DEMI 2023

16th International Conference on Accomplishments in Mechanical and Industrial Engineering

www.demi.mf.unibl.org

Banja Luka
1–2 Jun 2023.

Assessment of fractional order impact on performance of fractional ILC controller for upper limb exoskeleton

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Abstract *In this research paper, the application of Iterative Learning Control (ILC), an intelligent control method, is suggested in the form of a fractional-order PD-type controller. The main task of the ILC controller is to reject process model uncertainties, which are often present in complex systems such as various multibody systems, and to sequentially reduce a trajectory tracking error. As a control plant, an exoskeleton support arm with three degrees of freedom is used herein. The control scheme consists of feedback linearization compensating for the known part of the dynamics model and the feedforward ILC controller of the PDα-type. The feedback part of the control system is the classical PD controller. The feedforward control signal is filtered with a lowpass filter to avoid divergent behavior as iterations progress. Finally, simulation results are presented to demonstrate the proposed control system performance applied to the chosen control plant, as well as the achieved error convergence towards the steady-state value for various values of the fractional order.*

Keywords *Iterative learning control, feedback linearization, fractional derivative, trajectory tracking, exoskeleton*

1. INTRODUCTION

Trajectory tracking is one of the main problems in robotics, where the goal is to control a robot's motion to follow a predefined path or trajectory [1]. This problem also arises in various applications, such as industrial automation, autonomous vehicles, surgical and rehabilitation robots [2,3]. Appropriately achieved trajectory tracking enables robots to perform complex tasks accurately and efficiently.

The beginning of Iterative Learning Control (ILC) theory was introduced by Uchiyama and Arimoto in [4] and [5]. ILC can be defined as an intelligent memory type of control since it uses the information from the past experiences to improve the performance of the system [6]. ILC is particularly well-suited for systems that repeat a task or motion several times, such as robots. Mathematical models of the robot manipulators are idealized version of the real-world dynamics, leaving some degree of uncertainty in the model [7]. The advantage of the ILC algorithms is robustness to uncertainty and repetitive disturbances [8]. Combining model-based control such as feedback linearization and ILC can lead to improved trajectory tracking performance.

After fractional calculus gained popularity over the years, various control algorithms using fractional derivative or integral have been investigated [9]. In this study we investigated how fractional order derivative in ILC controller

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influences the error convergence over iterations and overall performance of the system.
In the Section 2, brief theoretical background for ILC and fractional calculus used in this study is laid out. Section 3 consists of controller design description divided into subsection describing feedback linearization and subsection describing fractional order iterative learning controller. Section 4 showcases simulation results of previously presented controller for various values of fractional order derivative in ILC.

2. THEORETICAL BACKGROUND

2.1 Iterative learning control

Iterative learning control algorithms utilize information from previous attempts to refine the control input and improve performance in subsequent iterations. The control system learns from its past errors and adapts the control inputs to achieve the desired output with higher accuracy in each subsequent iteration. The general form of iterative learning control [10, 11] can be given as:

$$\mathbf{u}_{k+1} = \boldsymbol{Q}(\mathbf{u}_k + \boldsymbol{L}\boldsymbol{e}_k), \tag{1}$$

where k is the iteration index, $\mathbf{u}$ is the control signal, $\mathbf{e}$ the error signal and $\boldsymbol{Q}$ and $\boldsymbol{L}$ are filter and learning function, respectively. There are many variations of the control law (1) and different approaches to modelling $\boldsymbol{Q}$ filter and learning function $\boldsymbol{L}$ [12,13]. In this paper we consider PD-type control algorithm with $\boldsymbol{Q}$ as lowpass filter:

$$\mathbf{u}_{k+1} = \boldsymbol{Q}\left(\mathbf{u}_k + \mathbf{K}_{\mathrm{p}}\mathbf{e}_k + \mathbf{K}_{\mathrm{d}}\dot{\mathbf{e}}_k\right), \tag{2}$$

where $\mathbf{K}_{\mathrm{p}}$ and $\mathbf{K}_{\mathrm{d}}$ are tuneable gain matrices.

