10th International Scientific Conference
IRMES 2022
"Machine design in the context of Industry 4.0 – Intelligent products"
26 May 2022, Belgrade, Serbia

CONTACT STRESS AND DEFORMATIONS IN ECCENTRICALLY LOADED THRUST BALL BEARING

Pavle LJUBOJEVIĆ
Radivoje MITROVIĆ
Tatjana LAZOVIĆ

Abstract: *Contact stress and deformations in the thrust ball bearing with a contact angle of 90 ° are considered in this paper. Ideally, an external axial load of this kind of bearings should be centric, but in reality, this force may act eccentrically. As a result, the load distribution between balls is inequal. The most unfavourable case is when the axial force intersects the radius vector of one of the balls. The load, contact stress and contact deformation of this ball are greater than the load of all rest of the balls and greater in the case of equal load distribution. It limits the load-carrying capacity of a real bearing, compared to the theoretical static load rating. Analyzes have shown that considered thrust ball bearings do not lose operational ability at low speeds, even if there is some load eccentricity when an external axial load is not more than 30- 40% of static bearing capacity. Otherwise, additional bearing capacity analyzes are required.*

Keywords: *thrust ball bearing, load distribution, contact deformations, contact stress*

1. INTRODUCTION

The thrust ball bearings can only withstand a pure axial load. Thereby single-row deep groove ball bearings of series 511, 512, 513 and 514, consisting of one row of balls and two washers - shaft washer and housing washer, transmit axial load in one direction.
The rotation speed of thrust ball bearings is limited, because at higher angular velocities, due to centrifugal force, the friction between the balls and the cage increases [1]. That is why these bearings are used in power transmissions, which transmit high torques at low speeds. They are also used in bearing arrangements of rotary cranes and crane hooks, in various types of rotary stands and rotary table feeders, generally for supporting vertical shafts. These bearings do not withstand radial loads. If a radial load component acts in the support, a radial bearing should be mounted together with an axial bearing in the bearing arrangement.
Ideally, the thrust ball bearing should be subjected to a centric load. This means that the direction of the axial force coincides with the bearing axis. Then the load distribution between rolling elements is equal, ie. all balls transfer the same part of the external axial load. However, very often in practice, for various reasons, the external load direction is shifted from the bearing axis by some value, called eccentricity. The reasons for eccentricity could be low manufacturing accuracy of the shaft and the housing, as well as improper installation.
The load distribution between rolling elements of the eccentrically loaded thrust ball bearing is unequal, and the degree of inequality depends on the eccentricity.
Eccentrically loaded ball bearing, as an issue recognized in practice, is also considered in the theory of rolling bearings [2]. The load distribution of an eccentrically loaded thrust ball bearing is mathematically described using Rumbarger integrals [2,3]. This is a well-known methodology applied in the conventional theory of load distribution between rolling elements of thrust ball bearings. However, although there is a mathematical model of load distribution in the thrust ball bearing, which takes into account the eccentricity, the standard recommendations for the calculation of thrust ball bearing's static load rating [4], as well as dynamic load rating and rating life [5] are not included. In these relevant and official documents, which are adhered to by bearing manufacturers, designers, end-users and university literature, the load ratings and rating life of thrust ball bearings are considered under the assumption of centric axial load. Also, a centric axial load is assumed in papers dealing with other types of thrust bearings, as well as with other phenomena (tribological, energy losses, etc.), [6-11]. In the case of eccentrically loaded thrust ball bearing, the accurate mathematical description of the load distribution is important for obtaining the exact load of every single rolling element in the bearing. Then load carrying capacity, as well as the service life of the entire bearing can be determined with great reliability [12-16].

An additional torque load occurs in the eccentrically loaded axial bearing. The influence of the torque on the load distribution between the rolling elements and on the service life of the axial ball bearing was analyzed in [18]. In this paper, contact stresses and deformations in an eccentrically loaded ball bearing with a contact angle of 90° and operating with rotational frequencies n ≤ 10 rpm will be analyzed in a slightly different way, by introducing two new quantities - load factor and eccentricity factor.

