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Izvorni naučni članak

EFFECTS OF HORIZONTAL INTERACTION ON REDISTRIBUTION OF FORCES OF PILES IN A GROUP

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ABSTRACT
This paper shows a horizontal interaction of soil-piles in a group. Different arrangement of piles and effects of the interaction on forces redistribution has been shown. Actual dimensions of piles have been considered with a focus on interaction, as well as the effects of stiffness on redistribution of forces with a certain probability.

KEY WORDS: Group of piles, vertical interaction, horizontal interaction, pile displacement, coefficient interaction

UTICAJ HORIZONTALNE INTERAKCIJE NA RASPODELU SILA KOD GRUPE ŠIPOVA

REZIME
U ovom radu prikazana je horizontalna interakcija šipova u grupi. Prikazan je različiti raspored šipova i uticaj interakcije na preraspodelu sila. Razmatrane su i relane dimenzije šipova uz kritički osvrt na interakciju, sa uticajem krutosti na preraspodelu.sa određenom verovatnoćom.

KLJUČNE REČI: Grupa šipova, vertikalan interakcija, horizontalna interakcija, ugib šipova, koeficijent interakcije

INTRODUCTION

The analysis of redistribution of forces and stiffness of a group of piles fixed in a cap-slab is conducted. A horizontal interaction of a group of piles is considered. The limited extent

of this paper does not allow treating the vertical interaction, and it should be discussed in a separate paper.
The interactions is defined as a deformation „displacement" of an unloaded pile, due to the action of an adjacent loaded pile, or as additional displacement of a loaded pile due to the action of the load on the adjacent pile. Interaction factor:

$$\alpha = \frac{Displacement\ caused\ by\ unit\ action\ on\ an\ adjacent\ pile}{Diplacement\ of\ the\ pile\ under\ unit\ head\ action}$$

Here, inertial or kinematic interactions are not considered, but only the effect of interaction under the action of a static horizontal and vertical force on piles in a group. The dynamic effects can be obtained from the *p-y* curves, when redistribution coefficients are applied on *p-y* curves for each individual pile.

LATERAL STIFFESS/FLEXIBILITY OF A PILE FOR THREE SOIL MODELS

Slika 1 Raspodela krutosti po dubini profila tla: konstantna, parabolična i linearna
Figure 1 Distribution of stiffness along the depth of soil profile: constant, parabolic and linear

The soil having constant stiffness with depth is typical for overconsolidated clays, stiffness linear change is characteristic for soft, normally consolidated clays and sand for higher dilatation levels. Parabolic change modulus is characteristic for sand but for small dilatation levels.

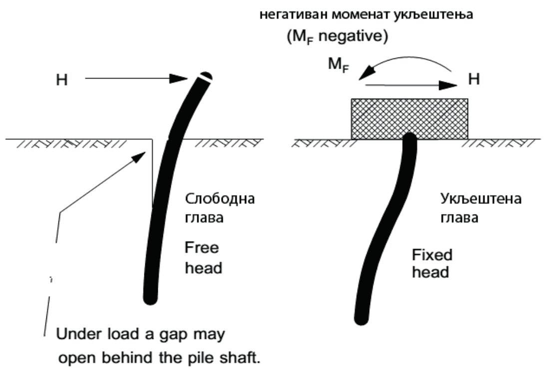


Figure 2 Free and fixed head piles.

Slika 2. Slobodna i uklještena glava šipa.

Figure 2. Free and fixed head pile.

A CONSTANT MODULUS OF SOILS

The equations for this soil model profile were given (Davies and Budhu) in the form:

$$K = \frac{E_P}{E_S} \tag{1}$$

If the actual length is greater than the active length (Pender), the pile is "long" (flexible) and the following expressions are used for the coefficients of flexibility:

$$f_{\mathrm{uH}} = \frac{1.3K^{-0.182}}{E_S D};\ f_{\mathrm{uM}} = f_{\theta H} = \frac{2.2K^{-0.455}}{E_S D^2};\ f_{\theta M} = 9.2\frac{K^{-0.727}}{E_S D^3} \tag{2}$$

In the paper (Pender, 1993) showed that the Winklers model and the elastic continuum model gave similar results.

