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# A post-processor for the five-axis machine MultiProDesk based on inverse kinematic transformation 

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#### Abstract

This paper shows the development of a post-processor for a five-axis table tilting machine. Postprocessing software transforms a machining toolpath in CL file into $G$-code, which is necessary for the machining process in most CNC. The subject of this analysis is the MultiProDesk machine with two rotary axes on the machine table. The kinematic structure of the proposed machine is $C^{\prime} A^{\prime} Y^{\prime} \mathrm{OXX}^{\prime}$. The machine can be modeled with two kinematic chains, one carrying the workpiece and the other carrying the tool. Equations needed to transform the CL data into the displacement of the machine's axis were derived using the inverse kinematic model. Currently, the machine has a three-axis configuration, and the model was extended using simplified versions of the two rotary axes. The inverse kinematic equations were implemented in a post-processing algorithm in the Matlab software. The calculated data was tested in the form of a G-code on a virtual model of the machine in the Vericut software. The simulated workpiece machined in the virtual surrounding matched the original workpiece used to calculate the toolpath, verifying the developed post-processor for the considered machine.


Keywords Post-processor, inverse kinematics, verification, simulation, virtual machine tools

## 1. INTRODUCTION

Five-axis machining is widely used today for machining parts with complex surfaces. What differentiates five-axis machining from standard three-axis machining is the addition of two rotational axes, which enables two more degrees of freedom for the machine. The additional rotational axes reduce the set-up time needed for complex parts, as most complex parts can be manufactured using one fixture. Using only one

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[^0]fixture per manufacturing process eliminates the misalignment of the workpiece during a single operation. Therefore, using five-axis machines in manufacturing complex parts increases accuracy. Multi-axis machining utilizes shorter tools, as it is easier to reach all the workpiece parts. Shorter tools minimize vibrations, which results in a better surface finish of the machined part. Last, the most important advantage of fiveaxis machines is their ability to machine highly complex surfaces that would not be possible to produce on a three-axis mill. Conventional threeaxis machines have three translational axes that are normal to each other. During the machining process, the orientation of the tool does not change, and it is always fixed in the direction of the machine's Z axis. With the addition of two rotational axes, five-axis machines enable the
change of the tool's orientation. Consequently, the programming of five-axis machining must be done using CAD/CAM software, as it is more complex than programming a three-axis machine. When creating a machining program using CAM software, the toolpath is first generated as a CL file that needs to be postprocessed to obtain a machining program that can be implemented in a machining process.
A CL file is a general machining program containing the tool's position and orientation regarding the workpiece coordinate system, as shown in Fig. 1. The data contained in a CL file cannot be used in a machining process directly, as it has no information about the layout of the axes of the machine. A post-processor is a program designed specifically for a certain machine that translates the CL file into a G-code suited for running on said machine.
Multiple solutions in developing the mathematical model of multi-axis machine kinematics were developed. Lee and She proposed an inverse kinematic model for three typical types of five-axis machines [1]. Later, the kinematics of five-axis machine types were developed based on a similar principle for fiveaxis machines with nutating head and table configurations [2-4]. The analysis of structural configuration and geometry of possible solutions for multi-axis machine tools was done by Chen in [5]. This paper proposes a method for deriving the kinematic equations for all feasible configurations of multi-axis machine tools based on inverse kinematic transformations using six elementary transformational matrices for translation and rotation. The method of using inverse kinematic transformation for developing post-processing kinematic equations was implemented and verified with experiments on multiple multi-axis machines [6-7].


Fig. 1. CL data representation

This paper describes a post-processing method based on inverse kinematic transformation for the machine MultiProDesk [9]. Currently, MultiProDesk is a 3-axis mill with two additional rotary axes in preparation. This paper proposes post-processing software for the five-axis version of the machine. This version of the machine has two rotary axes, $A$ and $C$, on the table of the machine. A post-processor software, based on the algorithm developed in [8], was developed using the derived kinematic equations. The kinematic model of the MultiProDesk was tested using the Vericut software. The three-axis machine model was augmented with a planned fourth rotary axes A and a simplified version of rotary axes C to test the five-axis kinematic equations.
This paper shows developing the inverse kinematic equations for a five-axis table-tilting machine MultiProDesk. The equations are derived using the inverse kinematic method by modeling the machine as two open kinematic chains. The inverse kinematic equations were implemented into a post-processing algorithm using MatLab software. The post-processing software was tested for a concave surface machining CL file. In order to verify the kinematic model of the machine, a five-axis machining simulation was configured and tested in the Vericut software.