2.2 Fractional calculus

Fractional calculus is a branch of mathematical analysis that extends the concepts of differentiation and integration to non-integer orders. In presented study ILC controller uses fractional order derivative or the error signal. Fractional derivative is calculated using Grunwald-Letnikov definition of fractional derivative [14]:

$$D_R^{\alpha} y(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \Delta_h^{\alpha} y(t)\,, \tag{3}$$

where h is the time step, D_R^{α} is the fractional derivative operator, $y(t)$ is the function to be differentiated and Δ_h^{α} is the finite difference notation with detailed explanation in [14].

3. CONTROLLER DESIGN

3.1 Application of feedback linearization control method for robot systems

Feedback linearization is a powerful model-based control technique that allows nonlinear systems to be transformed into fully or partly linear systems, so that they can be controlled using traditional linear control methods [15].
Equations of motion of a robot mechanism (i.e. robot dynamic model) are usually presented in canonical form in the joint-space formulation:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}, \tag{4}$$

where $\mathbf{M}(\mathbf{q})$ is the inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is the matrix containing centrifugal and Coriolis effects, $\mathbf{g}(\mathbf{q})$ is the vector of gravity forces and $\boldsymbol{\tau}$ is the vector of joint actuation torques. Equations of motion given by (4) can be rewritten in form where the model is split in nominal and uncertain part [7]:

$$(\mathbf{M}_N + \Delta\mathbf{M})\ddot{\mathbf{q}} + (\mathbf{C}_N + \Delta\mathbf{C})\dot{\mathbf{q}} + \mathbf{g}_N + \Delta\mathbf{g} = \boldsymbol{\tau}, \tag{5}$$

where nominal parts of the $\mathbf{M}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ and $\mathbf{g}(\mathbf{q})$ are denoted with subscript N and uncertain parts are prefixed with symbol Δ. Uncertainty is represented as an additive uncertainty [16]. Applying the feedback linearization method to the system (4) would result in linearized system with following equation:

$$\ddot{\mathbf{q}} = \mathbf{u}, \tag{6}$$

where $\mathbf{u}$ is the new control input. However, uncertainties of the mathematical model are not accounted for in such system, so in order to obtain the system model taking model uncertainties into account, the feedback linearization will be applied to the system represented with Eq. (4). For considered model it is assumed that feedback linearizing control law can only cancel out nominal nonlinearities leaving the uncertain parts. Regarding the previous sentence, linearizing control law is chosen in following form:

$$\boldsymbol{\tau} = \mathbf{M}_N \mathbf{u} + \mathbf{C}_N \dot{\mathbf{q}} + \mathbf{g}_N, \qquad (7)$$

where $\mathbf{u}$ is the new control input. Substituting equation Eq. (7) in Eq. (5) with assumption that uncertainties effects on inertia matrix are negligible, meaning that $\mathbf{M}^{-1}\mathbf{M}_N \approx \mathbf{I}$, new system of equations of motion is obtained:

$$\ddot{\mathbf{q}} = \mathbf{u} + \boldsymbol{\eta}, \qquad (8)$$

where $\boldsymbol{\eta}$ is vector representing uncertain part of the model and it is equal to:

$$\boldsymbol{\eta} = \mathbf{M}^{-1}(\Delta\mathbf{C}\dot{\mathbf{q}} + \Delta\mathbf{g}). \qquad (9)$$

Linearized system (8) can be further subjected to linear control laws which is the topic of the next subsection.