2. STRESS AND DEFORMATION IN THE BALL-RACEWAY CONTACT

In the unloaded ball bearing, there is point contact between the balls and the raceway groove. Transferring the load, balls and raceways are deformed in contact and the contact surface has an elliptical shape. The size of the contact ellipse depends on the elastic properties of the material, the dimensions of the contact surfaces and the load. Contact stresses and deformations in the ball-raceway contact can be determined using Hertz theory [1,2]. According to this theory, the dimensions of the contact ellipse can be determined, as well [17]. The equations for major and minor semiaxes of the contact ellipse at the place of every single i-th ball are:

$$a_{\mathrm{i}} = n_a \sqrt[3]{\frac{3}{2}\frac{(1-\nu^2)F_{\mathrm{i}}D_{\mathrm{w}}}{E(2-\xi)}}$$
$$b_{\mathrm{i}} = n_b \sqrt[3]{\frac{3}{2}\frac{(1-\nu^2)F_{\mathrm{i}}D_{\mathrm{w}}}{E(2-\xi)}} \qquad (1)$$

where: F_{i} is normal load in the i-th ball-raceway contact; E is the modulus of elasticity of bearing's parts; ν is Poisson's ratio; n_a, n_b are functions of elliptic integrals [2]; $\xi = R_{\mathrm{w}}/R$; $D_{\mathrm{w}} = 2R_{\mathrm{w}}$ is the ball diameter; R_{w} and R are radii of the ball and raceway groove, respectively.

The quantities n_a and n_b in expression (1) are given in appropriate tables [2], as a function of the auxiliary quantity $\cos\tau$, which depends on the bearing type. For ball bearings, it can be determined from the expression [17]:

$$\cos\tau = \frac{\xi}{2-\xi} \qquad (2)$$

The maximal ball-raceway contact stress is given by the expression [2]:

$$\sigma_{\max,\mathrm{i}} = \frac{3}{2}\frac{F_{\mathrm{i}}}{\pi a_{\mathrm{i}} b_{\mathrm{i}}} \qquad (3)$$

The contact deformation in the ball-raceway contact is determined based on the expression [2]:

$$\delta_{\mathrm{i}} = C_F F_{\mathrm{i}}^{2/3} \qquad (4)$$

The C_F constant in expression (4) depends on the bearing's internal geometry, modulus of elasticity, and Poisson's ratio. For thrust ball bearing, it can be determined based on the expression [17]:

$$C_F = \frac{3}{2}\left(\frac{2K}{\pi n_a}\right)\sqrt[3]{\frac{2}{3}\left(\frac{1-\nu}{E}\right)^2 \frac{2-\xi}{D_{\mathrm{w}}}} \qquad (5)$$

where $2K/\pi n_a$ is an auxiliary quantity given in appropriate tables [2] depending on $\cos\tau$.

The initial assumptions for the development of a mathematical model of load distribution in an axially loaded thrust ball bearing are:

- balls and washers are of ideal shape and dimensions;
- volume deformations of bearing parts are neglected;
- the angular distance between balls is equal;
- the bearing is statically loaded (n ≤ 10 rpm)
- bearing load (axial force) is of constant direction and intensity;
- one washer has an ideal (perpendicular) position to the bearing axis (no tilting) and has absolute volume and surface stiffness, ie. stress and deformation analysis will be done for the contact of the balls and the other washer, tilting due to the eccentric axial load.

The axially loaded thrust ball bearing is shown in Figure 1. The axial force is acting at a distance e from the bearing axis. The most unfavourable case is when the axial force intersects the radius vector of one of the balls (the 0-th ball in Fig. 1). Centric axial bearing loading is a special case of loading when $e = 0$.
Due to the eccentric loading, a tilting moment occurs as an additional load. The washer tilts by an angle θ (Fig. 1) causing additional contact deformations of the balls on the side of a force acting. The static equilibrium conditions of the bearing shown in Figure 1 can be written as follows:

$$F_{\mathrm{A}} = \sum_{\mathrm{i}=0}^{Z-1} F_{\mathrm{i}}$$
$$F_{\mathrm{A}} e = \frac{D_{\mathrm{p}}}{2}\sum_{\mathrm{i}=0}^{Z-1} F_{\mathrm{i}} \cos(\mathrm{i}\gamma) \qquad (6)$$

where: Z is the number of balls in the bearing; e is eccentricity, ie. the distance between the F_{A} force line and the bearing axis; D_{p} is the bearing's pitch diameter [4,5].

