SIMULATION OF INVOLUTE GEAR TOOTH PROFILE SHAPING

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Abstract

The involute is the most common shape of the gear tooth profile due to a number of its advantages compared to other profiles used in practice. The main advantages are the same type of tool for making both meshed gears, simple geometry of the tool, simple control and measuring of the shape and dimensions of the teeth, as well as that the gear meshing is not sensitive to varying gears' centre distance. Involute gears made with the same cutting tool - toothed rack can mesh correctly, regardless of the number of teeth. This fact, as well as the simple shape of the blade (a straight line), contributed to the establishment of the toothed rack as the basic tool for making involute teeth. In engineering education in the field of machine design and machine elements, it is still important to consider in detail the theory of gear pairs. Students learn about the geometry of gears, the kinematics of gear meshing, the strength, calculation of load carrying capacity, service life and reliability of gear pairs in various applications. Regardless of modern information technologies, modelling and computer simulations, the classic, traditional way of learning (or possibly hybrid, as a combination of conventional and modern approaches) is still the most prevalent in teaching. At the same time, laboratory exercises are of essential importance. The existing design of the laboratory educational device for simulating the generation of gearing using a toothed rack is presented in this paper. Then its design was modelled in the SolidWorks program and an improved variant solution is suggested. Also, a simulation of the involute teeth shaping was performed using AutoCAD.

Keywords: gears; involute profile; gear tooth shaping simulation; gear shaping educational device.

1 INTRODUCTION

The beginning of the "science and art" of gearing dates back to the BC era. [1]. In the paper [1], the author divided the evolution of the gearing theory into two periods: pre-Eulerian (Antikythera mechanism, Aristotle, Leonardo da Vinci, A. Dürer, R. Hooke, G. Desargues, P. de la Hire, C.E.L. Camus) and post-Eulerian (T. Olivier, C. Gochman, G.B. Grant, V.A. Shishkov, S. Radzevich, F.L. Litvin and many other researchers up to the present time). These two periods are separated by the time in which Leonhard Euler, Swiss mathematician, physicist, astronomer, geographer and engineer, made a fundamental contribution to the theory of involute gearing and laid the foundations for further development. The involute function was known before Euler. It was discovered by Christiaan Huygens (Fig. 1a), a Dutch mathematician, physicist, engineer, astronomer, and inventor. Huygens presented and mathematically described the involute in 1673 in his work entitled "Horologium oscillatorium sive de motu pendulorum ad horologia aptato demonstrationses geometricae" [1]. Also, before Euler, there were attempts to define the conjugate action law. Charles Etienne Louis Camus (Fig. 1b), a French mathematician and mechanician, was the closest to the "conjugate action law" for gearing with parallel axes, but he cannot be considered to have discovered this law [1,2]. Namely, he did define the gearing condition in the form of continuous contact of two surfaces in relative motion, but he considered that the line of action of those two surfaces is a curve. However, the line of attack (LA) must be straight, the common normal to the two contact surfaces in relative motion, just as the force acts along a straight line [1,2]. Leonhard Euler (Fig. 1c) connected the gearing condition with the properties of the involute and proposed the involute gearing and the involute tooth profile, which best fits the practical needs of the industry until today. Accordingly, the "conjugate action law" for gearing with parallel axes is reduced to a short definition: "a pair of transverse gear tooth profiles are said to be conjugate if a constant angular velocity of one profile produces a constant angular velocity in the meshing profile" [6]. The involute tooth profile is the only tooth shape that can be used for conjugate gears. In 1760, Euler proposed the ideal involute gearing of gear teeth with parallel axes, and this is considered the beginning of the scientific theory of toothing [1].

a) b) c) d)

Fig. 1. a) C. Huygens (1629-1695) [3]; b) C.E.L. Camus (1699-1768) [2]; c) L. Euler (1707-1783) [4]; d) R. Willis (1800-1875) [5]

The fundamental theorem of gearing is the accomplishment of Euler and Felix Savary (1797-1841), a French mathematician and professor of astronomy. Félix Savary derived the Euler-Savary formula in its modern form. Savary's proof can be found in "Leçons et cours autographiés, Notes sur les machines, par le professeur F. Savary, Ecole Polytechnique, 1835–1836 (unpublished lecture notes; available in the Bibliothéque Nationale in Paris) [7]. Although the creators of the theorem were Euler and Savary, it became widely known in Europe only after its publication in the well-known book of Robert Willis, English Cambridge professor and mechanical engineer (Fig. 1d), "Principles of Mechanisms, Designed for the Use of Students in the Universities and for Engineering Students Generally" (London, 1841). According to Willis, this theorem reads: "The angular velocities of the two pieces are to each other inversely as the segments into which the "line of action" divides the line of centres, or inversely as the perpendiculars from centres of motion upon the line of action" [1]. That is why the name Willis theorem was used for this theorem. However, nowadays this theorem is most often referred to as the "Camus-Euler-Savary fundamental theorem of gearing" or, more simply, the "CES theorem of gearing" [1]. In this paper, after a theoretical introduction, the procedure for teeth shaping by the relative motion of the tool - the toothed rack and the gear being processed is discussed. The simulation of gear shaping in AutoCAD was explained as well. The laboratory educational device for simulating this procedure and the models that can be obtained from it are presented. The device was modelled in SolidWorks and the complete technical documentation of the variant solution with improved design and an increased number of possibilities for simulating the making teeth profile was created.

