

LOAD-CARRYING CAPACITY OF SHEAR-LOADED BOLTED JOINTS

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Abstract Bolted joints are the most common and primary form of joints in most machines. The directions of research in this area mostly relate to strength, stiffness, load-carrying capacity, causes of failure, temperature influences etc. Fitted and clamped group bolted joints shear-loaded with torque are analyzed in this paper. A comparative analysis of their load-carrying capacity was performed. When the factor of safety (FOS) ratio of fitted and clamped bolts was determined, the geometric characteristics were varied so that this ratio was reduced, i.e. their load-carrying capacities became close. The carried-out research aims to provide the constructor with relevant data for the selection and application of the appropriate type of bolted joint. Using an analytical research method, the nominal thread diameter of the bolts, the number of bolts in a group joint, and the pitch diameter on which the bolts are mounted were varied. Useful practical results have been obtained, based on which the designer can make a choice of the appropriate type of shear-loaded bolted joint (fitted or clamped) and combine different influence parameters of the group joint (number of bolts and pitch diameter) to achieve the same load-carrying capacity or, in the case of different load-carrying capacity, to know in what relation would be their FOS.

Keywords: Bolted joint; shear-loaded bolted joint; fitted bolts; clamped bolts; bolted joint FOS.

1. INTRODUCTION

Mechanical systems are mostly composed of a large number of components. Joining the components ensures the structural integrity of the machine, achieves the desired relative position of all components of assemblies and subassemblies and realizes the designed functional unit [1]. Understanding joining technologies is a key issue in production, and there is also the continuous development of new processes, as well as the improvement of existing processes. Joints can be mechanical (formed through a mechanical mechanism), chemical (formed through chemical reactions) and thermal (formed using thermal energy) [2]. Bolted joints are mechanical joints and are the primary form of joining parts and subassemblies of most machines [3]. This type of joint is the most common choice when it is necessary to achieve a simple and relatively cheap way of joining parts. A special advantage of using bolted joints is their disassembling and/or reassembling, relatively easy maintenance and remanufacturing, protection of machine assemblies from the penetration particles of pollution, liquids and moisture, sealing, reduction of vibrations and noise [1,2,4]. However, joints are the weakest element in most structures. These are always places where something "separates, leaks, slips, wears out" [5]. Degradation of bolted joints can reduce the safety, security and reliability of the machine system [3]. The connection of machine parts by threaded joints has been a practice for many years. Although there is no exact evidence, it is assumed that the first threaded joints existed at least 500 years ago [3,5]. During this period, bolted joints experienced intensive development in the field of construction, and an increase in load capacity and reliability, in the process of standardization. Despite this, machine constructors have problems with the selection and calculation of bolted joints. That is why bolted joints are still the subject of research and

development in the pursuit of increasing the efficiency and reliability of their application. Research is carried out in very different directions, such as strength, stiffness, load capacity, causes of failure, tightening, the influence of increased temperature, etc. In doing so, all scientific methods of research, analytical, numerical, and experimental, are applied. Part of the available results of published research is presented in the following literature review. Optimal bolt tightening in the case of dynamic loading of the bolted joint was discussed in the paper [6]. A simple spring-mass model was developed and an analytical solution to the problem was derived, based on which the preload was minimized for certain given cases of bolted joints. Experimental work and analytical models for experimental results presentation are given in the paper [7]. In this combined experimental and theoretical study, micro- and macro-slip phenomena in bolted joints under the effect of torsional dynamic loading are illustrated. An improved mathematical model for determining the stiffness of bolted joints is presented in the paper [8]. A more accurate calculation of bolted joint stiffness provides a more reliable prediction of the behaviour of the joint in the assembling process, as well as under the influence of operational load. The influence of all geometrical parameters of the bolted joint was analyzed. Experimental data were used to determine the envelope angle in the proposed analytical model. Modelling using the finite element method was used to assess the accuracy of the proposed stiffness model. The behaviour of longitudinally loaded steel bolted joints with increasing strain rate was studied experimentally in the paper [9]. Based on the obtained results, a model was proposed for predicting the maximum load and the failure mode of bolted joints at high strain rates. The authors concluded that the model is in good agreement with the results of experimental research. An experimental and numerical analysis of friction dissipation in a bolted joint was carried out in [10]. The results showed that the equivalent stiffness that corresponds to the macro slip mode, as well as the upper limit of the sticking mode of the bolted joint, is most influenced by the intensity of the pre-tightening of the bolts. These results can be useful in properly designed bolted joints that would be used according to specific stiffness and damping criteria, which would result in a reduction in joint vibration [10]. Analysis of the stress in the plate with the hole to estimate the fatigue life due to the combined effect of cold expansion and bolt tightening (clamping force) was carried out in the paper [11]. Three-dimensional FE models analyzed using ANSYS software were used for the study. In the paper [12], the effects of bending moment, shear force and axial force were considered in static experiments and numerical analyzes of bolted joints. To determine the bending stiffness and strength of bolted joints, a comparison of the behaviour of bolted joints with rigid joints was made. The main conclusions drawn are that the numerical model of the bolted joint has been verified by a prototype static experiment so that the numerical results can be considered correct. According to these authors, bolted joints can be considered out-of-plane, semi-rigid and in-plane pinned joints [12]. The self-loosening of a transversely loaded bolted joint conditioned by a variable transverse force is discussed in the paper [13]. The authors developed a simplified numerical model and performed appropriate analyses. They found that self-loosening occurs already during the first load cycles of the joint. They came to interesting findings that shear is the main cause of deformation leading to self-loosening of the assembly, and that bending of the bolt shank limits self-loosening. The aim of the study [14] was the development of a 2D FE model for determining the stress in a bolted joint, based on which an analytical calculation of the load of the bolt and joint members (joined plates) was performed depending on the operational axial load of the joint. The stress during the tightening and operation of the bolt, as well as the variable stress, were calculated using the finite element method. The results of analytical calculations and calculations using the finite element method were compared and a high degree of agreement was established. The influence of increased temperature on the

