10^{-3}$	~ 100	470	50.2	$1.2 \cdot 10^{-5}$	$7.85 \cdot 10^3$	$1.36 \cdot 10^{-5}$

Some chemical reactors require materials with a high thermal resistance and low heat conduction. In that case a suitable correlation between thermal conductivity and temperature gives multiple linear regression model as presented in Ref. [15]. For thermo-sensitive materials, a thermal conductivity

coefficient is dependent on temperature and reference [16] discussed corresponding problem of heat conduction in a half-space. The dependence of thermal conductivity on temperature for most materials can be represented by an equation analogous to equation (3), where the common value of the coefficient is about 0.00059 K^{-1} , so it will be neglected in this work.

2. The heat source definition based on the properties of the electromagnetic wave

Based on expressions (2) and (3), the active power of the wave absorbed by the plate depends on the temperature and is described by the following equation

$$P = \frac{H_0^2}{2} \sqrt{\pi f \mu \rho_0} \sqrt{1 + \alpha_e (T - T_0)}. \quad (4)$$

If the temperature change $T - T_0$ is denoted as θ , for small temperature changes (Taylor series) it can be written that

$$\sqrt{1 + \alpha_e \theta} \approx 1 + \frac{\alpha_e}{2} \theta + \dots \quad (5)$$

but for larger temperature changes, this approximation is not valid. To determine an acceptable linear approximation for the temperature, range from about 300 K to about 1300 K, a suitable diagram is drawn, as shown in Fig. 4.

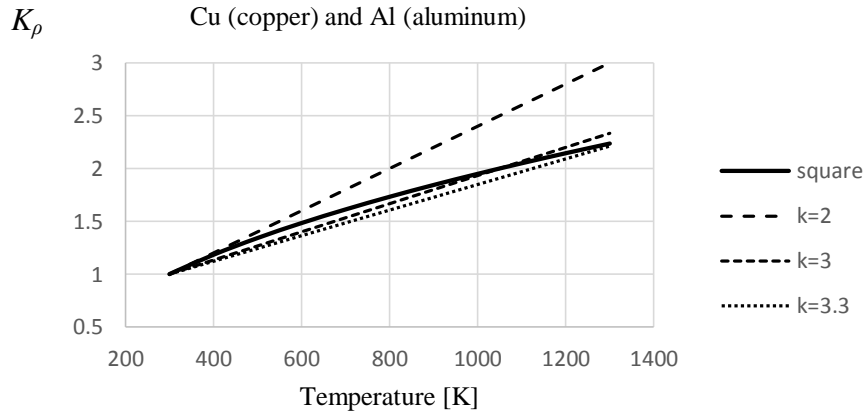


Figure 4 K_ρ with respect to temperature

The diagram in Figure 4 shows the relation between K_ρ and θ can be written as

$$K_\rho = \sqrt{\frac{\rho_t}{\rho_{0(293K)}}} = \sqrt{1 + \alpha_e \theta} \approx 1 + \frac{\alpha_e}{k} \theta. \quad (6)$$

It is evident that for temperature changes θ greater than 100 K, the best linear approximation is obtained by taking the factor k around 3.

To mathematically define the problem, it is necessary to calculate the power that the electromagnetic wave transmits to plate and to define spatially and temporally Joule heating losses as a suitable source of heat. Based on relation (4), the total power of the losses is shown by the equation

$$P = \frac{H_0^2}{2} \sqrt{\pi f \mu \rho_0} \left(1 + \frac{\alpha_e}{3} \theta \right). \quad (7)$$

To spatially define the source of heat, it is necessary to first determine the skin depth of the waves into the conductive plate. Skin depth δ mostly depends on the wave frequency and is given by the following equation (Ref. [11])

$$\delta = \sqrt{\frac{\rho_t}{\pi f \mu}} = \sqrt{\frac{\rho_0(1 + \alpha_e \theta)}{\pi f \mu}}. \quad (8)$$