The system of two equations (6) has Z unknown quantities. This is load F_{i} of every single ball of Z balls in the bearing. To determine all unknown quantities, it is necessary to introduce additional equations for contact deformations. Under the centric axial load F_{A}, deformations (δ_{a}) in the ball-raceway contact are equal for all balls. Due to the washer tilting under moment $F_{\mathrm{A}}e$ moment, with tilting angle θ (Fig. 1), there is an additional contact deformation of the balls located on the side of the external eccentric load acting (Fig. 1). The balls on the opposite side are relieved (Fig. 1) and their contact deformations are reduced, so they are not the subject of consideration in this paper. The contact deformations of individual balls differ, depending on their position to the point of external load F_{A}.

Based on the previous considerations, an expression can be written for the contact deformation at the location of every single i-th ball:

$$\delta_{\mathrm{i}} = \delta_{\mathrm{a}} + \delta_{\theta\mathrm{i}} \tag{7}$$

The contact deformation at the position of the i-th ball due to the washer tilting with tilting angle θ can be determined as follows:

$$\delta_{\theta\mathrm{i}} = \delta_{\theta}\cos(\mathrm{i}\gamma) \tag{8}$$

The contact deformation at the place of the most loaded "0" ball (i = 0), due to the washer tilting can be determined from expression (8) and based on Figure 1:

$$\delta_{\theta 0} = \delta_{\theta} = \theta\frac{D_{\mathrm{p}}}{2} \tag{9}$$

Based on expression (4), the load of the i-th ball can be determined:

$$F_{\mathrm{i}} = C_F^{-3/2}\delta_{\mathrm{i}}^{3/2} = C_{\delta}\delta_{\mathrm{i}}^{3/2} \tag{10}$$

By substituting expressions (7) and (10) in expression (6), we obtain:

$$\begin{aligned} \frac{F_{\mathrm{A}}}{C_{\delta}} &= \sum_{\mathrm{i}=0}^{Z\text{-}1}\left(\delta_{\mathrm{a}} + \delta_{\theta}\cos(\mathrm{i}\gamma)\right)^{3/2} \\ \frac{F_{\mathrm{A}}}{C_{\delta}}\cdot\frac{2e}{D_{\mathrm{p}}} &= \sum_{\mathrm{i}=0}^{Z\text{-}1}\left(\delta_{\mathrm{a}} + \delta_{\theta}\cos(\mathrm{i}\gamma)\right)^{3/2}\cos(\mathrm{i}\gamma) \end{aligned} \tag{11}$$

For considered thrust ball bearing, when the rotation speed is $n \leq 10$ rpm, a new value can be introduced - the load factor. It is a relative load, ie. external axial load F_{A} reduced to the static load rating C_0:

$$k_F = \frac{F_{\mathrm{A}}}{C_0} \tag{12}$$

For further analysis, the axial load eccentricity factor is introduced. It is a relative eccentricity, ie. eccentricity reduced to the bearing pitch radius:

$$k_e = \frac{e}{D_{\mathrm{p}}/2} \tag{13}$$

By introducing the load factor k_{F}, determined by expression (12) and the eccentricity factor k_{e}, defined by expression (13), equations (11) get the form:

$$\begin{aligned} k_F\frac{C_0}{C_{\delta}} &= \sum_{\mathrm{i}=0}^{Z\text{-}1}\left(\delta_{\mathrm{a}} + \delta_{\theta}\cos(\mathrm{i}\gamma)\right)^{3/2} \\ k_F k_e\frac{C_0}{C_{\delta}} &= \sum_{\mathrm{i}=0}^{Z\text{-}1}\left(\delta_{\mathrm{a}} + \delta_{\theta}\cos(\mathrm{i}\gamma)\right)^{3/2}\cos(\mathrm{i}\gamma) \end{aligned} \tag{14}$$

Expressions (14) are a system of two equations with two unknown quantities (contact deformations δ_{a} and δ_{θ}). It is a system of nonlinear equations that are solved iteratively, using some of the numerical methods. In this case, the MathCAD program was used for that purpose.