Equations (2) is for the case of free head piles. For fixed head piles (3) the following displacement of a pile head is given:

$$u_F = f_{FH} H \quad f_{FH} = \frac{0.80K^{-0.18}}{E_S D} \tag{3}$$

LINEAR SOIL MODULUS VARIATION

Budhu and Davies provided the equation for linear soil modulus variation. For this case, the Young's modulus of soil and stiffness is:

$$E_S = mD \;;\; K = \frac{E_P}{E_S} = \frac{E_P}{mD} \tag{4}$$

Where m is rang increase of the Young’s modulus with depth. Budhu and Davies gave values *m* for different densities of sand. This is appropriate for static loading of piles. but not for dynamic excitation of piles embedded in loose saturated sands. Other coefficients, equations and theory, see in Folić 2017.

LATERAL STIFFNESS OF A GROUP OF PILES

Horizontal displacements of group containing more than two piles are obtained from an equation which is similar to the equation for vertical displacement of a vertically load group of piles. For a group of n piles having a free head, the horizontal displacement of pile *k* is:

$$u_k = u_1\left(\sum_{j=1}^{n} H_j \alpha_{uHkj}\right) \tag{5}$$

From the equilibrium conditions, the equation for horizontally loaded group of piles is:

$$H = \sum_{j=1}^{n} H_j \tag{6}$$

Randolf 1981 (Pender, 1993) proposed the following expression for the horizontal interaction coefficient α_{uF} :

$$\alpha_{uF} = 0.3\left(\frac{D}{s}\right)\left[2(1+\nu)K\right]^{0.143}(1+\cos^2\xi) \tag{7}$$

Where ξ is defined on figure 3.
The was another formula (Fleming, Weltman, Randolph and Elson 1998), where diameter D was replaced with radius r_0.

$$\alpha_{uF} = 0.6\rho_c\left(\frac{r_0}{s}\right)\left[2(1+\nu)K\right]^{1/7}(1+\cos^2\xi) \tag{8}$$

This is actually the same expression as (7), the difference is in the term ρ_C, which is a more general form, because it includes the soil modulus variation with depth. The coefficient ρ_C, for the sandy soil is 0,5. However (Klar, Spasojević and Soga, 2004) provided the following formula, which somewhat differed from the previous two ones.

$$\alpha_{uF} = 0.3\left(\frac{D}{s}\right)\left[\frac{2(1+\nu)}{1+0.75\nu}K\right]^{1/7}(1+\cos^2\xi) \tag{9}$$

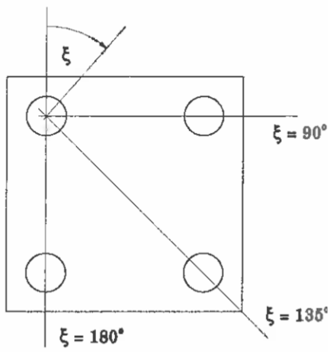
The results obtained by using the expression (9) differ from the result of the previous two expressions for 3 to 5% (the section in the square brackets under the square root). For the values of Poisson's ratio ranging from 0,3 and 0,5 and for the stiffness ratio K=1000 and K=800 the results in tables 1 and 2 are provided. This simplification favors safety, because it increases the effects of horizontal displacements due to interaction for about 4%.

Tabela 1 Vrednosti konstantnog dela jednačine koeficijenta horizontalne interakcije K=1000
Table 1 Values of the constant part in equation of lateral interaction K=1000

Poisson's ratio		Equation 7	Equation 8	Equation 9
ν =	0,5	3,14216	3,13857	2,99898
	%	100	99,89	95,44
ν =	0,4	3,11131	3,10779	2,99346
	%	100	99,89	96,21
ν =	0,3	3,07851	3,07506	2,98719
	%	100	99,89	97,03

Tabela 2 Vrednosti konstantnog dela jednačine koeficijenta horizontalne interakcije K=800
Table 2 Values of the constant part in equation of lateral interaction K=800

