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METODOLOGY OF ANALYSIS OF 2D RC FRAME MODELS WITH APPLICATION OF P-Y CURVES AS LINK ELEMENTS ON PILES.

Boris Folić*, Stanko Brčić**, A. Liolios***

* *University of Belgrade, Innovation Center, Faculty of Mechanical Engineering;* boris.folic@gmail.com
** *University of Belgrade, Faculty of Civil Engineering;* stanko.brcic@gmail.com
*** *Democritus University of Thrace, Dept. Civil Engineering, GR-67100 Xanthi, Greece, (e-mail:* aliolios@civil.duth.gr,

ABSTRACT
The paper presents the methodology of analysis of 2D frame model, founded on piles which represents a skeletal multi-storey building. Numerical analyses related to 2D RC frame are perfromed, with an accent on the pile-soil interaction. The results are a subject of another paper, while this paper is related to pile-soil interaction implemented by *p-y* curves. The *P-y* curves are deterimed according to Rees, Cox and Matlock model for sand. Using SAP 2000, *p-y* curves are applied on both sides of the pile, as link elements. Properties of link elements are assumed as the multi-linear plastic type, i.e. as hysteresis envelopes according to Takeda model. The linear part of link elements corresponds to the initial stiffness of the *p-y* curve. Curves are exposed to compression only and to almost negligible tension. Since it is the analysis of a seismic action, the curve coefficients are selected to correspond to repeated, i.e. cyclic load, and not for the static one. In such a way, numerical model asymptotically simulates the real actions and seismic response of structures in dynamic interaction of the pile-structure-soil system. This model offers a lot of potential for further development and study of seismic performance.

KEY WORDS: dynamic interaction, pile-structure-soil, *p-y* curves, link elements, 2D frame

METODOLOGIJA ANALIZE 2D AB RAMA PRIMENOM P-Y KRIVIH KAO LINK ELEMENATA NA ŠIPOVIMA

REZIME
U radu je prikazana metodologija analize 2D modela rama fundiranog na šipovima koji reprezentuje skeletnu višespratnu zgradu. Na 2D ramu su obavljene numeričke analize, sa naglaskom na interakciju šipovi-tlo. Rezultati su predmet posebnog rada, dok se ovaj rad odnosi na interakciju šip-tlo implementiranu primenom *p-y* krivih. *P-y* krive su određene prema Rees, Cox i Matlock modelu za pesak. Primenom SAP 2000, *p-y* krive su nanete na šip obostrano, kao link elementi. Link elementi su usvojeni kao više-linearni plastični tip,

odnosno kao histerezisne anvelope prema Takeda modelu. Linearni deo link elemenata odgovara inicijalnoj krutosti *p-y* krive. Krive su izložene samo pritisku i skoro zanemarljivom zatezanju. Kako se radi o analizi seizmičkog dejstva, izabrani su koeficijenti krive za ponovljeno, odn. ciklično, opterećenje, a ne za statičko. Time se težilo da model, asimptotski simulira realno dejstvo i seizmički odgovor konstrukcije u dinamičkoj interakciji sistema šip-konstrukcija-tlo. Ovaj model pruža niz mogućnosti za dalji razvoj i proučavanje seizmičkih performansi.

KLJUČNE REČI: dinamička interakcija, konstrukcija-šip-tlo, *p-y* krive, link elementi, 2D ram

INTRODUCTION

The structure-soil interaction was historically dealt with, directly or indirectly, by many researchers. Interaction was researched (according to Stevanović, 1999), among the first, by Zimmerman, 1888, who employed the Winkler hypothesis, published in 1876 for design of railways. Winkler introduced a model of mutually independent elastic springs, where settlement at one point depends only upon the force at that point. At a later date, the model of homogeneous elastic and isotropic semi-space (defined by the soil modulus of elasticity E_0 and by the Poisson's coefficient ν) was introduced, where the effects of a force in one point propagates across the entire semi-space. Tajimi, in 1969, developed a method of a seismic response, based on the elastic theory of wave propagation. Broms used the method of boundary equilibrium for two types of soil and three "lengths" of a pile, for the failure state. The structure-pile-soil system was researched by Hetenyi, Terzaghi, Housner, Penzien, Баранов, Горбунов-Пасадов, Ильчев, Синицын, Napatvaridze, Vesic, Pender, Prakash, Trifunac, Tazoh, Stewart, Makris, Badoni, Dobry, Budhu, Mylonakis (Novak, 1974), Nogami, (Poulos and Davis,1980). They formulated the problem using the systems of ordinary or partial differential equations, with solutions obtained through Bessel's or Hankel's functions, or using the models with springs and dampers. Gazetas used the model of kinematic and inertial interaction and model of a beam on dynamic Winkler foundation (BDWF), see (Folić, 2017). Wolf used both the frequency domain analysis and the mixed successive frequency-time analysis, considering the pile elements having the form of the disc-cone, etc.