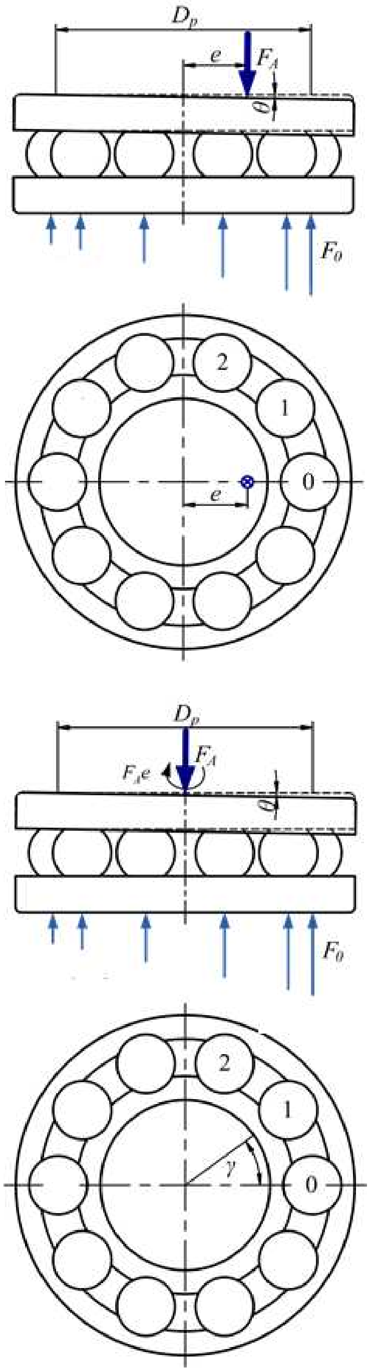


Fig.1. Eccentrically loaded thrust ball bearing

3. NUMERICAL EXAMPLE AND DISCUSSION OF RESULTS

The following analysis is carried out for thrust ball bearings with bore diameter d = 30 mm, of all standard series: 511, 512, 513 and 514 (Table 1).
Large and small semi-axes of contact ellipses, contact stress and deformations are determined using expressions (1)-(5). Four cases of bearing load were analyzed: 25%,

50%, 75% and 100% of the static load rating, ie. load factor values k_F = {0.25; 0.50; 0.75; 1.00}. Considered eccentricities are corresponding to the values of the eccentricity factor k_e = {0; 0.1; 0.2; 0.3; 0.4; 0.5}, including zero eccentricity loading (centric loading).

Table 1. The main parameters of thrust ball bearings with a bore diameter of 30 mm

Quantity		511	512	513	514
Bore diameter, mm	d	30			
Outer diameter, mm	D	47	52	60	70
Pitch diameter, mm	D_p	38.5	41	45	50
Number of balls	Z	18	12	10	8
Angle between balls, °	γ	20	30	36	45
Ball diameter, mm	D_w	6.00	7.98	10.32	15.08
Raceway radius, mm	R	3.23	4.29	5.54	8.12
$R_w/R = D_w/2R_w$	ξ	0.93			
Poisson's ratio	ν	0.3			
Modulus of elasticity, N/mm^2	E	$2.1 \cdot 10^5$			
Static load rating, kN	C_0	43	51	65	122
Auxiliary quantity	$\cos\tau$	0.82			
Constant, 10^{-4} mm/N$^{2/3}$	C_F	1.28	1.17	1.07	0.95
Constant, 10^5 N/mm$^{3/2}$	C_δ	6.9	7.9	9.0	10.8

Contact deformations caused by both external axial load and the washer tilting moment due to eccentricity were calculated by solving the equation system (14). The sum of these deformations is the contact deformation of the most loaded ball (the load distribution is unequal due to eccentricity). Then, using expression (10), the normal load of the most loaded ball was determined.
Based on expression (1), the dimensions of the contact ellipse at the position of the most loaded ball are determined. And, finally, the maximum contact stress of the most loaded ball is determined from expression (3). Results are shown by diagrams in Figures 2-8.