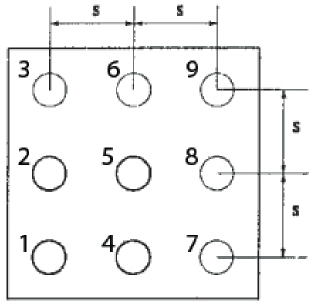
Poisson's ratio		Equation 7	Equation 8	Equation 9
ν =	0,5	3,043478	3,040096	2,904891
	%	100	99,89	95,45
ν =	0,4	3,013599	3,01028	2,899541
	%	100	99,89	96,22
ν =	0,3	2,981831	2,978578	2,893465
	%	100	99,89	97,04



Slika 3 Ugaoni odnosi između pojedinih šipova u grupi 2x2.

Figure 3 Angle relations between individual piles in a group of 2x2.

Example 1. Group of 3x3 piles linked by a cap with fixed head. The piles distribution is shown on figure 4. The distance pile axis is 5D, in both axes. Soil has a constant modulus E_S=25 MPa (overconsolidated clay). The pile is long (flexible), diameter is D=0,75m. The pile modulus is E_P=25 GPa. Poisson's ratio ν=0,50.



Slika 4 Raspored šipova u grupi 3x3
Figure 4 Group of piles in 3x3 arragment

A soil profile with a constant modulus, displacement and bending moment for a pile with a fixed head can be calculated using the following equations:

$$M_F = I_{MH} DH \quad I_{MF} = 0.24 K^{0.27} \tag{10}$$

$$u_F = f_{FH} H \quad f_{FH} = \frac{0.80 K^{-0.18}}{E_S D} \tag{11}$$

Stiffness of the piles with fixed head, under the action of a lateral force may be calculated like inverse coefficient flexibility:

$$K_{\mathrm{HF}} = 25 \times 0.75 / 0.8 \times 1000^{0.18} = 81.3\, MN/m \; or \; (kN/mm) \tag{12}$$

The stiffness of a group of 4 piles at a distance of 5D, is:

$$K_{\mathrm{HFG}} = 4 \times 81.3 / (1 + 0.377 + 0.189 + 0.20) = 184.1\, (kN/mm) \tag{13}$$

The coefficient factor interaction of a soil with uniform modulus, is calculated using Randolf.

The equation for calculation of interaction coefficient depends on the angle in the form $\mathrm{Cos}^2\xi$, by calculating the angle in relation to the vertical axis, because it is the direction of force action. Regarding that the cosine function is raised to second power (square), the direction of measuring the angle in relation to the vertical (direction of force action) is not important. This can prove important for the rate of calculation of interaction coefficients of a large group of piles. If symmetry can be employed, the rate can be greatly increased, and the time required for calculation can be almost halved for uniaxial symmetry.. In the case of a biaxial symmetry in the distribution of a group of piles, the calculation is reduced to approximately one quarter. The case of multi-axial and polar symmetry (multiaxial, circular or ring-like) are the special cases of groups of piles, which are mostly used for bridges and industrial structures, but they are not discussed here.

The displacements of the piles heads are equal. Stiffness of a group of 3x3 is:

$$K_{HGF}=289{,}77 \text{ kN/mm} \tag{14}$$

H_i (kN) Distribution of lateral forces after redistribution caused by interaction (arrang.3x3) is:

138,09	85,83	138,09
110,95	54,08	110,95
138,09	85,83	138,09

OC Clay 5D; 075; *max Hi* =138,09; *min Hi* =54,08; *Δ extr Hi*=84,01.