Since it is necessary to harmonize the theory with the practice and to determine conditions and areas of application, there are two approaches. One is based on experiences acquired during inspection or testing of the buildings and structures after earthquakes, and the other is based on experiments using the shaking tables or centrifugal tests. The knowledge about this field, about dynamic properties of soil and soil-structure interaction, considerably increased during the 20th century (Japanese Society of Civil Engineers, 1996 and 2000). After a number of strong earthquakes in Japan and USA, detailed researches of the new phenomena were undertaken. One of them is the Niigata earthquake in 1964 in Japan, when the liquefaction mechanism was observed and definitely explained. Even though the mechanism was discovered, the problem is rather complex and there is still no generally accepted theory which includes all the aspects of this phenomenon.

The model of RC 2D frame with RC piles (Fig. 1, left) is presented in this paper. The method of calculation and application of *p-y* curves, derived from the experimental research (Reese et al. 2001) and the time history (TH) analyses, including dynamic structure-pile-soil system interaction (DSPSI), is the main subject of the paper. This model of *p-y* curves may be used both for the buildings and for the bridges. Piles are imbedded in the soil which is modeled using horizontal *p-y* curves, on each side of the pile, at each meter of depth. In the vertical direction (friction along the surface and pile base) the support is considered to be linearly-elastic. Piles, generally, may be either clamped or elastically supported at the base. In this paper clamped support at the pile base is used. *P-y* curves are determined (according to Mosher, 2000) based upon the research by Reese, Cox, Koop and Matlock for the model of sand and for repeated loads.

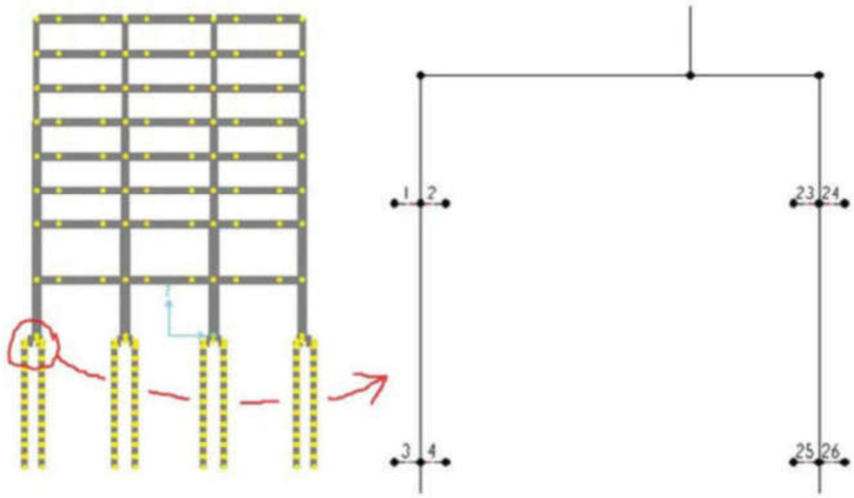

Figure 1. Left: Model of 2D RC frame founded on RC piles, with joints and FRAME elements. Right: numbers of link elements (p-y curves)
Slika 1. Levo: Model 2D AB rama fundiranog na AB šipovima sa čvorovima i grednim elemenatima. Desno: Numeracija link elemenata (p-y krive)

ABOUT THE MODEL

A 2D RC frame model, which is extracted from the regular symmetrical 3D model of the building, is used in the paper. At the top and at the bottom of each story, plastic hinges are introduced in columns. There are two types of beam reinforcement, one above the support point (intersections with other beams) and the other along the span. For each floor, the necessary reinforcement of the beams is determined. Considering that it is a regular, symmetrical building, the reinforcement is the same for all beams within the same floor, but different for support zones and for spans. The length of the support zones of beam elements is 0.20 L (the offset zone to be deducted from it), while the zone of the span is 0.60L. Plastic hinges in the beams are introduced at ends of reinforcement zones.

P-y curves are introduced as link elements, on each side of the pile (Figure 1, right), at each meter of the pile depth. The piles are condensed in the 2D plane, so the group of 3 piles is represented with 2 piles in plane, one pile being individual (1D60 in figure 2), and the other having modified properties, because they are multiplied with the coefficient 2 (2D60 in Figure 2). Analogously, the group of 4 piles is represented with 2 2D60 piles.

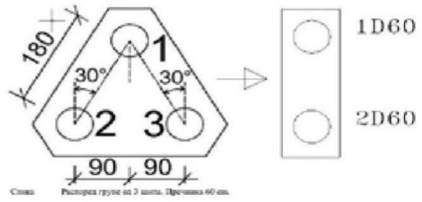

Figure 2 „Condensation“ principle of a group of 3 piles into the plane group of 2 piles (1D60 – individual pile, 2D60 – double pile).
Slika 2 Princip „kondenzacije“ grupe od 3 šipa u ravansku grupu od 2 šipa (1D60 samostalni šip, 2D60 dvostruki šip).