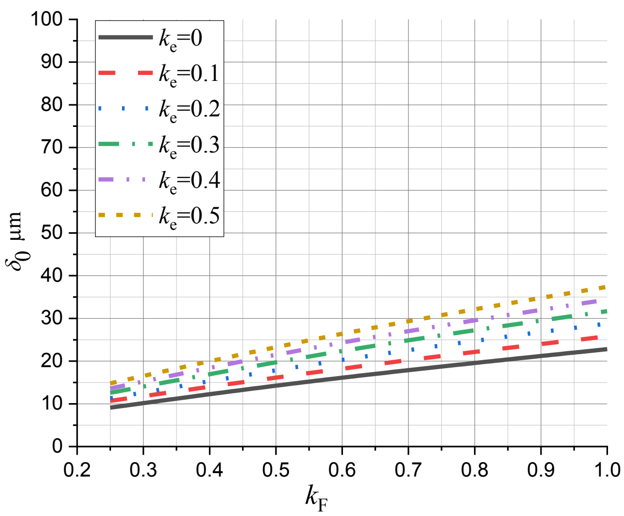


Fig. 2. Contact deformation of the most loaded ball of the eccentrically loaded bearing 51106, depending on the load and eccentricity

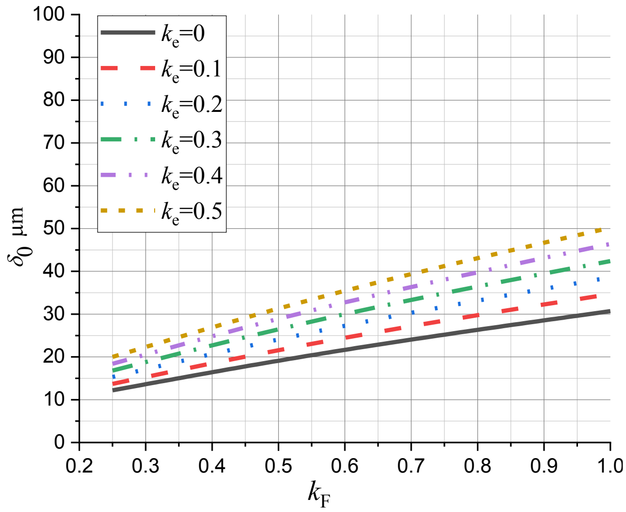


Fig. 3. Contact deformation of the most loaded ball of the eccentrically loaded bearing 51206, depending on the load and eccentricity

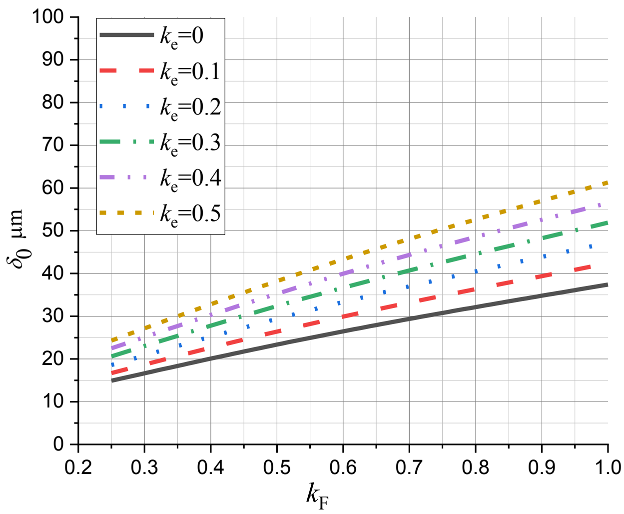


Fig. 4. Contact deformation of the most loaded ball of the eccentrically loaded bearing 51306, depending on the load and eccentricity