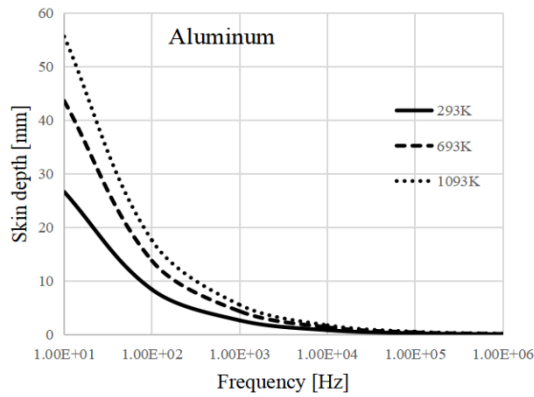
The frequency of electromagnetic waves in microwave ovens is of the order of 10^{10} Hz, while the frequency of gamma radiation is of the order of 10^{20} Hz. For induction heating, a frequency of up to 25 kHz is used, for welding around 25 MHz, and for microwave ovens it is usually around 2.5 GHz. The average power of electromagnetic radiation at the top of the atmosphere is about 1400 W m^{-2} .

Skin depths for the observed materials at a temperature of 20°C , and for different frequencies, are shown in Table 2.

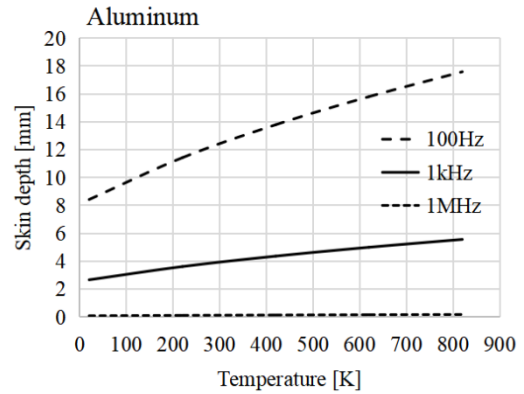
Table 2 Skin depth at 20°C

	10Hz	100Hz	1kHz	1MHz	1GHz
Al	26.63 mm	8.42 mm	2.66 mm	84.22 μm	2.66 μm
Cu	20.6 mm	6.52 mm	2.06 mm	65.23 μm	2.06 μm
Steel	6.02 mm	1.9 mm	0.6 mm	19.03 μm	0.6 μm

Diagrams from Figures 5a and 5b show the change in skin depth for an aluminum plate depending on the temperature and frequency of the incident wave.



a)



b)

Figure 5 a) Skin depth with respect to frequency (aluminum); b) Skin depth with respect to temperature (aluminum)

Based on the presented analysis, for high-frequency waves, the skin depth is very small, so the position of the heat sources that generate conduction currents can be shown by applying the Dirac δ function. According to the coordinate system attached to the plate and shown in Figure 1, the heat losses caused by the conduction currents due to the action of the high-frequency wave are treated as a heat source described by the equation

$$W(x,t) = \frac{H_0^2}{2} \sqrt{\pi f \mu \rho_0} \left(1 + \frac{\alpha_e}{3} \theta \right) \cdot \delta(x) \cdot H(t), \quad (9)$$

where the Dirac function $\delta(x)$ describes the position of the heat source, and the Heaviside function $H(t)$ defines the corresponding time change (t is time).

3. Mathematical modeling of the problem. Temperature field

The differential equation that describes the temperature field in the observed conductive plate, for the uncoupled problem, is as follows, (Ref. [10,11])

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{\kappa} \frac{\partial}{\partial t} \right) \theta = -\frac{W}{\lambda}. \quad (10)$$

In the case of thin plates, the assumption that the temperature changes linearly along the thickness of the plate is introduced, and for the adopted coordinate system it is represented by the following equation

$$\theta(x, y, z, t) = \tau_0(y, z, t) + \tau_1(y, z, t) \cdot \left(x - \frac{h}{2} \right), \quad (11)$$

in which τ_0 is the temperature of the middle cross section of the plate, and τ_1 is the temperature gradient across the plate thickness. As the intent of this work is to determine the influence of the change of specific electrical resistance on the thermal behavior of electrically conductive plates, in the further discussion only the determination of the temperature of the middle cross section of the plate will be shown. In the case of thin plates that are thermally insulated on both sides ($x=0$ and $x=h$), the temperature gradient across the thickness is small, and terms that have no direct impact on the final purpose of this work are removed from the differential equation. In that case, the differential equation describing the temperature of the middle cross section of the observed plate has the following form