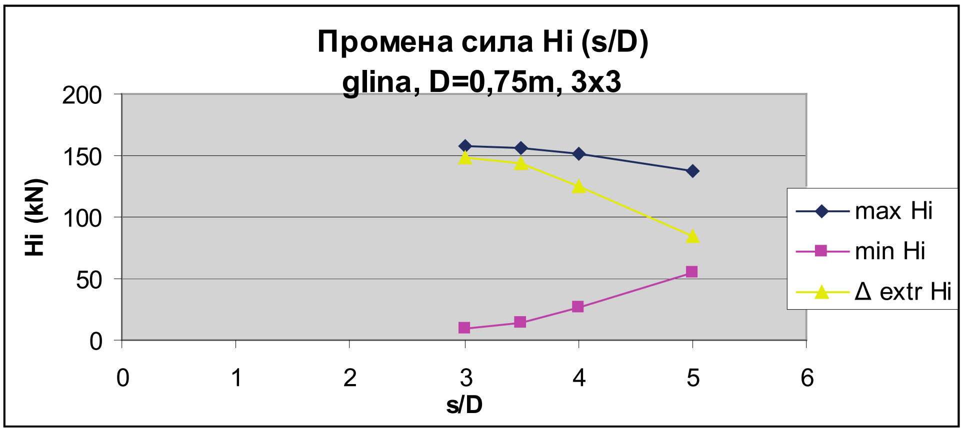
The ratio of the forces normalized in relation to the maximum force after redistribution is:

1	0,622	1	Σ 2,62		
0,803	0,392	0,803	Σ 2,00	Σ Σ 7,24	0,80
1	0,622	1	Σ 2,62		

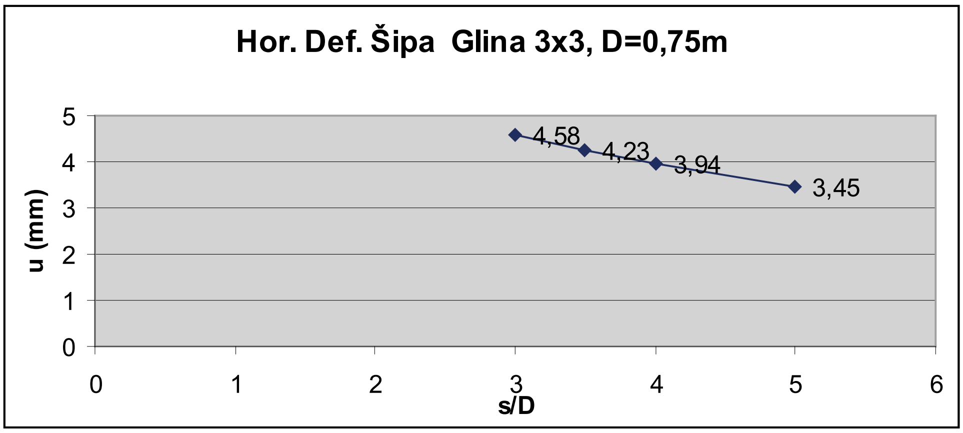
If the efficiency is observed in relation to the maximum force, then it is here 80%.

Displacement of a group of piles is:	3,45	(mm)
Stiffness of a group of piles is:	289,77	(kN/mm)
Efficiency of a group piles stiffness is	39,62	%

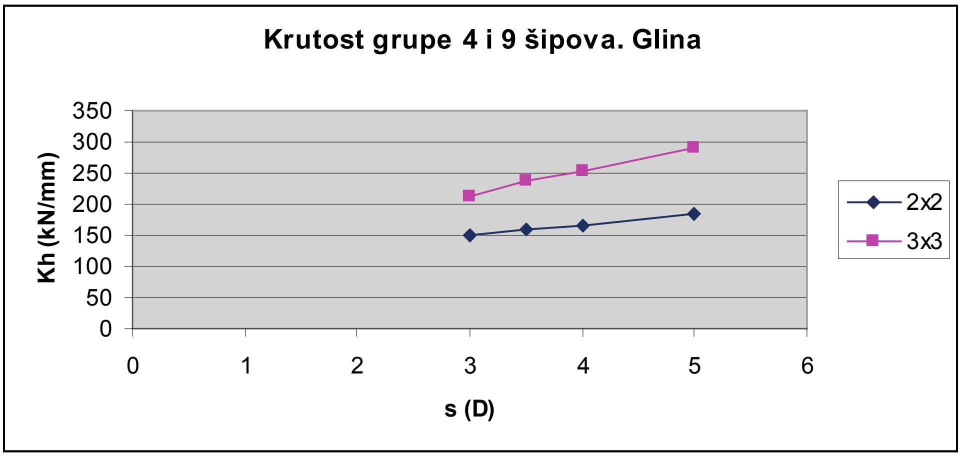
The efficiency of a group of piles in 3x3 distribution, having diameter 0,75m founded in overconsolidated clay, expressed through stiffness is only 50% of the efficiency expressed through maximum force. This holds for the distance of the pile axes of 3, 3.5, 4 and 5 *D*.