3.2 Iterative learning controller with fractional derivative

Iterative learning controllers have proven to be successful at improving trajectory tracking and uncertainty rejection. Here, we consider PD-type controller in feedback and feedforward PD$^{\alpha}$-type control action to improve trajectory tracking and ensure robustness of the system to uncertainties. Linear control law can be divided into feedback and feedforward sections:

$$\mathbf{u}_{k+1} = \mathbf{u}_{k+1}^{\mathrm{ff}} + \mathbf{u}_{k+1}^{\mathrm{fb}}, \qquad (7)$$

where feedback $\mathbf{u}_{k+1}^{\mathrm{fb}}$ and feedforward $\mathbf{u}_{k+1}^{\mathrm{ff}}$ terms are defined with following equations respectively:

$$\mathbf{u}_{k+1}^{\mathrm{fb}} = \mathbf{K}_{\mathrm{p}}^{\mathrm{fb}} \mathbf{e}_{k+1} + \mathbf{K}_{\mathrm{d}}^{\mathrm{fb}} \dot{\mathbf{e}}_{k+1}, \qquad (8)$$

$$\mathbf{u}_{k+1}^{\mathrm{ff}} = \boldsymbol{Q}\left(\mathbf{u}_k + \mathbf{K}_{\mathrm{p}}^{\mathrm{ff}} \mathbf{e}_k + \mathbf{K}_{\mathrm{d}}^{\mathrm{ff}} D^{\alpha} \mathbf{e}_k\right), \qquad (9)$$

where k is the iteration index, $\mathbf{e}$ is the error vector, $\dot{\mathbf{e}}$ is derivative of error vector, D^{α} is α-th order fractional derivative operator $\mathbf{K}_{\mathrm{p}}^{\mathrm{fb}}$ and $\mathbf{K}_{\mathrm{d}}^{\mathrm{fb}}$ are diagonal positive-definite proportional and derivative feedback gain matrices, $\mathbf{K}_{\mathrm{p}}^{\mathrm{ff}}$ and $\mathbf{K}_{\mathrm{d}}^{\mathrm{ff}}$ are diagonal positive-definite ILC learning gain matrices and $\boldsymbol{Q}$ is the lowpass filter. Lowpass filter is incorporated to filter out high frequencies which emerge in feedforward signal after apparent error convergence is achieved [17]. Filtering feedforward signal with lowpass filter will cause convergence towards non-zero steady-state value.

4. SIMULATION RESULTS

This section presents the simulation results of the assessment of fractional order impact on error norm convergence and validity of proposed controller. The numerical simulation is carried out in Matlab and Simulink using 3DoF model of exoskeleton for upper limbs. The exoskeleton model is defined in URDF (Unified Robot Description Format) and it consists of two truncated cones representing upper arm link and forearm link and chamfered cube representing shoulder link (Fig 1.). In Fig 1. $\mathrm{O}_{\mathrm{xyz}}$ is the reference coordinate frame, $\mathbf{e}_i$ are unit vectors representing the joint axes of rotations and C_i are centres of inertia of links both expressed in reference frame.

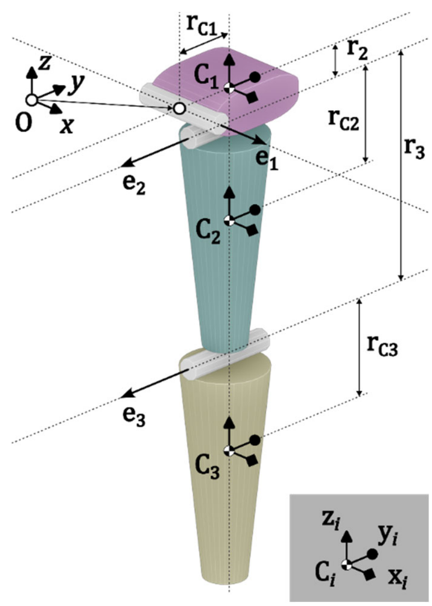

Fig. 1. Structure of the exoskeleton model

Distances between joint axes r_i, distances of centres of inertia from corresponding joint axes r_{Ci} and unit vectors $\mathbf{e}_i$ for exoskeleton model are given in Table 1.