behaviour of bolted joints was analyzed in the paper [15]. The authors performed experimental research on the influence of heat (from 400 °C to 700 °C) on the failure mode, deformation properties and impaired load-carrying capacity of bolted joints. The authors proposed models with different reliability for calculating strength loss under increasing temperature, which is of great importance for designing fire-resistant joints and structures. In the paper [16], a comparative analysis of the load-carrying capacity (factor of safety) of shear-loaded fitted and clamped bolted joints was performed under the same geometric and operational conditions. The analysis showed 60 times (!) greater load-carrying capacity of fitted bolted joints. Based on these results, this paper analyzed the influence of the parameters of fitted and clamped shear-loaded bolted joints on the ratio of their load-carrying capacities (factor of safety). The goal of these analyzes is to provide the constructor with relevant data for the selection and application of the appropriate bolted joint, depending on the geometry and type of bolt installation.

2. RELATIVE LOAD-CARRYING CAPACITY OF FITTED AND CLAMPED BOLTED JOINTS

The objective of shear-loaded bolted joints is to avoid the relative sliding of joint members. The operational load of a group shear-loaded bolted joint can be force or torque. A flanged joint that transmits torque T through a group bolted joint is shown in Figure 1. The group bolted joint consists of n bolts evenly placed on a pitch circle of diameter D . The load per bolt is:

$$F_1 = \frac{T}{D/2} \xi_{ie}, \quad (1)$$

where $\xi_{ie} = (1.3 \dots 1.5)$ is a factor of load distribution inequality.

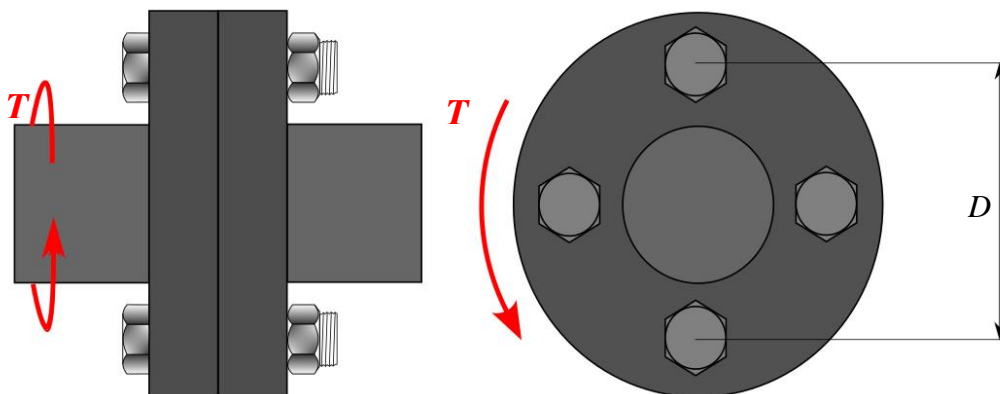


Figure 1. Group shear-loaded bolted joint [16].

There are two types of shear-loaded bolted joints (Figure 2): fitted and clamped ones. In the fitted bolted joints, the load is transferred from one joint member to another one through the unthreaded part of the bolt body (bolt shank) which behaves like a pin. In the clamped bolted joint, the relative movement of the joined members is prevented by the frictional force between them, dependent on the clamping force, ie. tightening torque. The operating load F_1 of the shear-loaded bolted joint, given by Eq. 1 is perpendicular to the axis of the bolt (Figure 2).