Slika 5 Dijagram preraspodele sila u **glini** usled horizontalne interakcije.
max H_i; min H_i; Δ extr H_i. 3x3. D=0.75 m.
Figure 5 Diagram of forces redistribution in overconsolidated clay caused by lateral interaction.
max H_i; min H_i; Δ extr H_i. 3x3. D=0.75 m.



Slika 6 Dijagram horizontalne deformacija grupe šipova usled horizontalne interakcije. 3x3. Glina. D=0.75 m.
Figure 6 Diagram of a lateral settlement of a group piles caused by lateral interaction. 3x3. Clay. D=0.75 m.



Slika 7 Dijagram promene krutosti za grupu od 2x2 i 3x3 šipova u glini zavisnosti od međusobnog rastojanja D=0.75m.
Figure 7 Diagram of stiffness variation of 2x2 and 3x3 groups of piles E_S=const. (in clay) depending on the mutual distance D=0.75m.

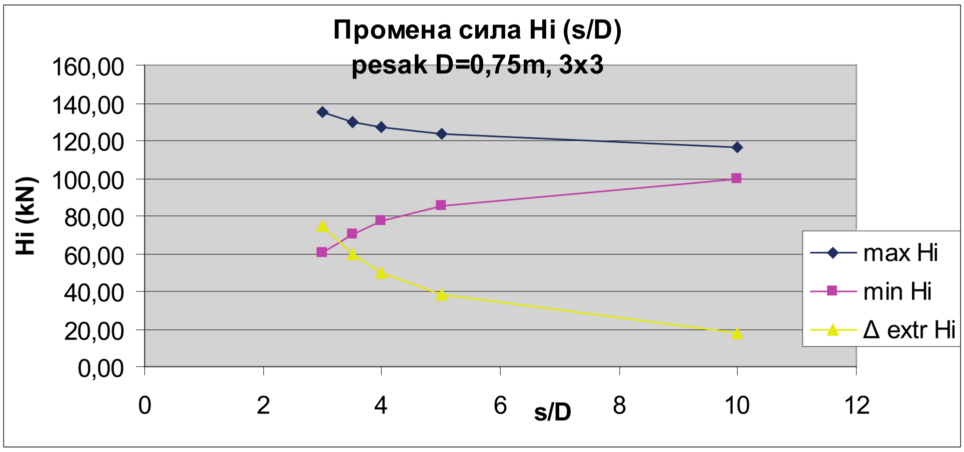
Example 2 With the pile diameter D=0.75m, which is a modified example from (Pender,1983), because the soil modulus is linearly variable by depth (soil-sand). The soil modulus adopted is at the depth of on one diameter of the pile. s/D=10. m=16,3 MPa/m; $E_S=m\cdot D$=12,23 MPa.

Here, m=16.3 MPa/m which for this pile results in the soil modulus $E_S=m\cdot D$=12,23 MPa. Therefore, only at the soil depth of 1,53 m (or 2,05 D) , the soil-pile modulus equals 25 MPa.