## 2. KINEMATIC MODELLING OF THE MULTIPRODESK MACHINE

Machine tools can be defined as open kinematic chains of serially connected links with rotational or translational joints. In compliance with this definition, the kinematic equations of a machine tool can be acquired by defining the relative positions of these links in respect of one another. The main objective in deriving these equations is to determine the displacements of the machine's axes to get the desired position and orientation of the tool regarding the workpiece's coordinate system. This kinematic model is obtained by assigning each joint of the machine a coordinate system and using them to define the relative motions between the adjacent joints. By representing each axis of the machine with its coordinate system, the relative motions of the axes can be defined with an elementary transformation matrix, in which a simple translation or rotational transformation is defined. The machine's configuration is

C'A'Y'OXZ, which determines that the Z and X axes drive the tool motions, and the workpiece is moved using the $\mathrm{Y}, \mathrm{A}$, and C axes. An illustration of the machine's axes and the necessary transformation matrices is shown in Fig. 2. First, a base coordinate system is defined. This coordinate system will be used as a starting point for the kinematic chains of the workpiece and tool. The position of the base coordinate system is chosen to be at the intersection of the axes of the machine's rotary components with axes matching the direction of the machine tools' axes, shown in Fig. 3. The workpiece kinematic chain is comprised of one translation and two rotational axes.
Each moving element of the machine is assigned a coordinate system. The coordinate systems of the workpiece kinematic chain are $O_{w 1} x_{w 1} y_{w 1} z_{w 1}, \quad O_{w 2} x_{w 2} y_{w 2} z_{w 2}, \quad$ and $O_{w 3} x_{w 3} y_{w 3} z_{w 3}$, shown in Fig. 4. A transformation matrix is defined between the workpiece coordinates system $O_{w} x_{w} y_{w} z_{w}$ and the C axes' coordinate system to close the kinematic chain. The first elementary transformation matrix, $T_{w 1}^{0}$, is defined between the base coordinate system and the coordinate system attached to the Y axes of the machine.

$$
T_{w 1}^{0}=\left[\begin{array}{cccc}
1 & 0 & 0 & x_{w 1}  \tag{1}\\
0 & 1 & 0 & Y \\
0 & 0 & 1 & z_{w 1} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The transformation matrix displayed in Eq. 1. defines the elementary translation transformation along the Y axes of the machine. Values $x_{w 1}$ and $z_{w 1}$ represent the constant distances along the X and Z axes of the machine between the base and the first coordinate system in the workpiece kinematic chain.


Fig. 2. Schematic diagram of machine tool kinematic chains


Fig. 3. The position of the base coordinate system
The value $Y$ is the only variable in the first matrix, defining the machine table's movement along the machine's $Y$ axes. The next two axes of the machine are the A and C rotary axes. Their transformation matrices are defined as follows:

$$
\begin{align*}
T_{w 2}^{w 1} & =\left[\begin{array}{cccc}
1 & 0 & 0 & x_{w 2} \\
0 & \cos (A) & -\sin (A) & 0 \\
0 & \sin (A) & \cos (A) & z_{w 2} \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{2}\\
T_{w 3}^{w 2} & =\left[\begin{array}{cccc}
\cos (C) & -\sin (C) & 0 & 0 \\
\sin (C) & \cos (C) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \tag{3}
\end{align*}
$$

In Eq. 2. and Eq. 3, the variables A and C represent the angles of the machine's rotary axes. Values $x_{w 2}$ and $z_{w 2}$ represent the constant distance between the coordinate systems $O_{w 1} x_{w 1} y_{w 1} z_{w 1}$ and $O_{w 2} x_{w 2} y_{w 2} z_{w 2}$ along the X and Z axes of the machine.


Fig. 4. The position of the coordinate systems of the workpiece kinematic chain

In order to simplify the resulting kinematic equations, coordinate systems $O_{w 2} x_{w 2} y_{w 2} z_{w 2}$ and $O_{w 3} x_{w 3} y_{w 3} z_{w 3}$ were placed at the intersection of the machine's rotary axes. Last, a transformation matrix defining the workpiece coordinate system regarding the C -axis coordinate system is defined. This matrix has no variable values, as the workpiece is fixed to the machining table.

$$
T_{w}^{w 3}=\left[\begin{array}{cccc}
1 & 0 & 0 & x_{w}  \tag{4}\\
0 & 1 & 0 & y_{w} \\
0 & 0 & 1 & z_{w} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The second kinematic chain contained in this machine is the tool kinematic chain. The coordinate systems of the tool kinematic chain are shown ih Fig. 5. The tool kinematic chain comprises the X and Z axes, so modeling the tool kinematic chain begins with defining the transformation matrix $\left(T_{t 1}^{0}\right)$ between the machine's $X$ axis and the base coordinate system.

$$
T_{t 1}^{0}=\left[\begin{array}{cccc}
1 & 0 & 0 & X  \tag{5}\\
0 & 1 & 0 & y_{t 1} \\
0 & 0 & 1 & z_{t 1} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