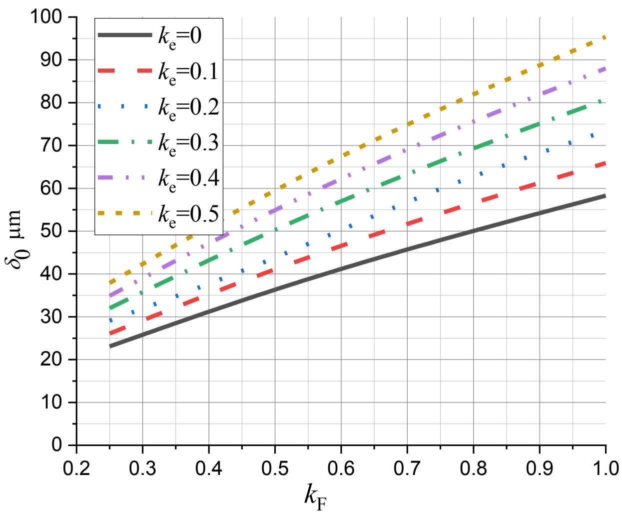


Fig. 5. Contact deformation of the most loaded ball of the eccentrically loaded bearing 51406, depending on the load and eccentricity

Based on diagrams for contact deformation δ_0 of the most loaded ("0") ball in contact with the raceway of eccentrically loaded thrust ball bearings (series 51106, 51206, 51306 and 51406), shown in Figures 2-5, it can be concluded that the contact deformation of the "0"-ball increases 2.5 times with increasing load (k_F = 0.25…1) and 1.64 times with increasing eccentricity (k_e = 0…0,5).
Diagrams in Figure 6 shows how all deformations, due to axial load, tilting moment and total one, change due to eccentricity. Axial deformations slightly decreases, but tilting deformation increasing is significantly. The reason of that is unequal load distribution in loaded zone. In this case all balls in this zone additionally take part of external load, grater than in the case of equal load distribution, ie. centric loading.

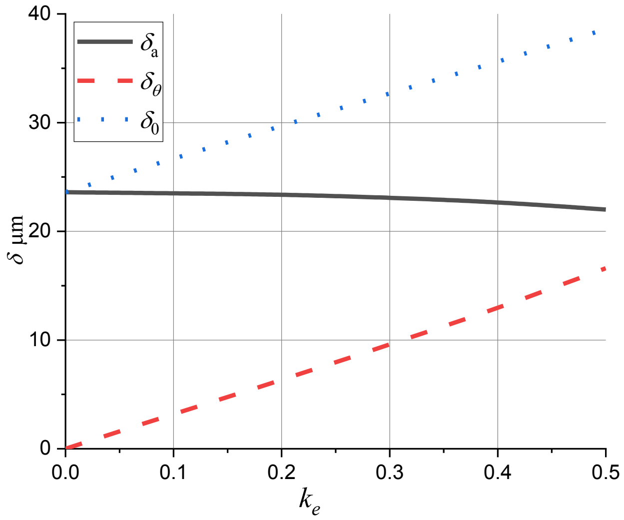


Fig. 6. Contact deformations caused by axial load and tilting moment, depending on eccentricity (for thrust ball bearing 51206 loaded with F_A =0.5C_0)

Based on diagrams of the maximum stress $\sigma_{max,0}$ in the ball-raceway contact in eccentrically loaded thrust ball bearings of series 51106 (the lighter one) and 51406 (the havier one), shown in Figures 7 and 8, it can be concluded that contact stress increases with increasing load and eccentricity. In the thrust ball bearing under the centric external axial load, limited contact stress of 4200 N/mm^2 is determined by the static load capacity C_0 [4].The values obtained in this paper, for the condition when the external axial load is equal to the static load capacity (F_A = C_0, ie. k_F = 1), are approximately 4500 N/mm2. This is an acceptable deviation since this difference is influenced by the constant C_δ, given in Equation (10) and Table 1, depending on bearing parts' material properties and geometry. These are also the parameters influenced by bearing manufacturer. Digarams from Figures 7 and 8 show that for considered bearings, contact stress does not exceed 4200 N/mm^2 for k_e ≈ (0.3 ... 0.4) and small eccentricities of k_e < 0.3.
By comparing bearings of different series, from light to heavy one, the load of the balls is increased, but the maximum contact stress between the balls and the raceway is approximately the same for all series (Fig. 7 and 8). This is due to the increase of contact surfaces between the balls and the groove of raceway – contact ellipses. [17].