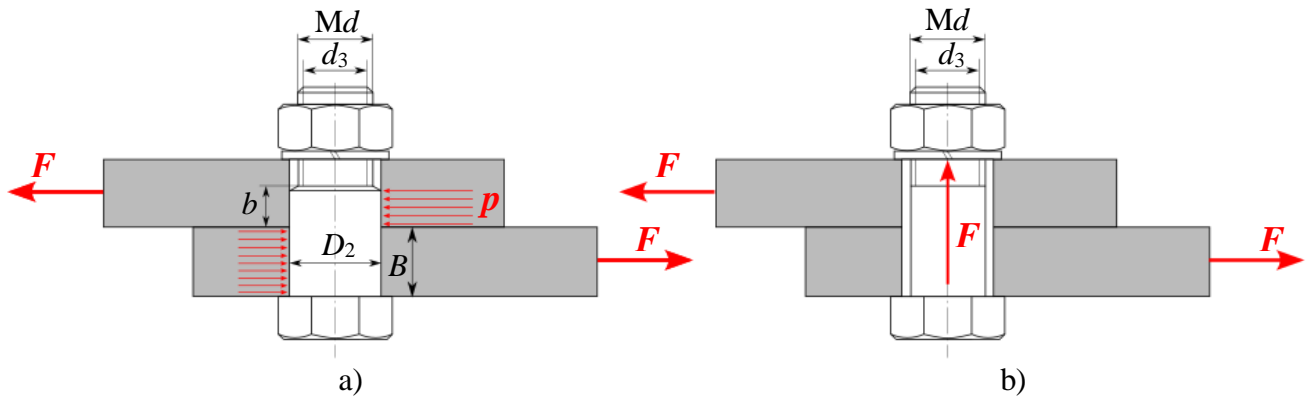


Figure 2. a) Fitted bolted joint; b) Clamped bolted joint [16].

Resistance to shear and contact pressure limits the load-carrying capacity of fitted bolted joints. The length of thread contact with one of the joined plates, which corresponds to the equal surface and volume load carrying capacity of the fitted bolted joint, was determined in the paper [16] and is $b = 0.4581D_2$, where D_2 is the diameter of the bolt shank (Figure 2). In this case, factors of safety (FOS) of shear (S_τ) and surface pressure (S_p) are given by the expression [16]:

$$S_\tau = S_p = \frac{0.2749R_e D_2^2 n D}{T \xi_{ie}}, \quad (2)$$

where R_e is the yield stress of bolt material.

Clamped bolted joints are loaded in the axial direction by the clamping force needed for the prevention of joint members' slip. The appropriate tension FOS is [16]:

$$S_\sigma = \frac{R_e d_3^2 \pi \mu n D}{8 T S_\mu \xi_c \xi_{ie}}, \quad (3)$$

where d_3 is minor thread diameter; μ is friction coefficient between joint members; S_μ is slip FOS; ξ_c is the joint clamping factor.

The ratio between load-carrying capacities of fitted and clamped bolts is the ratio between their FOS's (Eq. 2 and Eq. 3) [16]:

$$\frac{S_f}{S_c} = 0.7C \left(\frac{D_2}{d_3} \right)^2. \quad (4)$$

The constant C in Eq. 4 is given by expression [16]:

$$C = \frac{S_\mu \xi_c}{\mu}. \quad (5)$$

The values of the constant C depend on the clamping parameters: $S_\mu = (1.2 \dots 1.8)$; $\mu = (0.1 \dots 0.3)$; $\xi_c = (1.5 \dots 2.0)$, so C can be named a clamping constant. For the specified values of these parameters, the range of the clamping constant is $C = (6 \dots 36)$. In that case, Eq. 4 can be written as follows:

$$\frac{S_f}{S_c} = 0.7(6 \dots 36) \left(\frac{D_2}{d_3} \right)^2 = (4.2 \dots 25.2) \left(\frac{D_2}{d_3} \right)^2. \quad (6)$$

In addition to other influencing factors, the ratio of the load-carrying capacity of fitted and clamped bolts also depends on the ratio of appropriate diameters: the diameter of the fitted bolt shank and the minor thread diameter of the clamped bolt. Table 1 gives the values of these dimensions for standard bolts with a metric thread from M6 to M24.

Table 1. Minor thread diameter and fitted bolt shank diameter depending on standard metric thread.

Standard metric thread										
	M6	M8	M10	M12	M14	M16	M18	M20	M22	M24
Minor thread diameter d_3 (mm)	4.773	6.466	8.160	9.853	11.546	13.546	14.933	16.933	18.933	20.319
Fitted bolt shank diameter D_2 (mm)	7	9	11	13	15	17	19	21	23	25
D_2 / d_3	1.467	1.392	1.348	1.319	1.299	1.255	1.272	1.240	1.215	1.230

The dependences of the minor thread diameter d_3 and the diameter of the fitted bolt shank D_2 on the bolt thread size Md , based on the data from Table 1, are shown in Figure 3a. The dependence of the diameter ratio D_2/d_3 on the bolt thread size Md is shown in Figure 3b. The values of those diameters increase with the increase of the thread nominal diameter as a linear function. At the same time, the influence of the nominal thread diameter is slightly greater on the change of the fitted bolt shank diameter (Figure 3a). However, the diagram in Figure 3b shows that the ratio of these two diameters decreases with increased bolt size. For smaller bolts, the gradient of this change is greater. For bolts M6 ... M14 it is within the limits (1.30 ... 1.47), and for bolts from M16 to M24 within the limits (1.21 ... 1.27).