P-Y CURVES

Originally, the idea of modeling the soil using *p-y* curves was created using an analogy with the soil behavior curves in triaxial test. Meymand stated that it was most likely first introduced by McClelland and Focht, as early as in 1958. They recommended the procedure for correlation of data of triaxial stress-strain test with the pile force-displacement curves for specified depths, via the expected modulus of reaction of subsoil, for each soil layer. The *p-y* curve construction is presented in Figure 3. Both designations of the characteristic points of *p-y* curves, which can usually be found in literature, are written in Fig. 3.

Reese, on the basis of experiments, observed and displayed the concept of the soil failure in the shape of a wedge which occurs close to the soil surface. If the pile is sufficiently long two forms of the soil failure would appear. The first, upper one, has a wedge-like form, while the other, lower one, has a block form. The depth at which the soil failure changes form is called the critical depth.

P-y curve in Figure 3, according to the original method (Reese at all, 2001) and to others consists of four parts, those being:

- ☐ The first part, where there is a linear dependence of displacements and stresses, from the point 0 to the point y_k/p_k (a different designation can also be encountered: y_a/p_a),
- ☐ The second part, where there is a parabolic dependence of the stress and displacement from the point y_k/p_k to the point y_m/p_m (a different designation is y_b/p_b). Parameters of the parabola in this part are coefficients *C* and *n*. It is not the same *n* as the *n* which is applied in formula (10).
- ☐ The third part of the curve is an inclined straight line from the point y_m/p_m to the point y_u/p_u.

- ☐ The fourth part is the constant stress: after reaching displacement y_u there is no further increase in stress, i.e., with the increase of displacement after this point, the stress retains the constant value p_u.

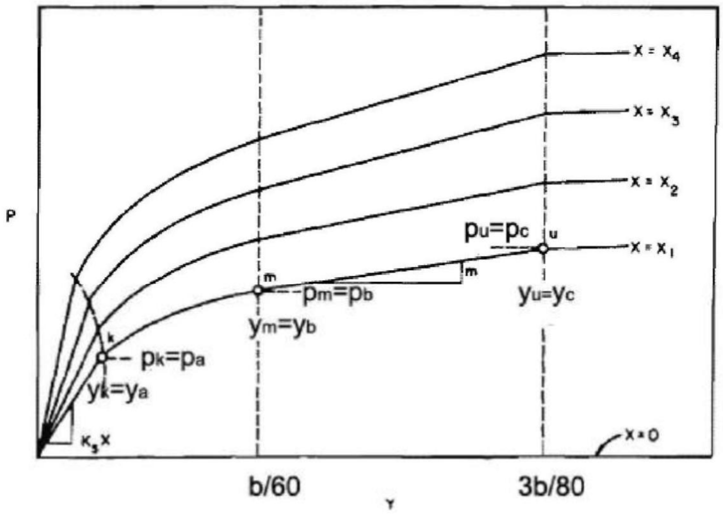

Figure 3. Construction of the characteristic shape of *p-y* curve (Folić 2017)
Slika 3. Konstrukcija karakterističnog oblika *p-y* krive (Folić 2017)

The points y_m and y_u depend exclusively on geometry of the pile, and have a fixed value, *b/60* and *3b/80* respectively, while the value of the deformation at the boundary of linearity y_k is determined by calculation, as well as the stresses p_m and p_u.

where:

b – width of the pile normally to direction of load action.

The calculation of characteristic points of the curve y_k/p_k, y_m/p_m and y_u/p_u, is performed in a number of steps. First, the coefficients C_1, C_2 i C_3 are determined (formulae (4), (5) and (6), or Figure 6, which can be used as a control). The coefficients depend exclusively on the angle of internal friction and the self weight of the soil, so their employment yields the values of lateral resistance of the soil, per unit of length of a pile. Then follows determination of the inclination k_0 of the initial part of *0-k* (*0-a*), according to formula:

$$k_0 = k \cdot z \qquad (1)$$

The coefficient *k* in Eq. (1) is determined according to the table 1 (limits of k_0 for dry and submerged sand with respect to the relative density in %, are provided in Figure 7).