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{\kappa} \frac{\partial}{\partial t} \right) \tau_0 = -\frac{W_0}{\lambda h}, \quad (12a)$$

$$W_0 = \int_0^h \left[\frac{H_0^2}{2} \sqrt{\pi f \mu \rho_0} \left(1 + \frac{\alpha_e}{3} \theta \right) \cdot \delta(x) \cdot H(t) \right] dx = C_w H(t) \int_0^h \left[\left(1 + \frac{\alpha_e}{3} \theta \right) \cdot \delta(x) \right] dx$$

$$W_0 = C_w H(t) + C_w H(t) \frac{\alpha_e}{3} \theta_{x=0} \approx C_w H(t) + C_w H(t) \frac{\alpha_e}{3} \tau_0$$

$$C_w = \frac{H_0^2}{2} \sqrt{\pi f \mu \rho_0}$$

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{\kappa} \frac{\partial}{\partial t} + \frac{\alpha_e C_w}{3 \lambda h} \right) \tau_0 = -\frac{C_w}{\lambda h} H(t). \quad (12b)$$

If the boundary conditions are adopted so that the temperature at the edges $y=0$, a and $z=0$, b is constant ($\theta=0$), a double finite Fourier transformation (symbol τ_{0nm}) can be applied in the directions of the y and z axes, so that the equation takes the following form

$$\left(-\alpha_n^2 - \alpha_m^2 - \frac{1}{\kappa} \frac{\partial}{\partial t} + \frac{\alpha_e C_w}{3 \lambda h} \right) \tau_{0nm} = -\frac{C_w}{\lambda h} H(t) \int_{y=0}^a \int_{z=0}^b \sin \alpha_n y \sin \alpha_m z dy dz, \quad (13)$$

$$\alpha_n = \frac{n\pi}{a}, \quad \alpha_m = \frac{m\pi}{b}.$$

As

$$\int_{y=0}^a \sin \alpha_n y dy = -\frac{1}{\alpha_n} \cos \alpha_n y \Big|_0^a = \frac{1}{\alpha_n} (1 - \cos n\pi) = \frac{2}{\alpha_n}, \quad n = 1, 3, 5 \dots$$

equation (13) takes the following form

$$\left(-\alpha_n^2 - \alpha_m^2 - \frac{1}{\kappa} \frac{\partial}{\partial t} + \frac{\alpha_e C_W}{3\lambda h} \right) \tau_{0nm} = -\frac{4}{\alpha_n \alpha_m} \frac{C_W}{\lambda h} H(t). \quad (14)$$

Since a dynamic problem is considered, it is necessary to apply the Laplace transformation (label *), so the function τ_{0nm}^* can be determined by arranging the equation below

$$\left(-\alpha_n^2 - \alpha_m^2 - \frac{p}{\kappa} + \frac{\alpha_e C_W}{3\lambda h} \right) \tau_{0nm}^* = -\frac{4}{\alpha_n \alpha_m} \frac{C_W}{\lambda h} \frac{1}{p}, \quad (15)$$

in which p is the Laplace transform parameter, and the initial condition is $\theta(t=0)=0$.

The transformed function has the form

$$\tau_{0nm}^* = \frac{\frac{4}{\alpha_n \alpha_m} \frac{C_W}{\lambda h} \frac{1}{p}}{\alpha_n^2 + \alpha_m^2 + \frac{p}{\kappa} - \frac{\alpha_e C_W}{3\lambda h}} = \frac{4C_W \kappa}{\alpha_n \alpha_m \lambda h} \cdot \frac{1}{p \left[p - \kappa \left(\frac{\alpha_e C_W}{3\lambda h} - \alpha_n^2 - \alpha_m^2 \right) \right]}. \quad (16)$$

When the following notation is introduced

$$K_1 = \frac{4C_W \kappa}{\lambda h}, \quad K_2 = \frac{\alpha_e C_W}{3\lambda h}, \quad (17)$$

equation (16) takes the following form

$$\tau_{0nm}^* = \frac{K_1}{\alpha_n \alpha_m} \cdot \frac{1}{p [p - \kappa (K_2 - \alpha_n^2 - \alpha_m^2)]}. \quad (18)$$