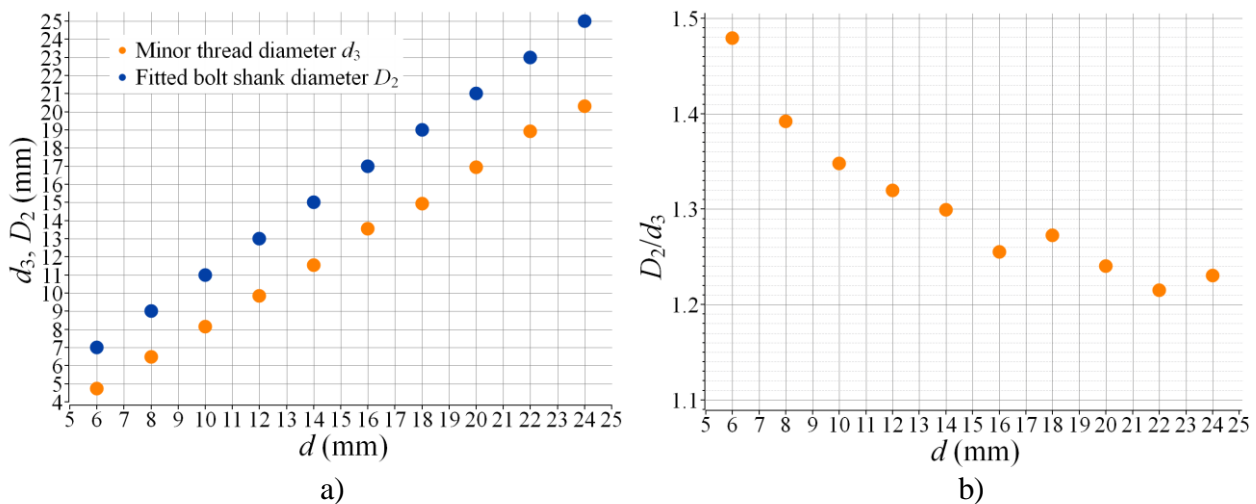


Figure 3. Minor thread diameter (a), fitted bolt shank diameter (a) and their ratio (b) depending on bolt size.

As it can be seen from Eq. 6, the load-carrying capacity ratio of fitted and clamped bolt S_f/S_c under the same geometric conditions, load conditions and strength of the bolt material depends on the square of the ratio D_2/d_3 . Given that the clamping parameters S_μ , μ and ζ_c are empirically determined in certain ranges, the influence of the geometry of fitted and clamped bolts with the same nominal thread diameter on the ratio of their FOS's is within certain limits. The dependence of the load-carrying capacity ratio, i.e. the FOS of the fitted and clamped bolted joints depending on the bolt size is shown in Figure 4. For the smallest considered bolt M6, the load carrying capacity ratio is within the limits of approx. (9 ... 54), and for the largest considered bolt M24 it is approx. (6 ... 37). Based on these data, it can be concluded that in most cases, the constructor should choose a fitted bolted joint because it will achieve a much higher load-carrying capacity compared to a clamped joint, under the same geometric conditions (the same number of bolts in the group bolted joint, the same bolt size, the same the pitch diameter on which the group of bolts is installed). However, regardless of the higher load-carrying capacity of fitted bolted joints, there are cases when preference is given to clamped bolted joints to shear-loaded joints (ease of installation, and thus the price). That is why the subject of this paper is an attempt to approximate the load-carrying capacity of fitted and clamped bolted joints, by varying the geometrical parameters of the joint. It is the number of bolts in a joint and the associated diameter of the pitch circle on which the bolts are mounted.

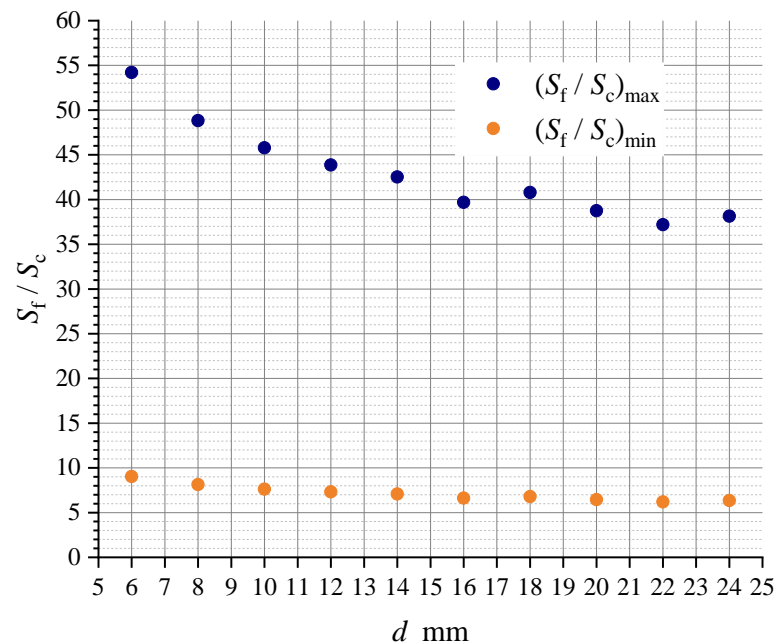


Figure 4. Extreme values of fitted and clamped bolts' FOS ratio influenced by bolt size.