In Eq. 4. the variable $X$ represents the necessary motions of the machine's X axes. Next, the Z-axis coordinate system is defined regarding the Xaxis coordinate system. The defined transformation matrix in Eq. 5 contains the constant distance values between the coordinate systems along the machine's X and Y axes.

$$
T_{t 2}^{t 1}=\left[\begin{array}{cccc}
1 & 0 & 0 & x_{t 2}  \tag{6}\\
0 & 1 & 0 & y_{t 2} \\
0 & 0 & 1 & Z-z_{t 1}-H_{t} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The last coordinate system that is defined for the kinematic model of the 5 -axis MultiProDesk machine is the tool coordinate system $O_{t} x_{t} y_{t} z_{t}$. The last transformation matrix is an elementary translation matrix containing the tool length correction.

$$
T_{t}^{t 2}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0  \tag{7}\\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & H_{t} \\
0 & 0 & 0 & 1
\end{array}\right]
$$



Fig. 5. The position of the coordinate systems of the tool kinematic chain

As previously stated, the CL file contains information about the position and orientation of the tool coordinate system regarding the workpiece coordinate system. This dependence can also be defined as a transformation matrix containing the CL data $-T_{t}^{w}$. The position and orientation of the tool tip vector regarding the base coordinate system can be obtained in two ways, either using the workpiece kinematic chain or the tool kinematic chain, presented in the following equations:

$$
\begin{gather*}
r_{t}^{0}=T_{w 1}^{0} \cdot T_{w 2}^{w 1} \cdot T_{w 3}^{w 2} \cdot T_{w}^{w 3} \cdot T_{t}^{w}  \tag{8}\\
r_{t}^{0}=T_{t 1}^{0} \cdot T_{t 2}^{t 1} \cdot T_{t}^{t 2} \tag{9}
\end{gather*}
$$

These equations are equivalent and can be presented in the following way:

$$
\begin{equation*}
T_{w 1}^{0} \cdot T_{w 2}^{w 1} \cdot T_{w 3}^{w 2} \cdot T_{w}^{w 3} \cdot T_{t}^{w}=T_{t 1}^{0} \cdot T_{t 2}^{t 1} \cdot T_{t}^{t 2} \tag{10}
\end{equation*}
$$

Eq. 9. can be transformed to get the $T_{t}^{w}$ matrix as a function of the elementary transformation matrices previously defined, shown in Eq. 10.

$$
\begin{gather*}
T_{t}^{w}=\left(T_{w}^{w 3}\right)^{-1} \cdot\left(T_{w 3}^{w 2}\right)^{-1} \\
\cdot\left(T_{w 2}^{w 1}\right)^{-1} \cdot\left(T_{w 1}^{0}\right)^{-1} \cdot T_{t 1}^{0} \cdot T_{t 2}^{t 1} \cdot T_{t}^{t 2} \tag{11}
\end{gather*}
$$

The $T_{t}^{w}$ matrix contains the CL data, and the elementary transformation matrices contain the length and angles of the machine's axis motions. Using Eq. 10, the machine axis variables X, Y, Z, A, and $C$ can be defined as the functions of the CL
data. The resulting matrix transformation $T_{t}^{W}$ has the following form:

$$
T_{t}^{w}=\left[\begin{array}{cccc}
u_{x} & v_{x} & w_{x} & p_{x}  \tag{12}\\
u_{y} & v_{x} & w_{y} & p_{y} \\
u_{z} & v_{z} & w_{z} & p_{z} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The resulting matrix describes the tool coordinate system position and orientation regarding the workpiece coordinate system. The third column represents the cosines of the tool coordinate systems, deriving the equations for calculating the necessary angles of the rotational axes.

$$
\begin{gather*}
w_{x}=K_{x}=\sin (A) \cdot \sin (C)  \tag{13}\\
w_{y}=K_{y}=\sin (A) \cdot \cos (C)  \tag{14}\\
w_{z}=K_{z}=\cos (\mathrm{A}) \tag{15}
\end{gather*}
$$

The tool coordinate system position, corrected to compensate for the movements of rotational axes, is defined in the fourth column of the resulting matrix and defines the following equations:

$$
\begin{align*}
& p_{x}=Q_{x}=a_{1} \cos (C)+X \cos (C)+a_{2} \sin (A) \sin (C) \\
& +\left(Z-a_{3}\right) \sin (A) \sin (C)-Y \cos (A) \sin (C)  \tag{16}\\
& p_{y}=Q_{y}=b_{1} \sin (C)-X \sin (C)+b_{2} \cos (C) \sin (A) \\
& +\left(Z-b_{3}\right) \cos (C) \sin (A)-Y \cos (A) \cos (C)  \tag{17}\\
& p_{z}=Q_{z}=c_{1} \cos (\mathrm{~A})+Y \sin (\mathrm{~A})+\left(Z-c_{2}\right)-c_{3} \tag{18}
\end{align*}
$$