The inverse Laplace transform of the previous expression gives the following equation

$$\tau_{0nm} = \frac{K_1}{\alpha_n \alpha_m} \cdot \frac{e^{\kappa(K_2 - \alpha_n^2 - \alpha_m^2)t} - 1}{\kappa(K_2 - \alpha_n^2 - \alpha_m^2)}. \quad (19)$$

Finally, the inverse finite Fourier sine transforms must be applied, so the analytical equation in closed form for the temperature field in the middle cross section of the plate is as follows

$$\tau_0 = \frac{4K_1}{ab} \sum_{n=1,3,\dots}^{\infty} \sum_{m=1,3,\dots}^{\infty} \frac{1}{\alpha_n \alpha_m} \cdot \frac{e^{\kappa(K_2 - \alpha_n^2 - \alpha_m^2)t} - 1}{\kappa(K_2 - \alpha_n^2 - \alpha_m^2)} \sin \alpha_n y \cdot \sin \alpha_m z. \quad (20)$$

4. Numerical examples

All material characteristics related to this problem are shown in Table 1. To form a numerical model, the strength of the magnetic field and the frequency of the waves are defined. In induction heating, the field strength is around 1000 Am^{-1} , and the frequency is around 25 kHz, while in welding, the frequency is around 25 MHz. That was the reason for selecting the field from 500 Am^{-1} to 1000 Am^{-1} , and the frequency of 1 MHz ÷ 1 GHz. Since thermal conductivity of steel is four times lower than thermal conductivity of aluminum, and even seven times lower than thermal conductivity of copper (Table 1), in order to obtain the same temperature difference range for all the materials, plates of

different dimensions and electromagnetic waves of different frequencies are considered. Calculation parameters for considered materials are shown in Table 3.

Table 3 Calculation parameters (C_w , K_1 , K_2) with respect to H_0 and f

Material	C_w [Wm^{-2}]	K_1 [K/s]	K_2 [m^{-2}]
Al	$1.66 \cdot 10^{-7} \times H_0^2 \cdot f^{0.5}$	$1.63 \cdot 10^{-6} \times C_w/h$	$6.83 \cdot 10^{-6} \times C_w/h$
Cu	$1.29 \cdot 10^{-7} \times H_0^2 \cdot f^{0.5}$	$1.14 \cdot 10^{-6} \times C_w/h$	$3.38 \cdot 10^{-6} \times C_w/h$
Carbon steel	$3.76 \cdot 10^{-6} \times H_0^2 \cdot f^{0.5}$	$1.08 \cdot 10^{-6} \times C_w/h$	$3.78 \cdot 10^{-5} \times C_w/h$

Firstly, the heating of a steel plate with dimensions $a \times b = 1 \text{ m} \times 1 \text{ m}$ was considered for different values of plate thickness. The field strength and wave frequency are given in the diagram from Figure 6. The diagram shows the change in temperature of the central point of the plate while reaching the stationary state. The influence of the plate thickness on its heating is not linear.