If fitted and clamped group bolted joints have different numbers of bolts (n_f and n_c , respectively) mounted on the same pitch diameter, then, based on Eq. 2 and Eq. 3, the FOS ratio, given by Eq. 4, can be written as follows:

$$\frac{S_f}{S_c} = 0.7C \left(\frac{D_2}{d_3} \right)^2 \frac{n_f}{n_c} = 0.7C \left(\frac{D_2}{d_3} \right)^2 \xi_n. \quad (7)$$

In Eq. 7, a new quantity was introduced, the ratio n_f/n_c of the number of fitted and clamped bolts in a group bolted joint, which can be named as a factor of the bolt numbers (ξ_n). The minimum number of bolts in a group bolted joint is three. For the following analyses, it is assumed that the maximum number of bolts is 12, i.e. is in the range (3 ... 12). In the cases of all possible combinations of bolt numbers of fitted and clamped joints, the factor of the bolt numbers is within the limits $\xi_n = (0.25 \dots 1.00)$. This means that the FOS ratio can be reduced to a quarter of the initial values (Figure 4) if the fitted bolted joint has 3 bolts, and the clamped bolted joint has 12 bolts on the same pitch diameter. In that case, for the smallest considered bolt M6, the FOS ratio of fitted and clamped bolts is within the limits of approx. (2.25 ... 13.5), and for the largest considered bolt M24, the limits are approx. (1.5 ... 9.25). This is still a big difference in the load-carrying capacity of fitted and clamped bolted joints. In addition, there is also a construction problem, related to the dimensions and the mass of structures, the material used and the price. Namely, in the case of the same pitch diameter, in the extreme values of bolt numbers $n_f/n_c = 3/12$, i.e. values of the factor of the bolt numbers $\xi_n = 0.25$, disproportion would appear in the construction of the flange joint made with fitted and clamped joints (an example is shown in Figure 5). Therefore, the pitch diameters of fitted and clamped group bolted joints (D_f and D_c , respectively) should be adjusted to the number of bolts in the group joint.

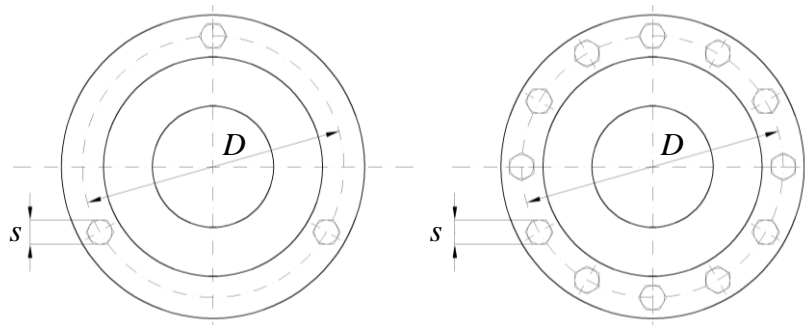


Figure 5. Group bolted joints with equal pitch diameter and considered extreme numbers of bolts.

If the pitch diameters are different, then Eq. (7) has the form:

$$\frac{S_f}{S_c} = 0.7C \left(\frac{D_2}{d_3} \right)^2 \xi_n \frac{D_f}{D_c} \quad (8)$$

Figure 6a shows the position of two adjacent bolts in a group bolted joint on the pitch diameter D . The diameter circle of the bolt head is determined by a standard measure - width across flats s . The bolts are distributed evenly, at an angular distance of $2\pi/n$. The arc distance between the centres of every two adjacent bolts is (Figure 6a):

$$l = \frac{2\pi}{n} \cdot \frac{D}{2} = \frac{\pi D}{n} \quad (9)$$

The length of the arc x in Figure 6 is given by the expression:

$$x = \gamma \frac{D}{2} \quad (10)$$

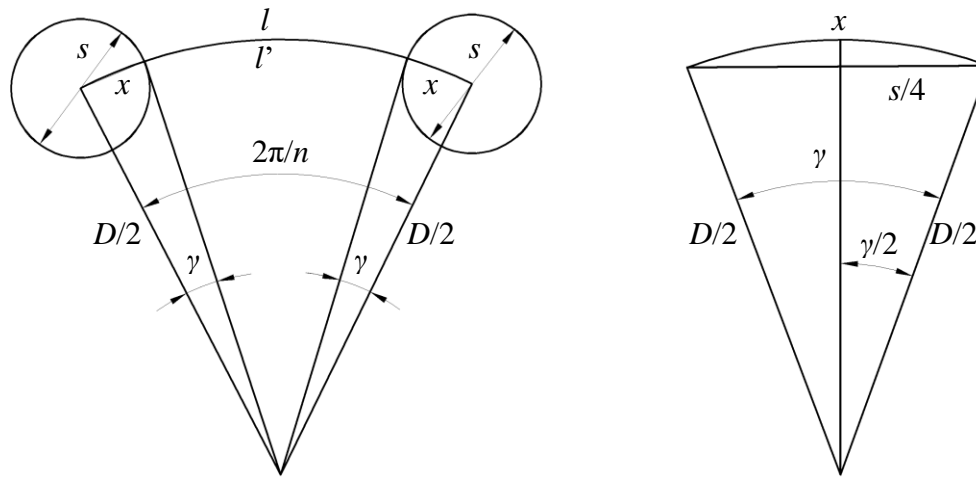


Figure 6. Geometrical parameters of the group bolted joint.

Based on Figure 6b it can be written as:

$$\sin \frac{\gamma}{2} = \frac{\frac{s}{4}}{\frac{D}{2}} = \frac{s}{2D} \rightarrow \gamma = 2 \arcsin \frac{s}{2D}. \quad (11)$$

By introducing the angle γ from Eq. 11 in Eq. 10, it is obtained:

$$x = D \arcsin \frac{s}{2D}. \quad (12)$$

The next step is to express the pitch diameter as a function of the number of bolts. For this purpose, another variable is introduced, which defines the distance between the bolts. The distance between the bolts can be written as the difference of the arcs shown in Figure 6a, and then expressed as a function of the arc length x (a multiple of x):

$$l' = l - 2x = ix, \quad i \geq 0 \quad (13)$$

The introduced quantity i cannot have small values (0, 1...) for practical structural and technological reasons. Namely, with small values of i , the distance between the bolts, and thus the pitch diameter, would be so small that constructively it would not be possible to realize a flange hub of the appropriate diameter, depending on the operational torque load. In addition, due to the technological conditions of assembly, it is necessary to have a certain distance between the bolts, i.e. free space, due to different types and dimensions of tools - wrenches for bolted joint clamping. The analysis of the dimensions of rigid couplings with installed $n = 3 \dots 12$ bolts up to size M24 [17], showed that the minimum value of size i is equal to 3. That is why cases when $i \geq 3$ will be considered in further analyses. Substituting Eq. 9 and Eq. 12 in Eq. 13 the pitch diameter as a function of the number of bolts is given by the expression:

$$D = \frac{s}{2 \sin \frac{\pi}{n(i+2)}}, \quad i \geq 3 \quad (14)$$

Based on the expression for the pitch diameter, in the case of the same size of bolts (width across flats of bolt heads is equal, ie. $s_f = s_c$) FOS ratio of the fitted and clamped bolted joints (Eq. 8) can be written in the form:

$$\frac{S_f}{S_c} = 0.7C \left(\frac{D_2}{d_3} \right)^2 \xi_n \frac{\sin \frac{\pi}{n_c(i+2)}}{\sin \frac{\pi}{n_c \xi_n(i+2)}} = 0.7C \left(\frac{D_2}{d_3} \right)^2 \xi_n K(i), \quad i \geq 3, \quad (15)$$

Table 2 shows the values of the auxiliary quantity K from Eq. 15 for $i = 1, 3$ and 6 . In doing so, the limit values of the factor of the number of bolts ξ_n were considered for all possible combinations of the number of bolts from 3 to 12. The highest value of the factor of the bolts numbers is ξ_n , when the number of fitted and clamped bolts is equal. In all other cases, the number of fitted bolts is less than the number of clamped bolts, so that $\xi_n < 1$. The lowest value of the factor of the number of bolts is in the case when, from the range (3 ... 12), the number of fitted bolts is the smallest, and the number of clamped bolts is the largest, that is $\xi_n = 3/12 = 0.25$ (Table 2).

Table 2. Auxiliary quantity K from Eq. 15 depending on number of bolts in group bolted joint.

n_f	3	4	5	6	7	8	9	10	11
n_c	12								
ξ_n	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833	0.917
$K(i=1)$	0.255	0.335	0.420	0.502	0.584	0.668	0.751	0.833	0.917
$K(i=3)$	0.252	0.334	0.418	0.501	0.584	0.667	0.750	0.833	0.917
$K(i=6)$	0.251	0.333	0.417	0.500	0.583	0.667	0.750	0.833	0.917