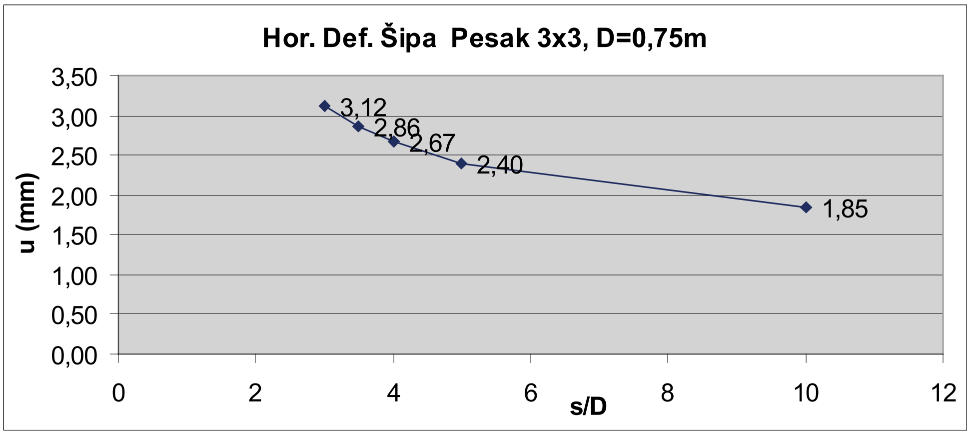
The coefficients of flexibility and stiffness of the pile differ, with the soil with linearly variable modulus by depth (labeled m) in comparison to the soil with a constant modulus. For the soil with linearly variable modulus by depth, displacement and momentum for the piles with fixed heads are calculated according to the following equations:

$$u^F = f_{FH} H \quad \mathrm{f}_{\mathrm{FH}} = \frac{1.35 K^{-0.333}}{m D^2} \tag{15}$$

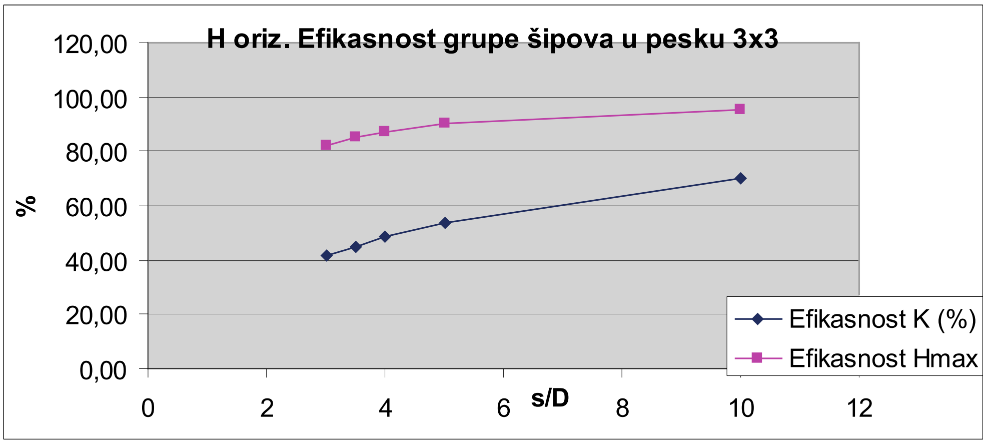
$$M^F = -I_{MF} H D \quad I_{MF} = 0.37 K^{0.222} \tag{16}$$



Slika 8 Dijagram preraspodele sila u pesku usled horizontalne interakcije.
max Hi; min Hi; Δ extr Hi. 3x3. m=16.3 MPa/m, D=0.75 m.
Figure 8 Diagram of redistribution of forces in sand due to lateral interaction.
max Hi; *min Hi*; *Δ extr Hi*. 3x3. *m*=16.3 MPa/m, *D*=0.75 m.