2 INVOLUTE GEARING

2.1 Involute

The shape of the teeth profile of one gear can be any. However, the shape of the meshed gear depends on the series of successive positions of the first gear tooth profile. At each point of contact, the profiles must have a common normal. To achieve this, the teeth profile of the second gear must be the envelope of a series of consecutive positions of the first gear teeth profile. An envelope is a line that touches all consecutive positions. The production of gears with tooth profiles of different shapes is technologically feasible but quite complicated and expensive (different tools of complex shapes are required). Therefore, the tooth profiles of both gears must be defined in the same way and described by the same curve. The curve that satisfies this condition is the involute. This was first observed by Euler and in his first papers he proposed an involute profile for the tooth profile of gears with parallel axes. Apart from the convenience of using the same tool with a simple geometry for both gears, the involute tooth profile also has advantages: both gears have the same tooth profile, the gearing is correct (the transmission ratio is constant), the gear mesh is not sensitive to changes in the axial distance, the dimension and form control is simple (i.e. universal control tools and instruments can be used). The involute of a circle is a curve described by a point on a straight line (tangent) that rolls around the circle without sliding (Fig. 2). The circle on which the tangent whose points describe the involute rolls is called the base circle and is designated "b". The tangent to the base circle t(b) is also the normal to the involute n(i), as shown in Fig. 2. This figure also shows the equations of the coordinates of the involute function (x, y) in the Cartesian coordinate system.

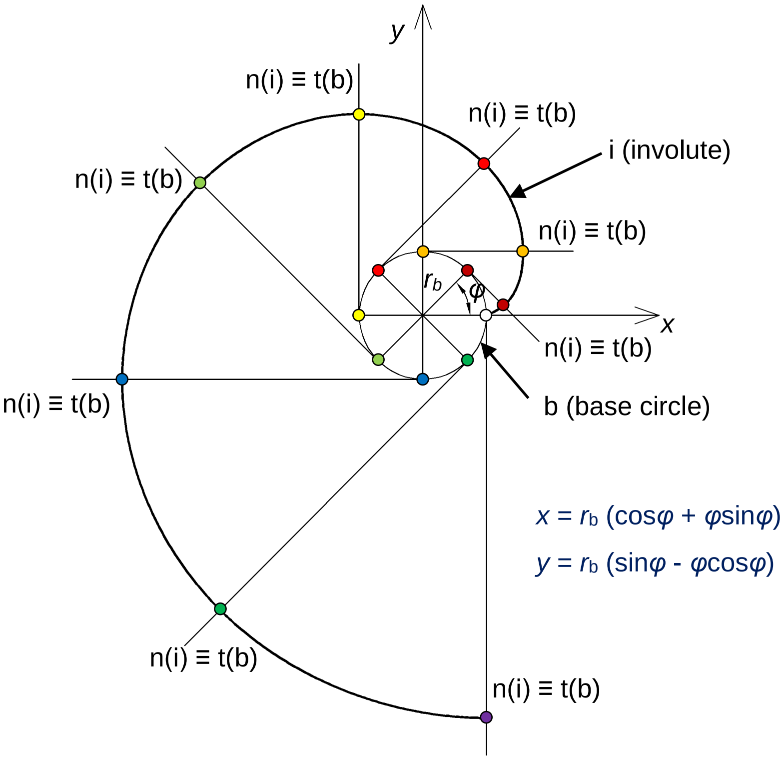


Fig. 2. Involute

The dimensions of the involute, used in the theory of involute gearing, are shown in Fig. 3. Point Y is a random point on the involute with the basic circle of radius r_b. The radius of the circle on which point Y is located is r_y. The angle between the radius vector of the involute start point A on the basic circle and the radius vector of the point Y is the basic parameter of the involute - the involute angle θ_y. The angle between the radius vector of the point where the tangent forming the involute touches the basic circle B and the radius vector of the point Y is the pressure angle of the involute α_y. It is the angle of the line of action, the normal to the involute and the tangent to the base circle for the point Y. Between these two angles there is a functional relationship called the involute function $\theta_y = \text{inv}\alpha_y = \tan\alpha_y – \alpha_y$ (Fig. 3).