Based on the results shown in Table 2 and the analysis carried out, it can be concluded that with the increase of i , the auxiliary quantity K tends to the value of the factor of the number of bolts ξ_n . The difference between the actual calculated value of K and ξ_n is the largest in the case of the smallest factor of the number of bolts ($\xi_n = 3/12 = 0.25$) and when $i = 1$. In this case, the difference is 2%. In the case of $i = 3$, the error is 0.8%, and for $i = 6$ it is 0.4%. In all other cases (larger values of ξ_n and i), the difference is even smaller. Errors in load rating up to 2%, and especially 0.8% or 0.4% are negligibly small. In addition, it was previously explained that the size $i < 3$ is not structurally applicable and justified. Consequently, the quantity K in Eq. 15 can be replaced by its limiting value, which is mathematically provably equal to ξ_n :

$$\lim_{i \rightarrow \infty} K(i) = \lim_{i \rightarrow \infty} \frac{\sin \frac{\pi}{n_c(i+2)}}{\sin \frac{\pi}{n_c \xi_n(i+2)}} = \left[\frac{0}{0} \right] \text{L'Hôpital's rule} = \lim_{i \rightarrow \infty} \frac{\left(\sin \frac{\pi}{n_c(i+2)} \right)'}{\left(\sin \frac{\pi}{n_c \xi_n(i+2)} \right)'} = \dots = \xi_n. \quad (16)$$

Eq. 16 for the FOS ratio after determining the limit value of the quantity K takes a simpler form:

$$\frac{S_f}{S_c} = 0.7C \left(\xi_n \frac{D_2}{d_3} \right)^2. \tag{17}$$

Based on this expression, relevant influences on the relative load-carrying capacity of fitted and clamped bolted joints can be analysed.

3. RESULTS, ANALYSIS AND DISCUSSION

The dependence of the minimum FOS ratio of the fitted and clamped group bolted joint on the bolt size and the ratio of the number of bolts in the joint (Eq. 7) is shown in Figure 7. 3D representation of the dependence of the ratio $(S_f/S_c)_{\min}$ on the factor of the number of bolts $\xi_n = n_f/n_c = (0.25 \dots 1.00)$ and on the diameter ratio D_2/d_3 , which are functions of the bolt size (Figure 3b) is given in Figure 8.

The S_f/S_c ratio has a minimum value when the clamping constant C from Eq. 5 is determined on the minimum clamping parameters ($S_\mu = 1.2$, $\mu = 0.1$ and $\xi_c = 1.5$), i.e. $C = 6$. The ratio S_f/S_c is selected for each number of fitted ($n_f = 3, 4, \dots, 12$) for which the factor of the number of bolts ξ_n is the smallest, i.e. when $n_c = 12$. For all other combinations of bolt numbers (when $n_c = 3, 4, \dots, 11$), the values of the minimum FOS ratio are higher. The ratio $(S_f/S_c)_{\min}$ has the highest value in all cases when the number of bolts in the compared joints is equal, i.e. when $n_f = n_c$.

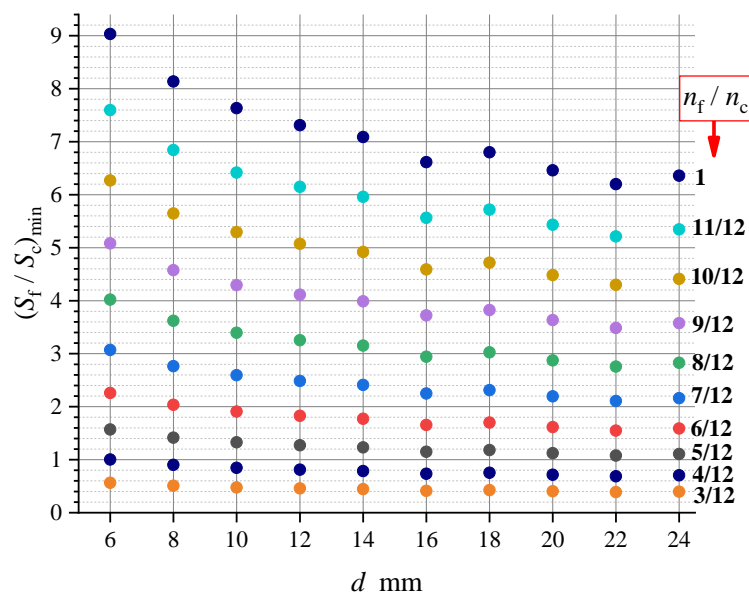


Figure 7. Minimum fitted and clamped bolts' FOS ratio depending on bolt size and number of bolts in group bolted joint.

Based on the diagrams in Figures 7 and 8, the following can be concluded:

- FOS ratio decreases with an increase of the bolt size and the ratio of their number in the group bolted joint;
- the influence of the number of bolts decreases with an increase in the bolt size, i.e. the dispersion of the FOS ratio value is reduced;

- of all the considered values, the smallest FOS ratio is in the case of the combination of bolt numbers $n_f/n_c = 3/12$ with bolt size M24 and is $(S_f/S_c)_{\min} = 0.4$ (Figure 7);
- the minimum FOS ratio has the highest value in all cases of an equal number of M6 bolts in a group bolted joint ($n_f = n_c$), regardless of the absolute value of their number (for example 3/3, 4/4, ... , 12/12) and is $(S_f/S_c)_{\min} = 9$ (Figure 7);
- in general, the combination of small bolts in an approximately equal number in a group bolted joint gives the greatest deviations in the comparative load-carrying capacity of fitted and clamped bolted joints (Figure 8);