Table 1. Joint parameters of the exoskeleton

Parameter	Joint 1	Joint 2	Joint 3
$\mathbf{e}_i$	[1,0,0]	[0, −1,0]	[0, −1,0]
r_i[m]	0	0.25	1.1
r_{Ci}[m]	0.25	0.493	0.493

Desired trajectory is defined in joint space coordinates as a fifth order polynomial:

$$q_d = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5, \quad (10)$$

The exoskeleton system is tasked with reaching certain point in joint space from zero position and returning back to zero for certain time period as shown in Fig 2. Waypoints of the desired trajectory for this simulation are summarized in Table 2.

Table 2. Waypoints of the desired trajectory

Waypoint no.	Joint 1 [rad]	Joint 2 [rad]	Joint 3 [rad]	Time period [s]
1	0	0	0	0
2	1	0.6	0.5	1
3	0	0	0	2

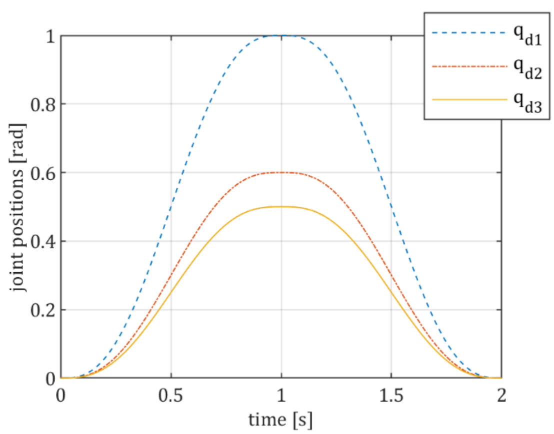

Fig. 2. Desired trajectory in joint space

Feedback gain matrices are chosen by trial and error:

$$\mathbf{K}_{\mathrm{p}}^{\mathrm{fb}} = diag[20,30,30], \mathbf{K}_{\mathrm{d}}^{\mathrm{fb}} = diag[10,15,15]$$

Learning gain matrices are chosen as:

$$\mathbf{K}_{\mathrm{p}}^{\mathrm{ff}} = diag[1,1,1], \mathbf{K}_{\mathrm{d}}^{\mathrm{ff}} = diag[1,1,1]$$

The cut-off frequency for lowpass filter Q is chosen to be $\mathrm{f_{cut-off}} = 1\mathrm{Hz}$.
The quality of the system response over iterations is evaluated by calculating infinite norm (also known as the max norm) of the error signal $\|\mathbf{e}(t)\|_\infty$ for each iteration and plotting it over iteration axis to see if there exists convergence trend towards zero or some non-zero steady-state value. Results from simulation show that fractional order ranging from zero to 1 in equation Eq. (9) exhibit similar behavior of an error norm over iterations as Fig 3. suggests. The convergence exists, but norms of error signal for different values of fractional order do not vary much between themselves. However, Fig 4. shows that fractional order in interval $\alpha \in [1,2]$ results in significant improvement of an error norm convergence. Increasing fractional order from 1 towards 2 error norm converges faster towards steady-state value. Going beyond fractional order of two, error norm displays divergence, i.e. system becomes unstable, Fig 5.

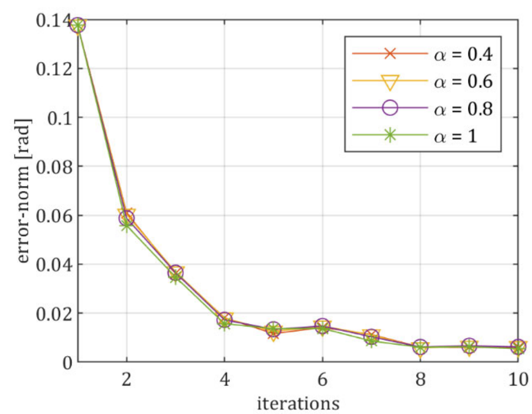

Fig. 3. Error norm for joint 1 for fractional order in interval $\boldsymbol{\alpha} \in [\mathbf{0}, \mathbf{1}]$