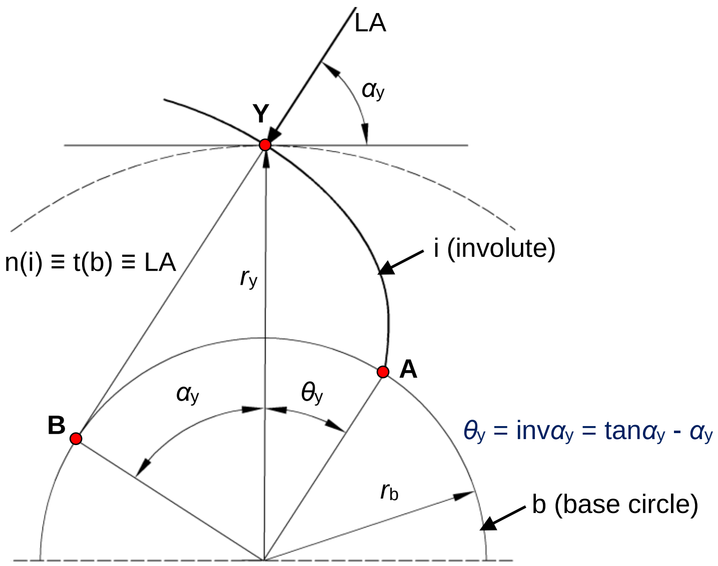


Fig. 3. Involute parameters according to the theory of involute gearing

2.2 Involute tooth shaping by rack-type cutter

In the International Standard ISO 54 [8], the standard basic rack tooth profile is defined, which provides geometric references for determining the tooth size of the involute gear system. This standard does not define the tool – gear cutter, but the dimensions of the tool can be defined according to the standard profile as a corresponding conformal profile (mating standard rack tooth profile). Such a tool with a conformal profile concerning the standard profile is shown in Fig. 4. Geometrical parameters of a standard basic rack are pitch – $p = \pi m$; tooth thickness – $s_P = 0.5p$; space width – $e_P = 0.5p$; pressure angle – α_P; tooth depth – h_P; dedendum of the tooth – h_{fP}; addendum of the tooth – h_{aP}; bottom clearance between standard basic rack tooth and mating standard basic rack tooth – c_P; fillet radius of the basic rack – $r_{fP} = c_P/(1\text{-sin } \alpha_P)$; module – m. The dimensions of the standard basic rack tooth profile use pitch line P-P as reference (datum) line.

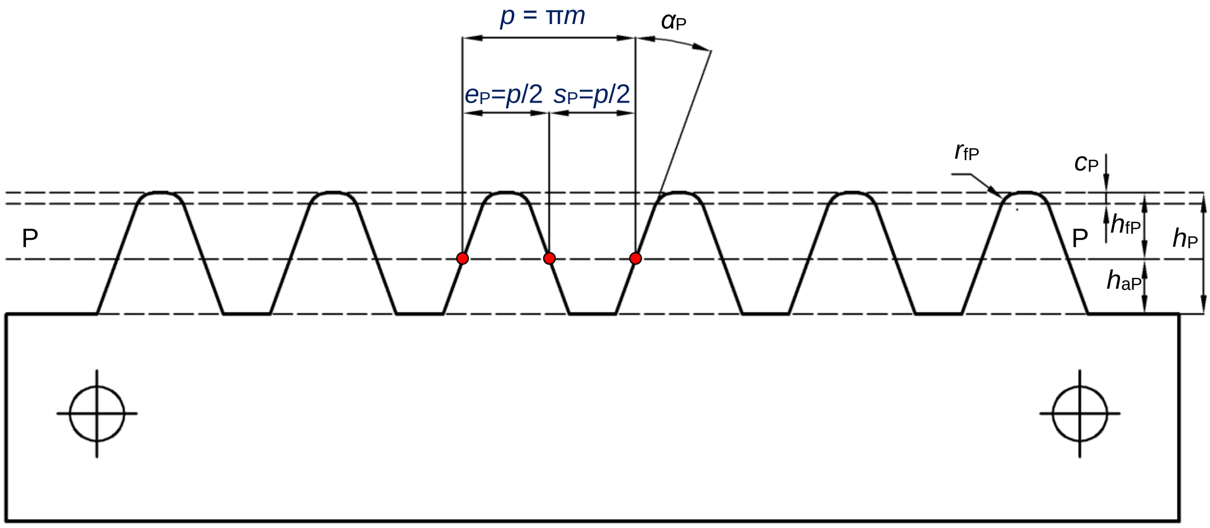


Fig. 4. Gear cutter with mating standard rack tooth profile

Values of the standard basic rack tooth profile characteristics are given in Table 1 [8].