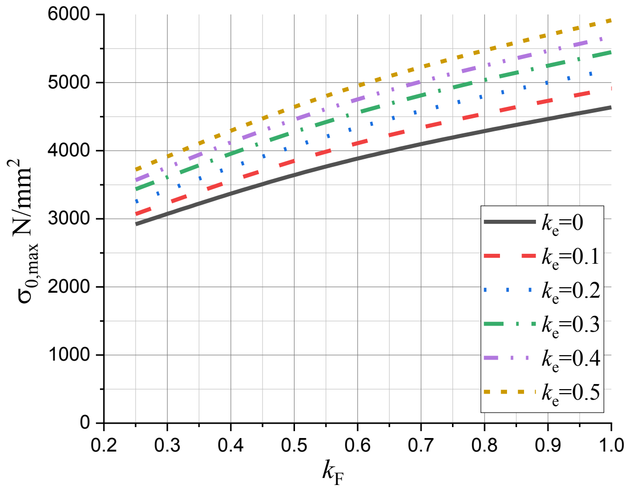


Fig. 7. Contact stress of the most loaded ball of the eccentrically loaded bearing 51106, depending on the load and eccentricity

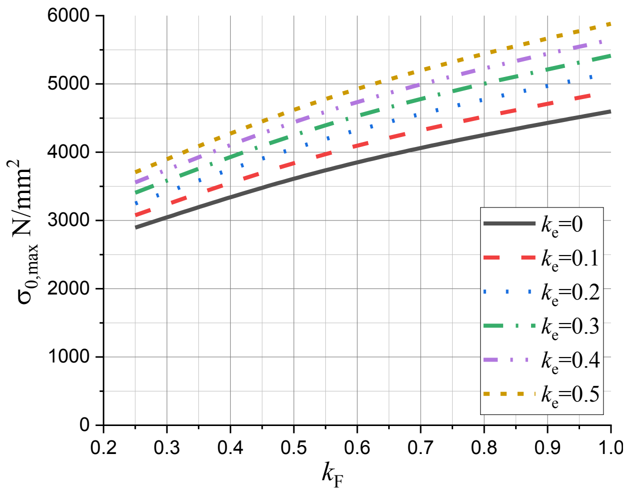


Fig. 8. Contact stress of the most loaded ball of the eccentrically loaded bearing 51306, depending on the load and eccentricity

4. CONCLUSION

In this paper, deformations and stresses in the contact of balls with the raceways of thrust ball bearings with a contact angle of 90° and operating with rotational frequencies n ≤ 10 rpm (statically loaded axial bearings) are considered. Ideally, the direction of the axial load should coincide with the bearing axis. In reality, there are minor or major deviations. It leads to unequal load distribution when one ball is loaded more than all others. Therefore, both the contact deformation and the stress at the location of this ball are the greatest. Their values limit the bearing capacity of a real bearing, which is less than the bearing capacity of an ideal centrically loaded bearing. Conventional roller bearing theory and corresponding ISO do not consider eccentrically loaded ball bearings. Analyzes have shown that the considered bearings 51106, 51206, 51306 and 51406 can perform their function smoothly at low speeds (n ≤ 10 rpm), even if there is semo eccentricity of the external load and if the external axial load is not more than 30- 40% of static load rate. Otherwise, additional bearing capacity analyzes are required. The carried out analyzes and the obtained results can be the basis for further research of the loading rate of thrust ball bearing under eccentric external axial load, ie unequal load distribution between the balls. In this case, the contact stress at the place of the most loaded ball can be higher than the limit value and can cause unforeseen premature and excessive raceway surface damage. The results can be used for a more accurate assessment of the thrust ball bearing capacity, as well as in case studies for failure analysis.