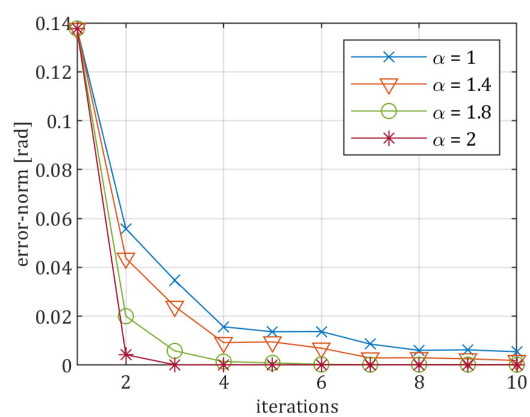

Fig. 4. Error norm for joint 1 for fractional order in interval $\alpha \in [1,2]$

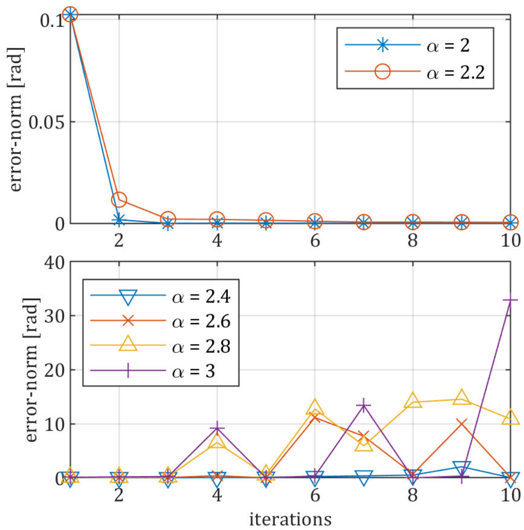

Fig. 5. Error norm for joint 1 for fractional order in interval $\alpha \in [2,3]$

Based on the previous results the best convergence properties, for tested exoskeleton system with uncertainties, achieves controller with fractional order in interval $\alpha \in [1.8,2.2]$.

5. CONCLUSION

The goal of the presented study was to assess the performance of the proposed fractional iterative learning controller of upper limbs 3DoF exoskeleton system with model uncertainties, taking into account the variation of fractional order values. The control scheme consists of feedback linearization compensating for the determined part of the nonlinear dynamics model and the feedforward ILC controller of the PD^{α}-type. Numerical simulations of the proposed control method have shown that a fractional order in certain interval between 1.8 and 2.2 gives significantly better control system performance compared to other values. Values of fractional order beyond 2.2 cause system to become unstable. The performance of the fractional order ILC controller in the determined order interval is superior compared to its integer order counterpart.

Acknowledgement

This research has been supported by the research grants of the Serbian Ministry of Science, Technological Development and Innovations, grant No. 451-03-68/2023-14/200066 and 451-03-47/2023-01/ 200105 from 03.02.2023.

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CIP - Каталогизација у публикацији
Народна и универзитетска библиотека
Републике Српске, Бања Лука

621.3(082)(086.76)
621(082)(086.76)

МЕЂУНАРОДНА конференција о достигнућима у машинству и индустријском инжењерству (16 ; 2023 ; Бања Лука)

Proceedings [Електронски извор] : DEMI 2023 / [16th international conference on accomplishments in mechanical and industrial engineering, Banja Luka, Jun 2023 ; editor in chief Petar Gvero]. - El. zbornik. - Banja Luka : Faculty of Mechanical Engineering, 2023. - 1 електронски оптички диск (CD-ROM) : слика ; 12 cm

Систематски захтјеви: нису наведени. - Насл. са насл. екрана. - Ел. публикација у ПДФ формату опсега 533 стр. - Опис извора дана 7. 06. 2023. - Библиографија уз сваки рад.

ISBN 978-99976-11-04-8

COBISS.RS-ID 138545409