Table 1. Standard profile parameters

Item	Standard basic rack value	
	Gears transmitting high torques	Gears for normal service
α_P	20°	20°
h_{fP}	1m	1m
h_{aP}	1.25m	1.2m
c_P	0.25m	0.2m
r_{fP}	0.38m	0.3m

The process of making tooth profile using a rack-type tool is called the relative rolling process and is based on the basic law of gearing. Relative rolling during gearing is the rolling of a pitch line on a pitch circle. A series of successive positions of this basic profile during the relative rolling of the rack tool and the gear forms the shape of the profile of the gear teeth, as an envelope of a series of successive positions of the rack cutter blade (Fig. 5).

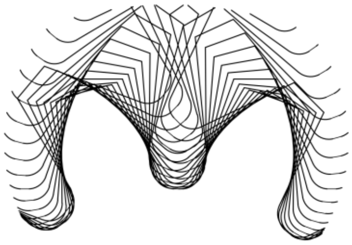


Fig. 5. Enveloping (AutoCAD simulation)

According to the described principle of relative motion of the rack-type tool and gear, the Swiss company MAAG has developed a technological method and produced a suitable machine (Fig. 6). In the video comment [9], the author states that “MAAG gear shaping machine is the most accurate gear cutting in the world. It is used to manufacture master gears or the gears with high accuracy and without sound.”

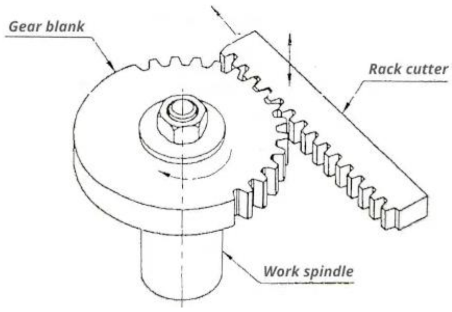


Fig. 6. MAAG gear shaper model designation (SH 75 K) [9] and MAAG method scheme [10]

The pitch line P-P of the standard profile is shown in Fig. 4. It is the middle line of the profile, which divides the straight part of the profile into two halves. Also on this line, the tooth thickness space width is equal (half the pitch). The zero position of the rack concerning the gear is the position where the pitch line of the rack is tangent to the pitch circle of the gear to be cut. The pitch diameter of the gear is equal to the product of the number of teeth and the module, i.e. $d = zm$. However, the teeth can also be cut when the rack moves away from the axis of the gear or closer to the axis of the gear by a certain value. The rack shift concerning the axis of the gear can be negative or positive and is expressed as a function of the module, i.e. as a product xm of the module and the shift coefficient (Fig. 7).

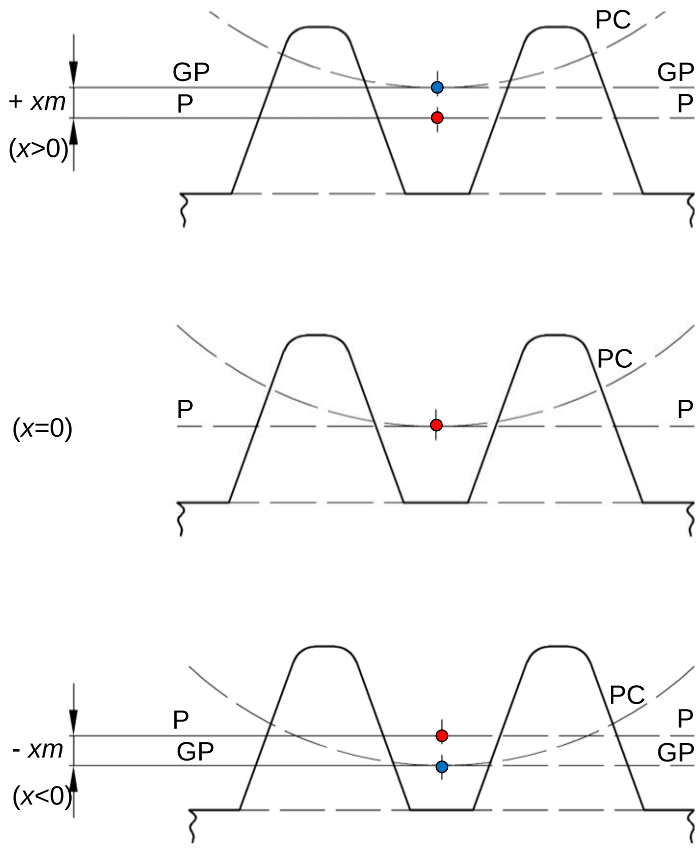


Fig.7. Rack cutter shifting: positive, zero (no shifting) and negative, respectively