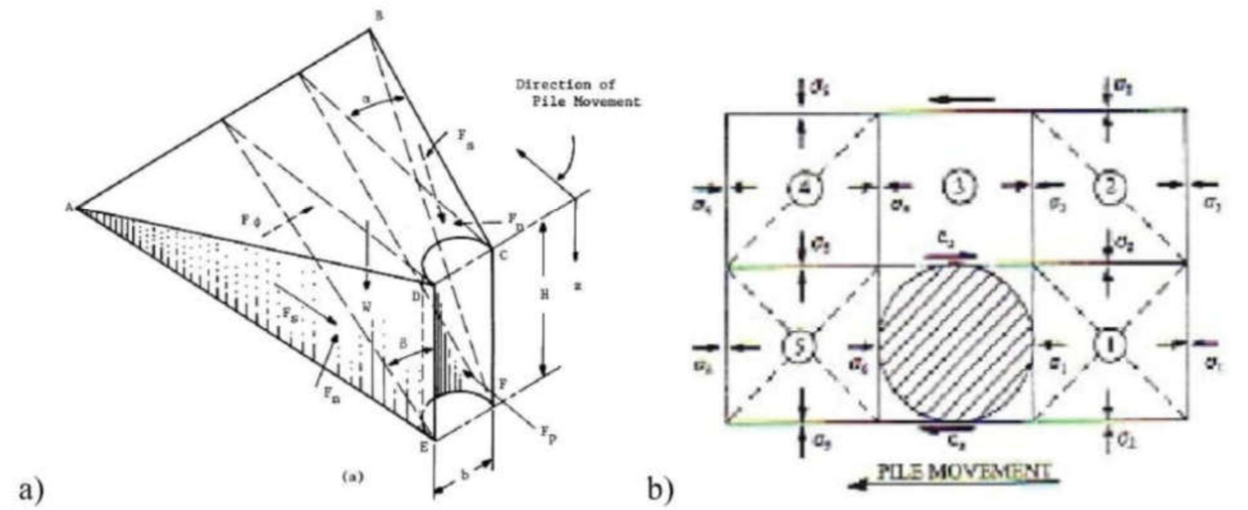

Figure 4. Two types of soil failure a) wedge mode b) block mode, or flow (according to Reese at all, 2001)

Slika 4 Dva tipa sloma tla: a) klinasti oblika, b) blok oblik, ili tečenje (prema Risu i dr, 2001)

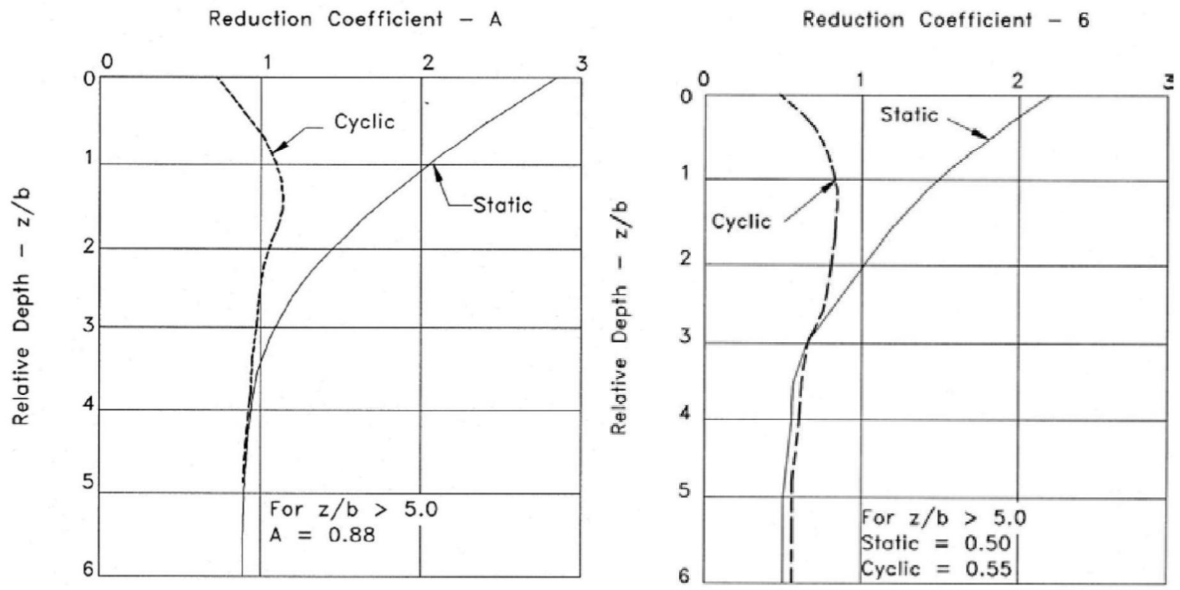

Figure 5. Coefficients of reduction of lateral resistance for sand - A and B (according to Mosher at all, 2001)

Slika 5. Koeficijenti redukcije horizontalne otpornosti za pesak - A i B (prema Mosher i dr, 2001)

Table 1. Coefficient of lateral soil reaction for sand. Initial modulus of *p-y* curve as function of relative compactness and the level of underground water table (WT)
Табела 1. Коефицијент хоризонталне реакције тла за песак. Почетни нагиб *p-y* криве у функцији релативне збијености и нивоа подземне воде.

	Soil modulus	Coefficient k	for sand
Relative compactness	Loose	Medium dense	(Hard) dense
Submerged sand	5,430 KPa/m	16,300 KPa/m	33,900 KPa/m
Sand above WT	6,790 KPa/m	24,430 KPa/m	61,000 KPa/m