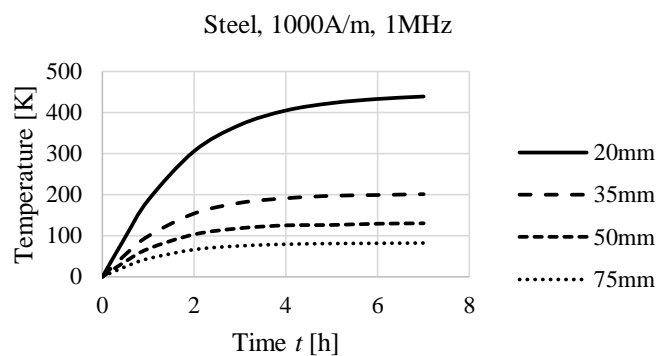
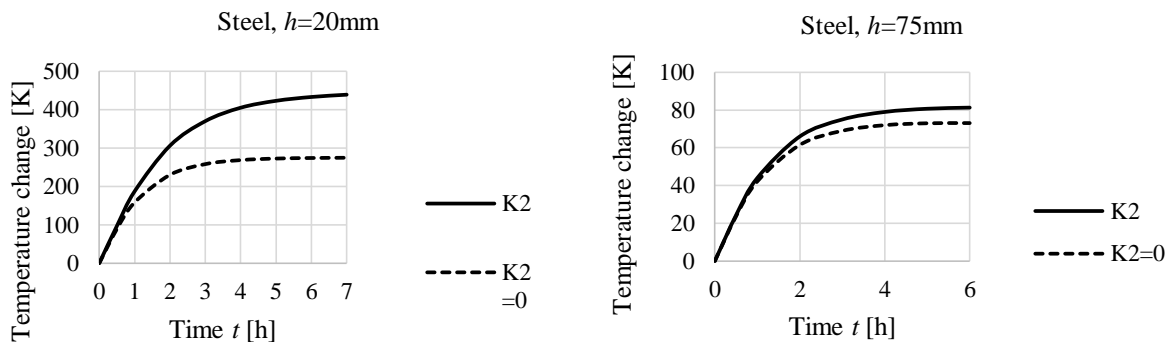


Figure 6 Temperature change with respect to plate thickness h ($a \times b = 1 \text{ m} \times 1 \text{ m}$)

To demonstrate the influence of the change of specific electrical resistance when heated by electromagnetic waves, first calculations were performed for a plate with dimensions $a \times b \times h = 1 \text{ m} \times 1 \text{ m} \times 20 \text{ mm}$. In this case, the temperature change is great, so that the difference of the calculation with factor K_2 (when the change in electrical resistance is considered) or without it is significant. From the diagram shown in Figure 7, the difference is of the order of 30 %.



a)

b)

Figure 7 Temperature change with respect to K_2 for the plate thickness of a) 20 mm ($a \times b = 1 \text{ m} \times 1 \text{ m}$); b) 75mm ($a \times b = 1 \text{ m} \times 1 \text{ m}$)

For plates of greater thickness, where the temperature is significantly lower, a much smaller difference in calculations was obtained as expected (less than 10 %), which is shown in the diagram in Figure 7b.

Diagram in Figure 8 shows the temperature change along one central line of the plate, at $y=500\text{mm}$, for the plates of different thickness.

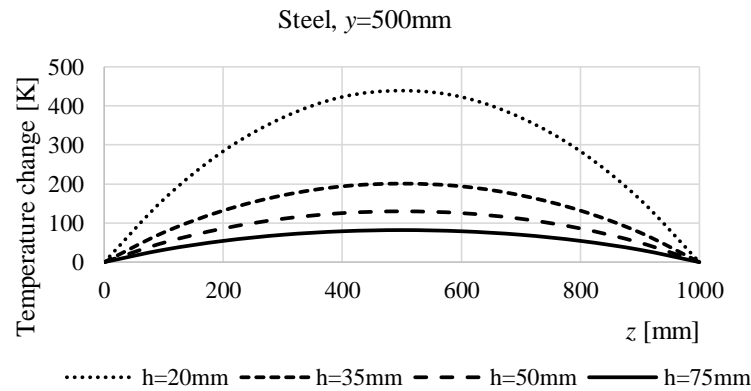


Figure 8 Temperature change with respect to coordinate z ($a \times b = 1 \text{ m} \times 1 \text{ m}$)

Due to the thermal boundary conditions, i.e., the cooling of the plate along the side edges, for copper and aluminum plates with dimensions $a \times b = 1 \text{ m} \times 1 \text{ m}$, the temperature change is relatively small. To observe and show the significance of the change in specific electrical resistance for these materials as well, panels with dimensions $a \times b \times h = 2 \text{ m} \times 2 \text{ m} \times 20 \text{ mm}$ were considered because, in the case of these dimensions, the cooling is much slower. Also, the wave frequency was raised to a value of 1 GHz.