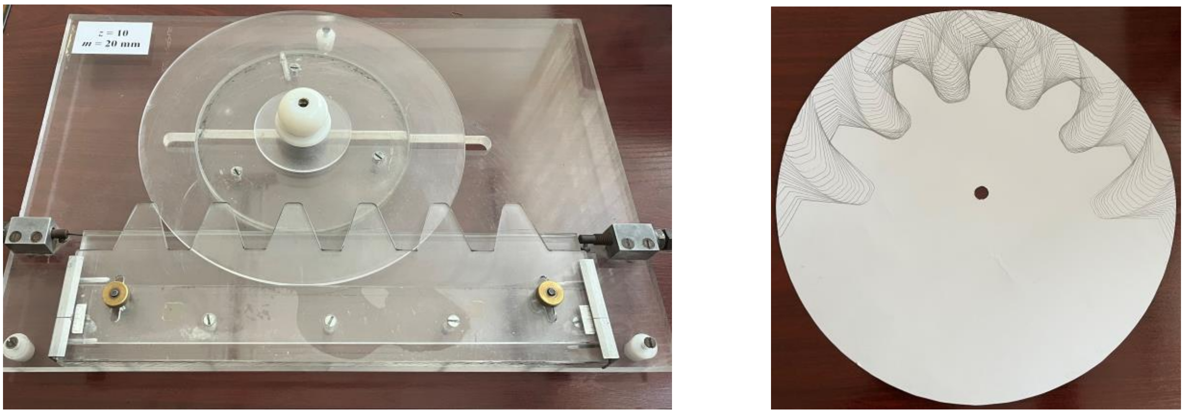
The dimensions of any rack are determined by the module (Table 1). Module values are standardized [11]. The diameter of the pitch circle of the gear depends on the number of teeth. The positioning of the rack concerning the axis of the gear depends on the size of the pitch circle. The three basic parameters that completely determine all the geometric dimensions of the gear pair teeth are the number of teeth, the module and the profile shift coefficient. The gear rack is positioned concerning the gear axis depending on the pitch diameter of the gear and the sign and value of the profile shift coefficient. The rack profile shift is the distance between the gear pitch line GP-GP (tangent to the PC gear pitch circle) and the rack pitch line P-P. Each number of teeth and module corresponds to the limit values of the rack profile shift coefficients (x_{max}, x_{min}), for which there are appropriate formulas and/or tables of recommendations. Regardless of the shifting, the number and module of the teeth of the gear will be the same, but the shape of the teeth profile will differ. If the shifting is negative, the tooth has a wide tip and an undercut root. A tooth made with a positive shifting has a wide dedendum with a large cross-section, but the tooth thickness at the top is thinner (see later).

3 EDUCATIONAL EQUIPMENT

3.1 Actual design

The training system for shaping simulation of involute gear tooth profile, used for several decades in laboratory work under the course "Machine elements" at the Faculty of Mechanical Engineering (University of Belgrade, Serbia), is shown in Fig. 7a. This device simulates the generating of gear with the number of teeth z = 10 and the module m = 20 (the pitch circle is d = 200 mm). The pitch of the rack and generated gear is p = 62.8 mm. According to Table 1, bottom clearance is $c_P = 0.2m$ = 4 mm, fillet radius is $r_{fP} = 0.3m$ = 6 mm, addendum of tooth is $h_{fP} = m$ = 20 mm and addendum of tooth is $h_{aP} = 1.2m$ = 24 mm. The height of the rack and gear teeth is $h_P = h_{fP} + h_{aP}$ = 44 mm. There are six teeth on the rack, and the lead of the turntable on which the gear is to be generated is located (paper circle diameter d = 300 mm) is 235 mm, so contours/envelopes of three teeth completely and two more teeth partially can be obtained on this device (Fig. 7b). If the lead of the turntable would be

630 mm, all teeth could be generated. On this actual device, it is possible to achieve this by removing the gear paper model, turning the paper to the blank side towards the rack and returning the turntable to the beginning of the lead, to repeat the cutting simulation procedure.



a) b)

Fig. 7. Educational equipment for simulation of involute gear shaping, based on MAAG method (a) and part of the gear model drawn on this equipment (b)

The rack starting position, when the rack pitch line is tangent of the pitch circle of the generated gear d = 200 mm, is marked on the device. In this case, the teeth would be cut with zero shift. There is a scale on the device to adjust the rack shift (positive and negative). For a gear with 10 teeth with module 20 mm, the limiting shift coefficients are $x_{\mathrm{min}} = 1 - 0.5z\sin\alpha_{\mathrm{P}} = 1 - 0.5 \cdot 10\sin 20° = 0.415$ (in practice it is possible to use the value $x_{\mathrm{min}} = 0.35$ with a negligible small undercut [12]) and $x_{\mathrm{max}} = 0.7$ mm [12]. Both rack shifts are positive. If the shift coefficient is less than 0.35, including values less than zero, the teeth would be significantly undercut, with a very weakened root, which is unacceptable in practice. Also, the maximum shift coefficient is determined from the condition that the thickness of the tooth at the tip should be at least 0.2m. In the case of this gear, the tip thickness is 4 mm. According to the limit values of shift coefficients, the recommended limit shift is from +7 mm to +9 mm. However, for educational purposes, it is possible to set rack shifts outside the permissible range on the laboratory device for the simulation of tooth formation. Examples of a pointed tooth tip due to a large positive shift (Fig. 8a), a slightly undercut tooth at zero shift (Fig. 8b) and a severe undercut tooth due to a large negative shift (Fig. 8c) are obtained using this educational equipment. All three gears in Fig. 8 have the same module and number of teeth, but the shape of the teeth differs significantly due to the tool shift in relation to generated gear axis.