Then a limit value of lateral resistance, as a lower value, is calculated for the wedge mode of soil failure, close to the ground surface, p_{st}, Eq. (2), or for the block mode of soil failure, at a greater depth, p_{sd} Eq.(3):

$$p_{st} = (C_1 z + C_2 b)\gamma' z \tag{2}$$

$$p_{sd} = C_3 b \gamma' z \tag{3}$$

$$C_1 = \frac{K \tan\varphi \, Sin\beta}{\tan(\beta-\varphi)\cos(\varphi/2)} + \frac{\tan^2\beta \, Sin(\varphi/2)}{\tan(\beta-\varphi)} + K\tan\beta(\tan\varphi \, Sin\beta - \tan(\varphi/2)) \tag{4}$$

$$C_2 = \frac{K\tan\beta}{\tan(\beta-\varphi)} - \tan^2(45-\varphi/2) \tag{5}$$

$$C_3 = K\tan\varphi\tan^4\beta + \tan^2(45-\varphi/2)(\tan^8\beta - 1) \tag{6}$$

where:
γ- effective unit weight of sand
z – depth with respect to the soil surface
K – coefficient of lateral soil pressure, selected value is 0.4
φ – angle of interior friction

$$\beta = 45 + \varphi/2 \tag{7}$$

z_{cr}- critical depth at which the soil failure from the wedge mode, Eq. (2), becomes the block mode, Eq. (3)

After that, lateral resistances for the transitional points of the *p-y* curve are calculated (the third part of the *p-y* curve, Fig. 3, y_m and y_u; or y_c and y_b), according to:

$$y_u = 3\cdot b/80\,,\;\; p_u = A\cdot(p_{st}, z < z_{cr}; p_{sd}, z > z_{cr}) \tag{8}$$

$$y_m = b/60\,,\;\; p_m = B\cdot(p_{st}, z < z_{cr}; p_{sd}, z > z_{cr}) \tag{9}$$

Reduction coefficients A and B depend on the type of load and for dynamic analysis the cyclic load curves are used. In this calculation, the estimated error of determination of A and B coefficients for the cyclic load, from the diagram in Fig. 5, is around ±1% and in an extreme cases it does not exceed ±3%. The A and B coefficients are provided in the diagram in the interval from 0 to 5 *z/b*. After the five pile depths, the curves representing coefficients A and B have a constant value, as given in Fig. 5, so in this calculation they are completely in agreement with the theory.

Construction of *p-y* curve according to API recommendations (according to Mosher, it was proposed by Murchison and O'Neill) is performed using the formula which is more convenient than the original Reese formula:

$$\frac{P}{p_u} = n \cdot A \tanh\left(\frac{kH}{n \cdot A \cdot p_u} y \right) \tag{10}$$

where:

p_u – is ultimate strength at the depth H (the same as p_c),
k – initial modulus of soil reaction (according to the table 1, for sand)
y – lateral displacement of the pile

The part $n{\cdot}A$ in Eq. (10) refers to the type of load and shape of the pile, so for the cyclic load and the constant cross-section of the pile (prismatic), the formula is:

$$P = 1.0\, p_u \tanh\left(\frac{k_0(z)}{0.9 p_u} y \right) \tag{11}$$

where $k_0(z)$ is determined according to the formula (1), and p_u from the set of previous formulae, conclusive with (8). Therefore, when using the formula (11), it is necessary to know only p_u and k_0 , at each meter of pile depth.