ACKNOWLEDGMENT

This work was supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia (Contract No. 451-03-68/2022-14/200135, dated 4 February 2022)

REFERENCES

[1] Krsmanović, V., Mitrović, R.: *Sliding and rolling bearings* (in Serbian), University of Belgrade, Faculty of Mechanical Engineering, Belgrade, ISBN 978-86-17-19284-4, 2015

[2] Harris, T.A.: Rolling Bearing Analysis, John Wiley and Sons, New York, ISBN 0-471-35457-0, 2001

[3] Rumbarger, J.: Thrust bearing with eccentric loads, *Machine Design*, 1962

[4] ISO 76:2006 Rolling bearings – Static load ratings

[5] ISO 281:1990 Rolling bearings – Dynamic load ratings and rating life

[6] Gold, P.W., Loos, J.: Wear resistance of PVD-coatings in roller bearings, *Wear*, 253, pp. 465-472, ISSN 0043-1648, 2002

[7] Aguirrebeitia, J., Avilés, R., Fernandéz de Bustos, I., Abasolo, M.: Calculation of general static load-carrying capacity for the design of four-contact-point slewing bearings, *Transactions of ASME Journal of Mechanical Engineering*, 132, pp. 064501/1-6, ISSN 0738-0666, 2010

[8] Cousseau, T., Graça, B., Campos, A. and Seabra J.: Friction torque in grease lubricated thrust ball bearings, *Tribology International*, 44, pp. 523-531, ISSN 0301-679X, 2011

[9] Ren, Z., Wang, J., Guo, F. and Lubrecht, A.A.: Experimental and numerical study of the effect of raceway waviness on the oil film in thrust ball bearings, *Tribology International*, 73, pp. 1-9, ISSN 0301-679X, 2014

[10] Cousseau, T., Graça, B.M., Campos, A.V. and Seabra J.H.O.: Influence of grease rheology on thrust ball bearings friction torque, *Tribology International*, 46, pp. 106-113, ISSN 0301-679X, 2012

[11] Fernandez, C.M.C., Marques, P.M.T., Martins, R. and Seabra J.H.O.: Gearbox power loss. Part I: Losses in rolling bearings, *Tribology International*, 88, pp. 298-308, ISSN 0301-679X, 2015

[12] Ristivojević, M., Mitrović, R.: *Load distribution – Gears and rolling bearings* (in Serbian), University of Belgrade, Faculty of Mechanical Engineering, Belgrade, ISBN 86-7558-112-2, 2002

[13] Lazović, T., Mitrović, R., Ristivojević, M.: *Load distribution between rolling elements of ball and roller bearing*, Proceedings of 3rd International Conference Research and Development in Mechanical Industry – RaDMI'03, 19-23 September, Herceg Novi, Montenegro, pp. 11-19, 2003

[14] Lazović, T., Ristivojević, M., Mitrović, R.: Mathematical model of load distribution in rolling bearing, *FME Transactions*, 36, pp. 189-196, ISSN 1451-2092, 2008.

[15] Lazović, T., Mitrović, R., Ristivojević, M.: Influence of internal radial clearance on the ball bearing service life, *Journal of the Balkan tribological association*, 16 (1), pp. 1-8, ISSN 1310-4772, 2010

[16] Lazović, T.: *Service life of ball bearings* (in Serbian), University of Belgrade, Faculty of mechanical engineering, Belgrade, ISBN 978-86-6060-082-2, 2021

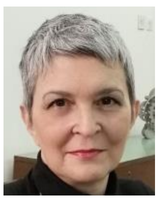
[17] Lazović, T., Varagić, S., Milović, LJ.: Contact stress and deformations in thrust ball bearing, *Journal of Machine Design*, 10, 3, pp. 85-92, ISSN, 2018

[18] Tianyu, L. and Jiwei, L.: *The influence of moment load on fatigue life of thrust ball bearing*, Proceedings of the International Conference on Mechanical Engineering and Material Science - MEMS, 28-30 December, Shanghai, China pp. 25-28, 2012

CORRESPONDENCE

Pavle LJUBOJEVIĆ, Ph.D. student
Teaching and research assistant
University of Belgrade
Faculty of Mechanical Engineering
Kraljice Marije 16, 11120 Belgrade
pljubojevic@mas.bg.ac.rs

Radivoje MITROVIĆ, Ph.D.
Full Professor
University of Belgrade
Faculty of Mechanical Engineering
Kraljice Marije 16, 11120 Belgrade
rmitrovic@mas.bg.ac.rs



Tatjana LAZOVIĆ, Ph.D.
Full Professor
University of Belgrade
Faculty of Mechanical Engineering
Kraljice Marije 16, 11120 Belgrade
tlazovic@mas.bg.ac.rs