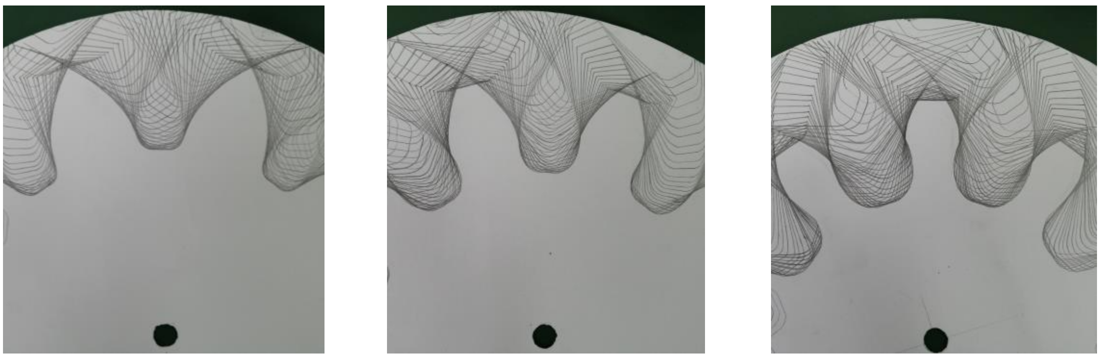
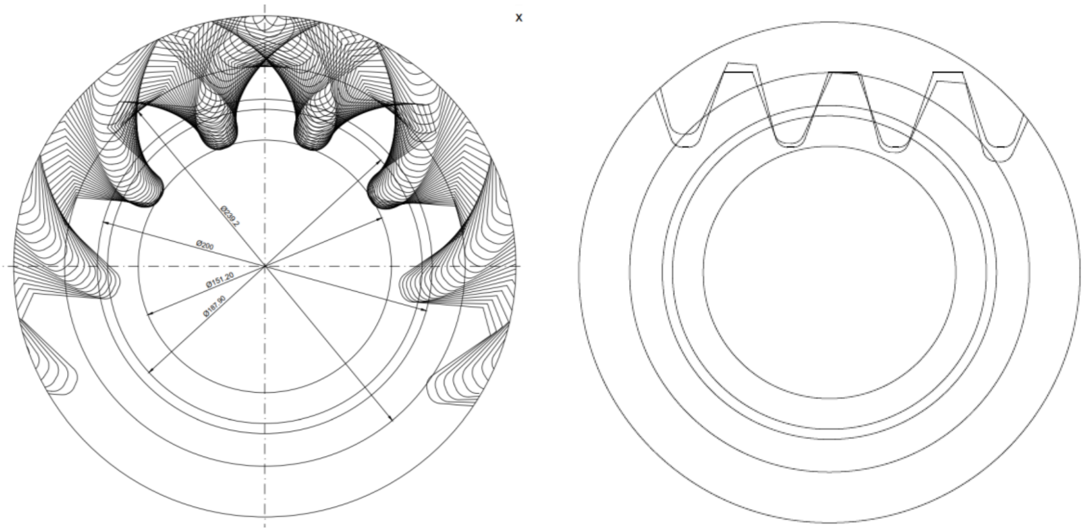


Fig. 8. Examples of gear models generated on laboratory equipment depending on rack tool shift: a) positive shift; b) zero shift; c) negative shift

A part of the gear model, the generation of which is simulated in AutoCAD, is shown in Fig. 9a. At first, the standard rack profile is drawn. Then a pitch circle is drawn so that the pitch line of the standard profile is tangent to it. It is the starting position of the tool profile relative to the gear to be drawn. In the example from Fig. 9, the profile is shifted towards the centre of the gear (negative shift). The contour of the rack profile, which goes outside the 300 mm circle, is deleted with the *Trim* command. To simulate the rolling of a pitch circle along the pitch line, without slip, the rotation angle of the disc γ is defined. In the *Rotate* command, an angle with a negative sign is entered, to turn the disk clockwise (the disk moves to the left by rolling). The displacement of the disk is obtained as the product of the radius of the base circle and the angle of disk rotation (in radians), i.e. $r_b\gamma$. After these steps, the new rack profile should be placed in the same position where the previous profile was located. By successively repeating the deletion, rotation, displacement and placement of the bar profile commands, the desired shape of the gear teeth is obtained. Two successive positions of the tool profile are shown in Fig. 9b.



a) b)

Fig. 9. Part of the gear model drawn in AutoCAD, simulating MAAG method of gear shaping

3.2 Improved design

A model of the variant solution with an improved way of relative rolling of the pitch circle and the pitch line is shown in Figure 10. Instead of a tensioned wire wrapped around the round plate on the pitch circle and positioned to coincide with the pitch line, the variant solution has a pinion-rack mechanism installed. In this way, maintenance of the device was facilitated (the wire required periodic retensioning), as well as rolling without slipping (which can happen in case of wire loosening).