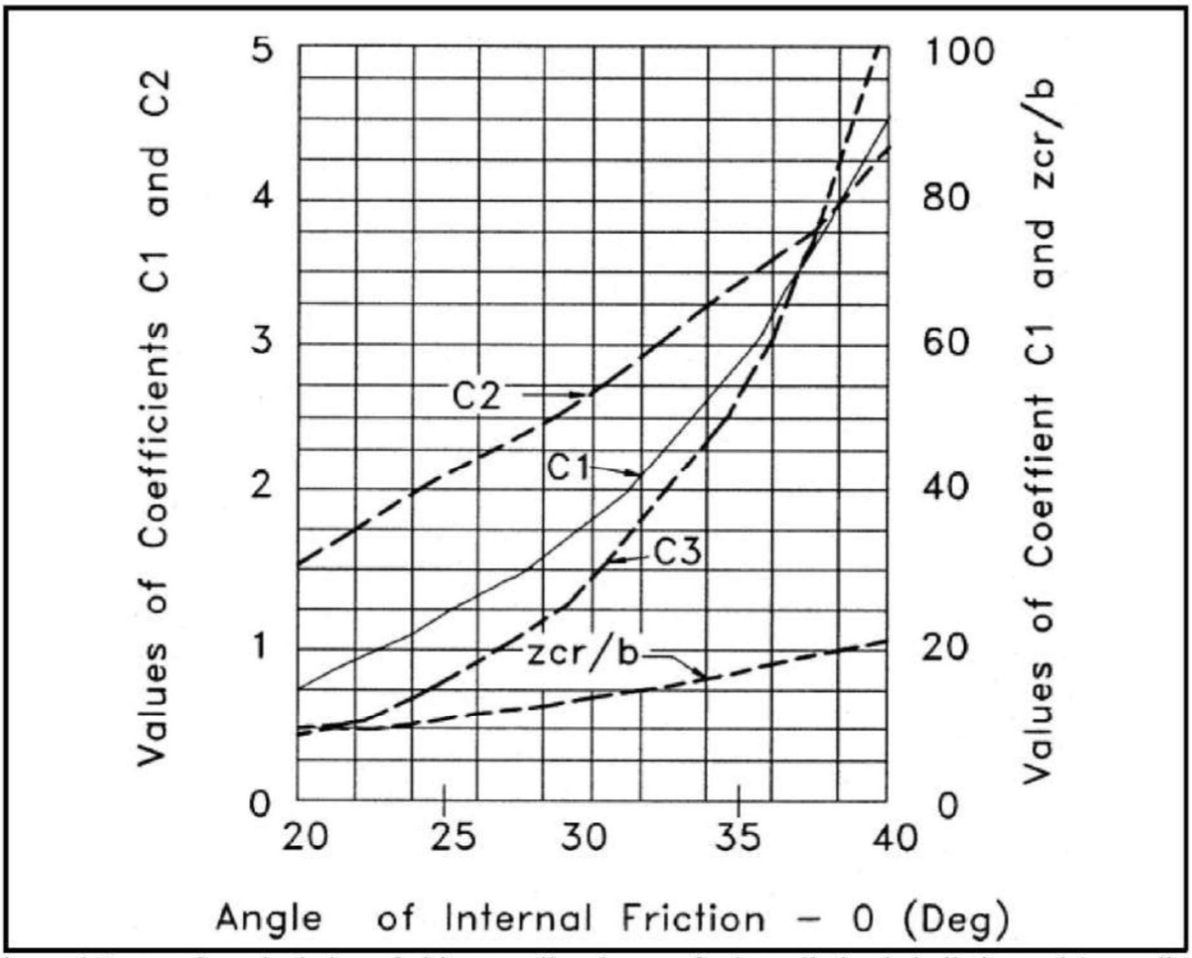

Figure 6 Factors for calculation of ultimate soil resistance for laterally loaded pile in sand (according to Mosher at all, 2001)

Slika 6 Faktori za sračunavanje granične otpornosti tla za horizontalno opterećen šip u pesku (prema Mosher i dr, 2001)

Table 2 presents parameters of *p-y* curve, which consists of 4 parts, depth of the pile - z, pile diameter 60cm, and soil parameters as given in the table caption. Both usual designations for characteristic points of *p-y* curve: p_a, p_b, p_c , i.e., p_k, p_m, p_u are given in the table.

In the upper part of table 3, parameters of the *p-y* curves for the first 4 meters of depth are given. In the lower part there are the pairs of columns of *y-p* values, ready for input into SAP2000, as link elements. Angle of φ=35° is the boundary value between the medium dense and dense sand, Figure 7.

Table 2. *p-y* curve: φ= 35°; D=0,60 m; γ= 10 kN/m^3; k= 33900 kPa/m.
Tabela 2. *p-y* kriva: φ= 35°; D=0,60 m; γ= 10 kN/m^3; k= 33900 kPa/m.

i;	z;	k_0;	y_a;	p_a=p_k;	p_b=p_m;	p_c=p_u
1	1	33900	8,15E-04	27,64	42,86	52,23
2	2	67800	7,00E-04	47,47	105,34	144,80
3	3	101700	3,53E-04	35,90	178,72	285,95
4	4	135600	5,63E-04	76,29	303,64	485,82
5	5	169500	8,18E-04	138,73	461,23	737,98
6	6	203400	1,12E-03	227,78	651,51	1042,41
7	7	237300	1,47E-03	347,96	874,45	1399,13
8	8	271200	1,86E-03	503,70	1130,07	1808,12
9	9	305100	2,29E-03	699,40	1418,37	2269,39
10	10	339000	2,77E-03	939,43	1739,34	2782,95
11	11	372900	2,88E-03	1074,71	1952,70	3124,33

Table 3. *p-y* curve: φ= 35°; D=0,60 m; γ= 10 kN/m^3; k= 33900 kPa/m.
Tabela 3 *p-y* kriva: φ= 35°; D=0,60 m; γ= 10 kN/m^3; k= 33900 kPa/m.

ko=	33900	ko=	67800	ko=	101700	ko=	135600
z=	**1**	Z=	**2**	Z=	**3**	z=	**4**
pc=	52,23	pc=	144,80	pc=	285,95	pc=	485,82
y (m)	p(kPa/m)	y (m)	p(kPa/m)	y (m)	p(kPa/m)	y (m)	p(kPa/m)
1,0E-06	1,0E-05	1,0E-06	1,0E-05	1,0E-06	1,0E-05	1,0E-06	1,0E-05
0	0	0	0	0	0	0	0
-0,001	-29,82	-0,001	-63,24	-0,001	-97,62	-0,001	-132,19
-0,002	-44,98	-0,002	-106,22	-0,002	-174,86	-0,002	-246,15
-0,003	-50,15	-0,003	-128,35	-0,003	-225,42	-0,003	-332,50
-0,0045	-51,93	-0,0045	-140,58	-0,0045	-263,57	-0,0045	-412,93
-0,005	-52,07	-0,005	-142,14	-0,005	-270,08	-0,005	-429,66
-0,007	-52,22	-0,007	-144,39	-0,007	-282,04	-0,007	-466,69
-0,009	-52,23	-0,009	-144,74	-0,009	-285,00	-0,009	-479,47
-0,01	-52,23	-0,01	-144,78	-0,01	-285,48	-0,01	-482,18
-0,015	-52,23	-0,015	-144,80	-0,015	-285,94	-0,015	-485,60
-0,02	-52,23	-0,02	-144,80	-0,02	-285,95	-0,02	-485,81
-0,0229	-52,23	-0,0229	-144,80	-0,0229	-285,95	-0,0229	-485,82
-0,09	-52,23	-0,09	-144,80	-0,09	-285,95	-0,09	-485,82
-0,18	-52,23	-0,18	-144,80	-0,18	-285,95	-0,18	-485,82