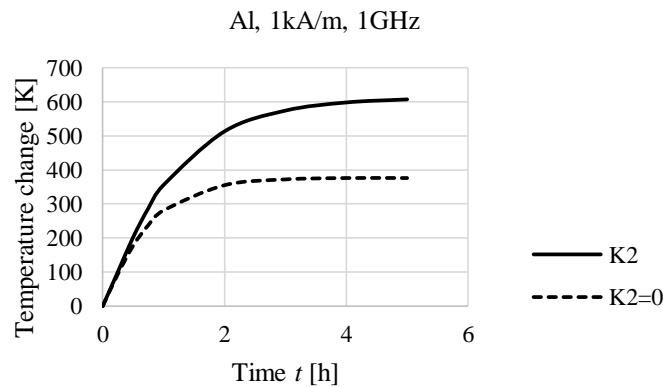


Figure 9 Temperature change with respect to K_2 for the aluminum plate thickness of 20 mm ($a \times b = 2\text{m} \times 2\text{m}$)

Figures 9 and 10 illustrate the temperature change over time for a point with coordinates $y=z=1 \text{ m}$. For the selected parameters and selected boundary conditions, the aluminum plate heats up much more easily than the copper plate. Since the wave frequency is also very high, the aluminum plate theoretically reaches a temperature of about 600°C . That is why the difference in the calculation results with and without considering the K_2 factor is over 30 %.

For the same parameters, the copper plate is heated to a maximum temperature of about 180°C, so in that case the influence of the change in specific electrical resistance with respect to temperature is less important.

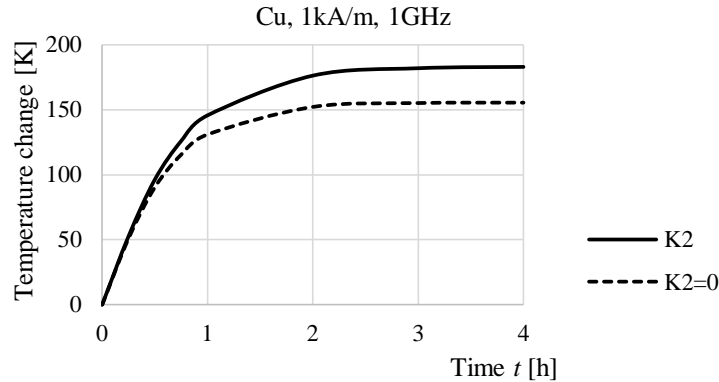


Figure 10 Temperature change with respect to K_2 for the copper plate thickness of 20 mm ($a \times b = 2\text{m} \times 2\text{m}$)

5. Conclusion

When high-frequency electromagnetic waves act on a plate made of electrically conductive material, the plate absorbs part of the energy of the waves and conduction currents appear. The absorbed wave energy is converted into a heat source whose power can be determined via the Poynting vector. For electro-conductive materials such as copper, aluminum and steel, the depth of wave penetration is very small, so the formed heat source can be spatially described by the Dirac function. Since the dynamic change of plate temperature is considered in the paper, the Heaviside function is also introduced to obtain the solution in an analytical form. Electrical resistivity is a characteristic of the material that figures in the Poynting vector, and to a significant extent depends on the temperature. It is shown that, during the analytical calculation, it is best to perform the appropriate linearization of the Poynting vector in the manner shown in the paper. By applying the technique of integral transformations, a solution is obtained in a closed analytical form. The importance of the influence of the change in specific electrical resistance is explained through appropriate numerical examples. In this paper, it is clearly shown that for temperature changes greater than 100 K, this dependence must be considered.

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Nomenclature

a, b, h [m]- plate dimensions	α_t [K^{-1}] - coefficient of thermal expansion
C_e [$\text{Jkg}^{-1}\text{K}^{-1}$] - specific heat at const. deformation	α_e [K^{-1}] - thermal coefficient of electric resistance
f [Hz] - wave frequency	λ [$\text{Wm}^{-1}\text{K}^{-1}$] - thermal conductivity
H_0 [Am^{-1}] - strength of the magnetic field	μ_0 [Hm^{-1}] - magnetic permeability

P [Wm^{-2}]- Poynting vector
 t [s, h] - time
 T [K, °C] - temperature

ρ [kgm^{-3}] - material density
 ρ_t [Ωm] - specific electric resistance
 $\theta=T-T_0$ [K, °C] - temperature change

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