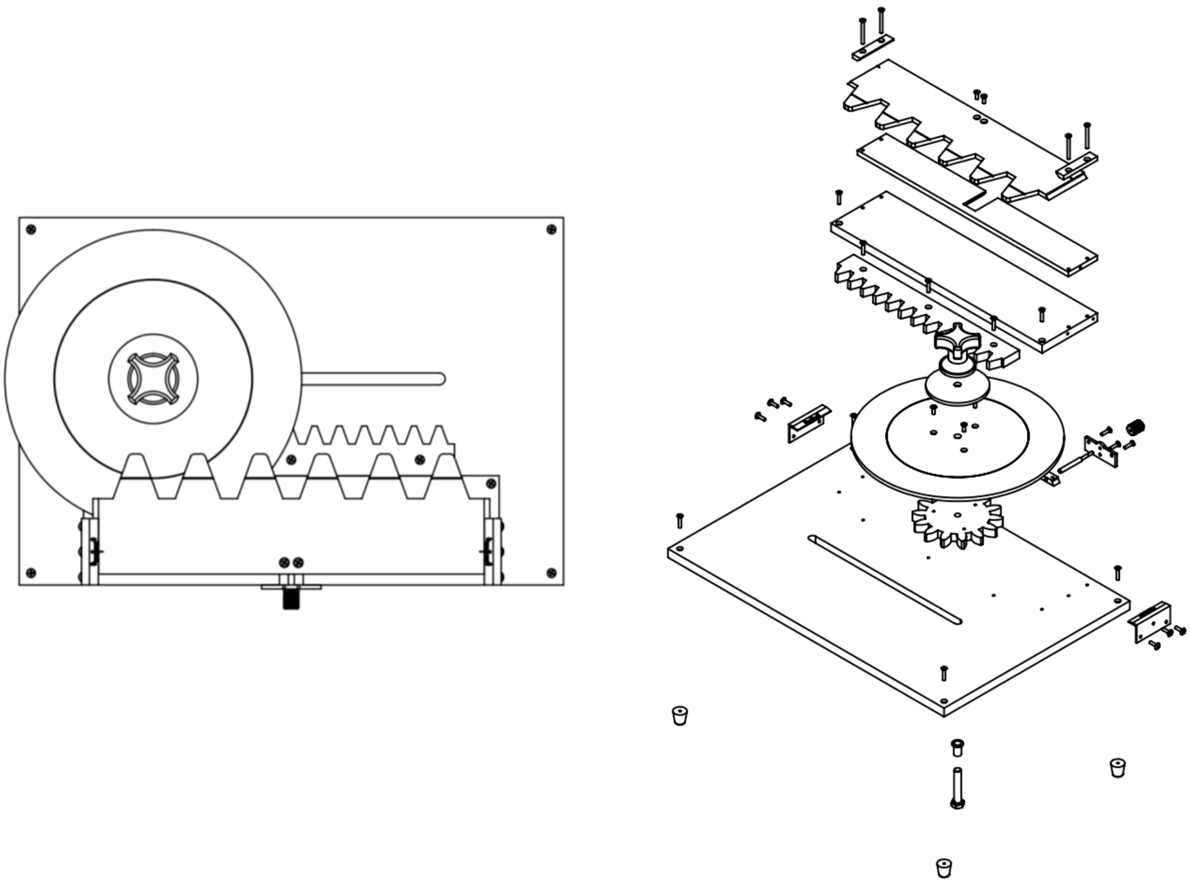


Fig. 10. A variant solution of equipment

Considering that tools - racks with different geometric parameters are used on real machines, to generate gears of different numbers of teeth and modules (Fig. 11), two additional variants of replaceable models of racks for laboratory equipment (with different modules and appropriate numbers of teeth) are presented in this paper (Fig. 12).