Using SAP 2000, the *p-y* curves are applied on both sides of the pile, as link elements. The link elements are assumed as a multi-plastic type, i.e. as hysteresis envelope according to the Takeda model. The link elements in SAP 2000 are applied as the two-node elements, so for each link element it is necessary to introduce another node, away from the pile, which, together with the link element, represents nonlinear soil. The first node is part of the pile in which two link elements are joined (Fig. 1, right). The link element itself has a linear and non-linear part, Figure 8. The linear part is used for linear analysis, and for determination of the first natural mode. In the nonlinear analysis both linear and nonlinear parts are used, in ratio which depends on the level of realized strains.

The linear part of link element corresponds to the initial stiffness of *p-y* curve. The curves are applied on both sides (Figure 1, right) because they are exposed to pressure only, and negligible tension, which simulates the property of the soil to convey only pressure. Therefore, asymmetry of the seismic response may be determined (as presented in another paper for this Conference). Since the seismic action is analyzed, the selected curve coefficients correspond to repeated (cyclic) load, rather than for a static one. It was attempted that this model asymptotically simulates the real actions and seismic responses of structures in dynamic interaction of the pile-structure-soil system. This model offers a great potential for further development and study of seismic performance, such as dissipation of energy along the depth of the pile etc. Another paper is presenting the change of the first natural mode and states of plastic hinges depending on PGA.

In the case of pushover analysis it is also necessary to use cyclic *p-y* curves, because the loads in the pushover analysis are quasi-static and not static. This quasi-static load attempts to cause the effects upon the structure which would be as similar as possible to the ones caused by an earthquake. Quasi-static load is used for determination of seismic (performances) properties of the structure and also to determine the sequence of formation of plastic hinges, when the structure-pile-soil system eventually becomes a mechanism. It should be mentioned that the initial plastic hinges, in pushover analysis and time history analysis can be significantly different. However, as the number of plastic hinges increases, the differences usually decrease.

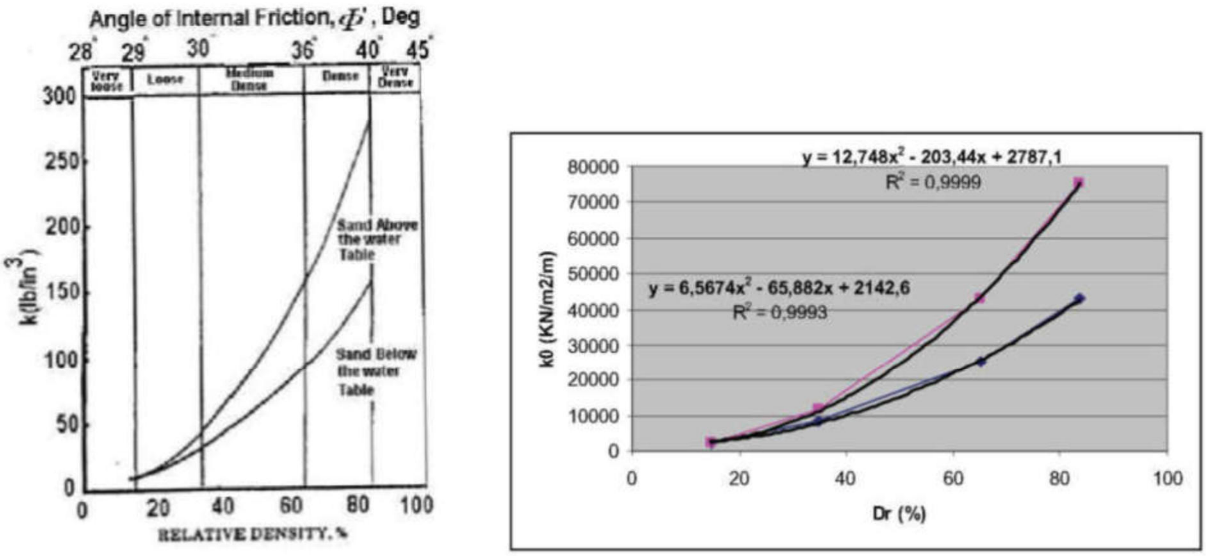

Figure 7. Initial modulus of lateral reaction of soil as function of the water table level, density and internal angle of friction, API (American Petroleum Institute), after (Folić B. i Folić R. 2018). Right curves k_0 as function of compactness, with formulae for aproximate calculation.
Slika 7. Početni modul horizontalne reakcije tla u zavisnosti od nivoa podzemne vode, zbijenosti i ugla unutrašnjeg trenja, API (Američki institut za naftu), prema (Folić B. i Folić R. 2018). Desno krive zavisnosti k_0 od zbijenosti, sa formulama za približan proračun.