In the previous equations, $a_{i}, b_{i}$, and $c_{i}(i=$ 1...3) are constant values resulting from the constant distances between the coordinate systems. Solving the system of trigonometric equations, Eq. 13-15 yields the functions of the machine's rotational axes angles in the form:

$$
\begin{array}{r}
A=\arccos \left(K_{z}\right) \\
C=\operatorname{atan} 2\left(K_{x}, K_{y}\right) \tag{20}
\end{array}
$$

The necessary lengths of the machine's translation axes movements are derived by solving the system of linear equations 16-18. This process yields the following equations:

$$
\begin{align*}
& X=\cos (c)\left(Q_{x}-a_{1} \cos (C)\right)-\sin (c)\left(Q_{y}-b_{1} \sin (C)\right)- \\
& -\sin (A) \sin (C) \cos (C)\left(a_{2}-a_{3}-b_{2}+b_{3}\right)+ \\
& a_{1} \cos ^{2}(C)+b_{1} \sin ^{2}(C) \tag{21}
\end{align*}
$$

$$
\begin{align*}
& Y=c_{3} \sin (A)-\cos (A)\left(Q_{x} \sin (C)+Q_{y} \cos (C)\right)+ \\
& \cos (A) \sin (A)\left(\cos ^{2}(C)\left(b_{2}-b_{3}-c_{1}+c_{2}\right)+\sin ^{2}(C)\left(a_{2}-\right.\right. \\
& \left.\left.a_{3}+c_{1}+c_{2}\right)\right)+\cos (A) \cos (C) \sin (C)\left(b_{1}+a_{1}\right)  \tag{22}\\
& Z=\sin (A)\left(Q_{y} \cos (C)+Q_{x} \sin (C)\right)+c_{3} \cos (A)- \\
& \cos ^{2}(A)\left(c_{1}-c_{2}\right)+\sin ^{2}(A)\left(\cos ^{2}(C)\left(b_{3}-b_{2}\right)+\right. \\
& \left.\sin ^{2}(C)\left(a_{3}-a_{2}\right)\right)-\cos (C) \sin (A) \sin (C)\left(a_{1}+b_{1}\right) \tag{23}
\end{align*}
$$

## 3. POSTPROCESSOR IMPLEMENTATION AND VERIFICATION

Derived functions for calculating the necessary angles and displacements of the machine's axes were implemented into a postprocessing program developed in the MatLab software. The algorithm used for this program is based on the postprocessing program developed in [8]. The CL file and G-code, besides the statements for tool motions, consist of other data needed for a manufacturing process. Since this test aims to verify the inverse kinematic equations, only the necessary data outside the motion statements in the CL file were translated into a G-code. The main algorithm implemented into the postprocessing software consists of an iterative process, where one by one, the sets of data that include to tool's position and orientation from the CL file are read and calculated using Eq. 1923. First, the current data set is read, then, using Eq. 19 and 20, the necessary angles of the machine's rotary axes are calculated. The lengths of the translation axis movements are computed by implementing the calculated angles into Eq. 21-23 calculated angles of the rotary axes. The transformed data set is written in the G-code syntax into a separate file. Given the tool motions that occur with multi-axis machining, only the linear interpolation G1 is used while constructing the G code.
The G-code generated using the beforementioned equations was tested using the Vercut software, where it is possible to simulate a material removal process based on a G-code program. For the experiment, a workpiece with a concave surface was formed to verify the correct engagement of both rotary axes. The design of the workpiece and writing of the manufacturing program was done in the PTC Creo software. The simulation of the manufacturing based on CL file is shown in Fig. 6.


Fig. 6. Toolpath generated using PTC Creo

This experiment can be defined as virtual machining, as the finished virtual workpiece can be saved in a standard CAD format and then compared to the original CAD model of the workpiece. The resulting shape of the workpiece matches the desired starting part, so it can be concluded that the experiment was a success. The simulation of the G-code in the Vericut software is shown in Fig. 7.

## 4. CONCLUSION

This paper presented a method for developing kinematic equations for a multi-axis postprocessor. Software for transforming CL data into G-code was developed for the multi-axis version of the machine MultiProDesk. The proposed machine's mathematical model was developed using the inverse kinematic methodology. Elementary transformational matrices described the motions between the machine's moving segments. The postprocessing software was then implemented to transform the CL file for concave workpiece machining. The formed G-code was tested on a virtual model of the machine MultiPro Desk in the Vericut software. The process was determined to be a success, as the workpiece machined during the simulation reflected the geometry of the desired workpiece. The proposed method can be utilized for various types of multi-axis machines. Further research can be extended by developing post-processing software for the four-axis version of the proposed machine or different configurations of the machine's rotary axis.


Fig. 7. Testing the G-code in Vericut

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