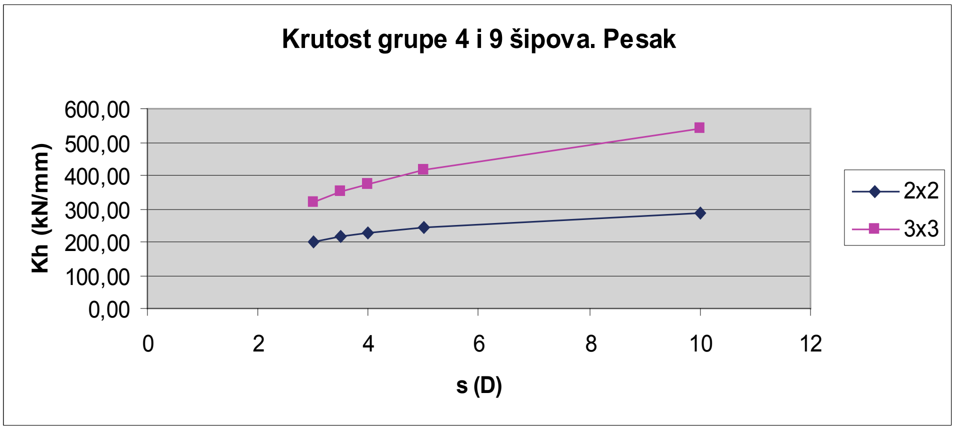


Slika 9 Dijagram horizontalne deformacije grupe šipova usled horizontalne interakcije. 3x3. Pesak. m=16.3 MPa/m, D=0.75 m
Figure 9 Diagram of horizontal deformation of a group of piles due to horizontal interaction. 3x3. Sand. *m*=16.3 MPa/m, *D*=0.75 m



Slika 10 Dijagram efikasnosti grupe šipova usled horizontalne interakcije: prema krutosti i prema maksimalnoj sili. Raspored 3x3. Pesak. Krutost naglo pada nakon smanjenja rastojanja ispod 5 prečnika. m=16.3 MPa/m, D=0.75 m

Figure 10 Diagram of efficiency of a group of piles due to horizontal interaction: according to stiffness and maximum force. Arrangement 3x3. Linearly variable soil modulus (sand). Stiffness decreases abruptly after distance is reduced under 5 diameters *m*=16.3 MPa/m, *D*=0.75 m



Slika 11 Dijagram promene krutosti za grupu od 2x2 i 3x3 šipova (ugrađenih) u pesku zavisnosti od međusobnog rastojanja. m=16.3 MPa/m, D=0.75 m.

Figure 11 Diagram variation stiffness for a group of piles 2x2 and 3x3 embedded in soil with linear variation modulus depending on mutual distance. m=16.3 MPa/m, D=0.75 m.

RESULT ANALYSIS

In the case of making foundations in the soil with linearly variable modulus by depth ($E_S=m \cdot D$=25 MPa) the horizontal stiffness of a group of piles having diameter 0,75 m in a distribution 3x3 is less different than the horizontal stiffness of the groups in the distribution 2x2, if the distance between the piles is 3; 3.5 or 4 *D*. The difference becomes more prominent only at 5*D*. According to the proposed calculation theory, if for dimensioning of a group of piles a dominant condition is horizontal stiffness (and not vertical bearing capacity) it is not cost efficient to increase the number of piles in a group with a distance of up to 4 *D* between them, but it is better to increase the axial distance between the piles to 5 *D* or to opt for larger distances and diameters of piles. When for the 3 *D* , the 2x2 distribution is changed to 3x3 distribution, the stiffness of the group due to interaction increases for only 59%, for 5 *D* it is 72%, and for 10 *D* it is 90%.

CONCLUSION

When selecting the number and distribution of piles in a group, if the horizontal stiffness does not have a crucial impact on the designing factor, but rather resistance to horizontal force (shear) of the pile heads, the increase of a number of piles in a group is an acceptable solution, event at distances between the piles smaller than 4 *D*. This is the consequence of the horizontal interaction of piles in a group. The increase of the number or diameter of piles is adequately implemented in seismically active areas, when design is produced according to the properties, that is, seismic capacity of the structure. Therefore, if in design of the piles, horizontal stiffness of a group has an important role, it is more favorable to use the axial distances of 4 or 5 D. Of course, in case of the diameters larger than 5 *D* one should consider dimensions and stiffness of a cap, because larger diameters of piles for larger mutual distances can be implemented for the foundation slabs, where there are no classic caps (in the case of buildings).

When designing foundations on the piles, the necessary condition, in addition to adoption of a number of piles is determination of axial distance. Axial distance is limited by the dimensions of caps and/or by the thickness of a slab (stiffness) In some cases of the 3x3 destruction, when the axial distances are small, such as 3 to 4*D*, due to a specific interaction it is cost effective to exclude the central pile, unless it is used for seismic function to increase plastic hinges and dissipation of seismic energy. It can be implemented even for more than 9 piles. The other conclusions are provided in the text and the result analysis.

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