The soil in this model is represented using coupled *p-y* springs (Figure 4 left), which, regarding that they are subjected to compression only, are placed on both sides of the pile. These link elements are of multi-linear plastic curve type, i.e. non-linear hysteresis Takeda model. The modeled multi-linear link elements also enable the onset of the gap effect of the hysteresis curve.

If instead of *p-y* curves the secant elastic stiffness is used, their upper boundary of inclination represents the initial stiffness of *p-y* curves (Figure 7 and table 1). The initial linear lateral stiffness of the p-y curves for sand increases proportionally with the depth *z*, (in this case) according to the formula: $k_x=33900 \cdot z$, but only up to the value p_a (p_k in Fig.3). Further, the *p-y* curve behaves as a hyperbole with a slant asymptote, while the maximum value of pressure p_c, according to the formula (11), represents a horizontal asymptote. Lateral stiffness of elastic springs is ostensibly smaller in comparison to the *p-y* curves, but only in relation to the initial stiffness of the p-y curves, because such elastic springs provide considerably higher soil resistance forces at higher values of displacements and do not have hysteretic loss of capacity, as is the case of *p-y* curves. It is the larger displacements which occur in the upper layers of soil. If the secant stiffness was used (Pando, 2013) for elastic springs, the result would be different, at least in the part referring to the displacements smaller than the intersection point of the *p-y* curve and the secant stiffness line. In (Folić, 2017), it was shown that the usage of secant stiffness can have a negligible effect, but also it can considerably change the form of failure of the structure and arrangement of plastic hinges, for the middle bridge frame. The middle bridge frame for different soil types, migration of plastic hinges and damping was discussed by (Suarez, 2005). The structural response largely depends on the used accelerogram, intensity of PGA, inclination of secant curve, change of natural mode of the structure-pile-soil system and response spectrum. Therefore, changes of natural modes of the SPS system, using various secant stiffnesses should be considered in relation to the response spectrum for a chosen accelerogram.

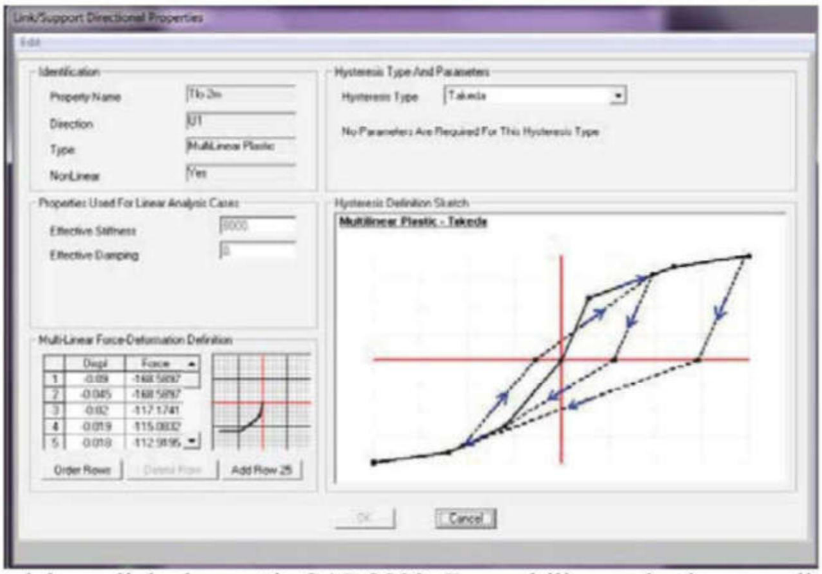

Figure 8. Soil model as a link element in SAP 2000. *P-y* multilinear plastic, according to Folić at all (2013).

Slika 8. Model tla kao link element u SAP 2000. *P-y* višelinearno plastično, prema Folić i dr. (2013).

The *p-y* curves from table 3 should be entered into the properties of link elements, in the lower left part of the dialog form, as seen in Fig. 8.

It is important to emphasize at the end of this section, that unless sand is clean, and has a considerable cohesion, the mentioned procedures cannot be implemented. Instead, specially constructed curves, which include these effects as well, should be used (Meyer at all, 1979, Reese at all 2001, Folić B. 2017).

CONCLUSION

The presented model of 2D RC frame founded on RC piles, with the soil modeled with horizontal *p-y* curves, offers a range of options for numerical research of seismic performances of the structure-pile-soil system. This model of *p-y* curves is applicable for buildings and bridges, for small pile diameters, and also for the bridges with pile diameters of 150-180cm. The algorithm of calculation of the *p-y* curves for sand is provided in detail, as well as the method for application on the piles of 2D models of RC frames.

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