Fig. 11. Pack-type cutters [13]

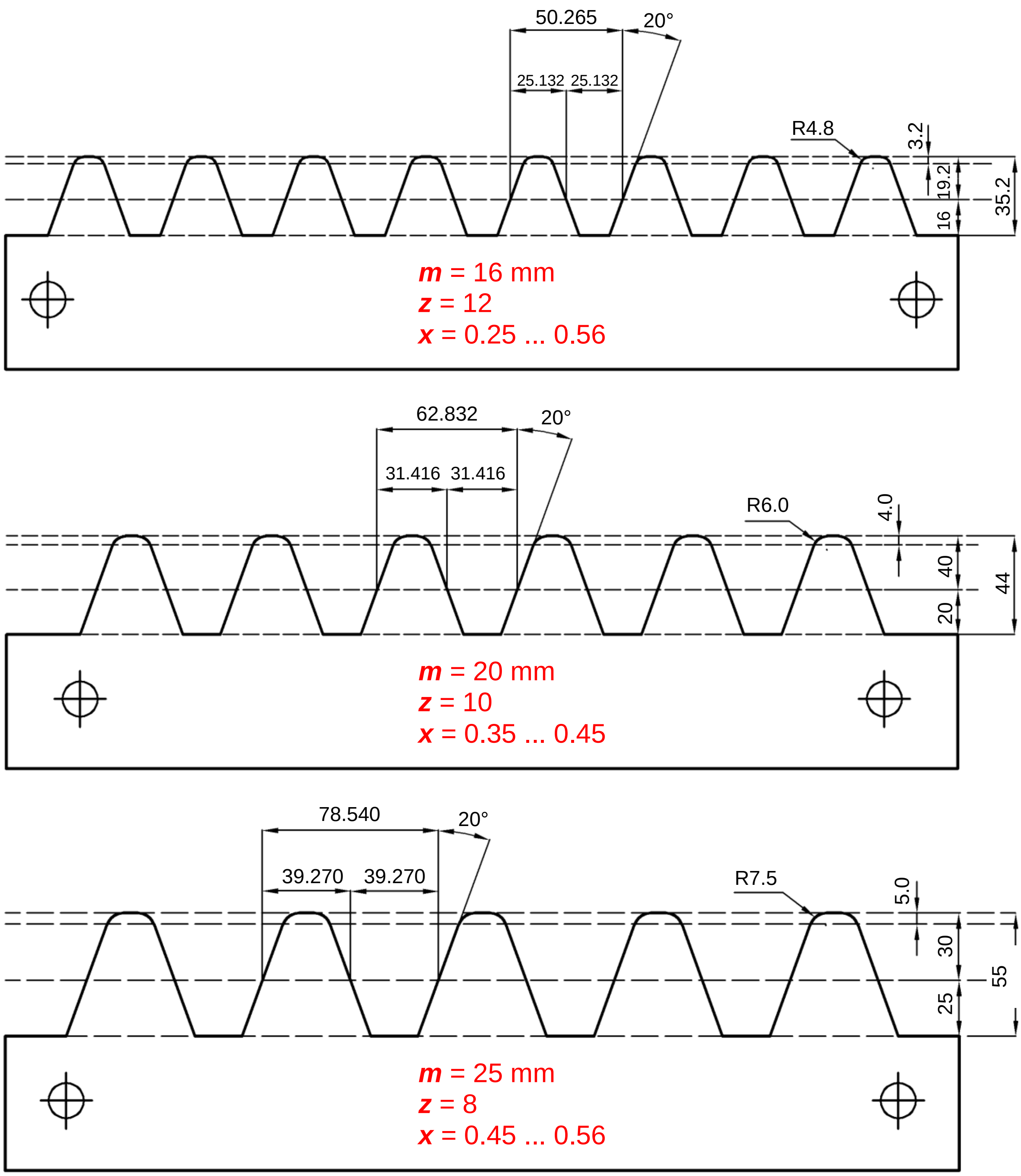


Fig. 12. Rack variants for different combinations of modules and number of teeth

Models of rack tools according to Fig. 12 can be mounted on an existing device. The limit values of shift coefficients were determined based on the recommendations in [12].

4 CONCLUSION

Involute gearing is the most common in industrial practice. That is why it is an object of interest in the theory of mechanisms and machine elements, i.e. the theory of mechanical power transmissions. For students, to learn the basics of the technological process of generating involute teeth, appropriate laboratory devices for simulation were developed. Nowadays, the presentation of the simulation of gear generating is easily done in e-form, using various software for 3D modelling and computer simulation. The Internet is full of simulations, but the methodical effect is stronger when the student him/herself, in reality (using "paper and pencil"), simulates the cutting of teeth by outlining the contour of the tool with a pencil. In this process student from the geometrically simple shape of the rack (with a straight profile) gets teeth of a complex involute profile (the method of relative motion based on the basic theorem of gearing). This paper presents an existing design solution for such a laboratory device. A variant solution of the improved construction is also given, which is technically more reliable and with the possibility of simulating several geometrically different shapes of teeth. A complete understanding of the gear manufacturing process and the geometry of the gearing is the basis for the kinematics, strength, service life and reliability of the gear pairs. That is one of the main goals of engineering education in the field of designing mechanical power transmissions.

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