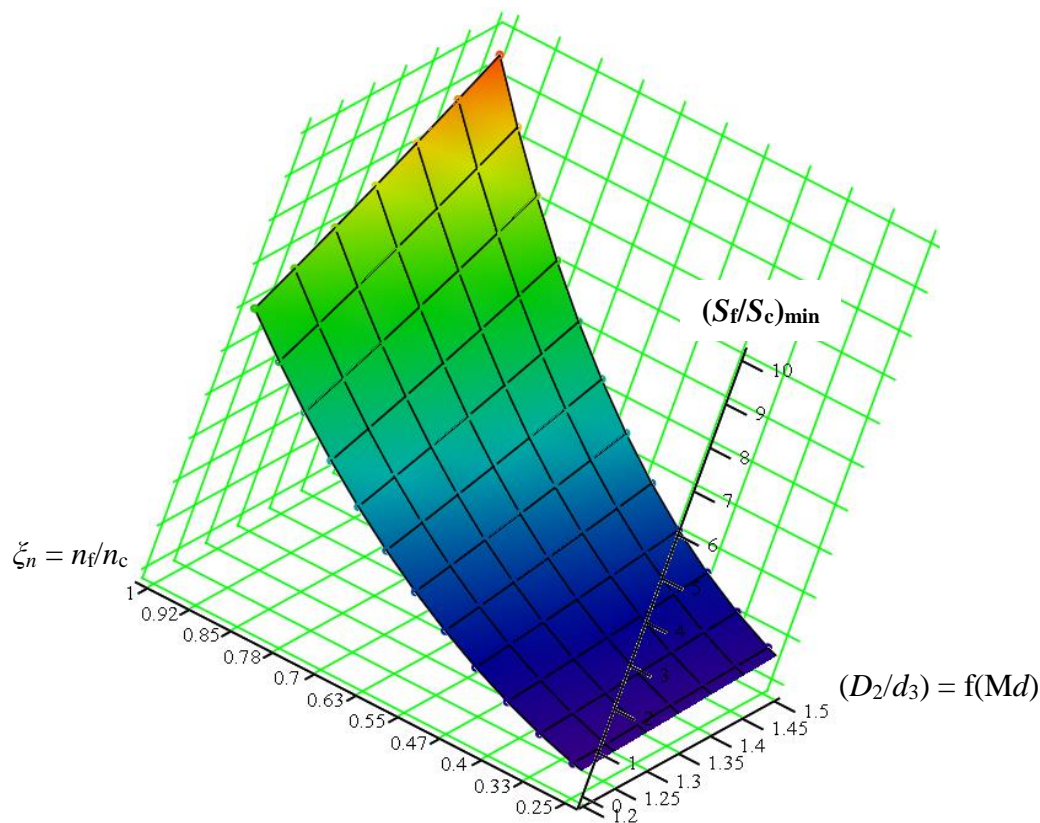


Figure 8. Simultaneous effect of bolt size and number of fitted/clamped bolts on their relative load-carrying capacity.

- in the combination of large bolts with a large difference in their number, the difference in load-carrying capacity of the fitted and clamped joints decreases (Figure 8);
- in earlier analyzes [16] it was shown that the load-carrying capacity of fitted bolted joints is always and many times higher than the load-carrying capacity of clamped bolted joints, with the same number of fitted and clamped bolts in group bolted joints with the same pitch diameter, from 6 to 54 times, depending on the clamping conditions and bolt sizes (Figure 3);
- further analysis, carried out in this paper, showed that by introducing the difference in the number of bolts and the diameter of the fitted and clamped group bolted joints, the difference in their load-carrying capacity can be reduced;

- also, the FOS ratio can be less than 1 (Figure 7), which means that in a certain combination of size and number of bolts, the load-carrying capacity of the fitted joint is lower than the load-carrying capacity of the clamped joint;
- the minimum FOS ratio is not higher than 1 in cases where $(n_f/n_c)_{\min} = 3/12$ or $4/12$ (Figure 7).

Now, the question is for which combinations of numbers of bolts and their size the load-carrying capacity of the fitted and clamped joints shear-loaded bolted joint is the same, i.e. FOS's are equal ($S_f = S_c$). This function can be obtained from Eq. 17, based on the conditions:

$$\frac{S_f}{S_c} = 0.7C \left(\xi_n \frac{D_2}{d_3} \right)^2 = 1. \quad (18)$$

Based on Eq. 18, it is obtained:

$$\xi_n = (0.7C)^{-0.5} \left(\frac{D_2}{d_3} \right)^{-1}, \text{ i.e.} \quad (19)$$

$$\xi_n \approx (0.2 \dots 0.5) \left(\frac{D_2}{d_3} \right)^{-1}, \text{ when } C = 6 \dots 36 \text{ (Eq. 5)}$$

The function that relates the ratio of the number of bolts of fitted and clamped bolted joints under the condition of their equal load-carrying capacity is shown in Figure 9. The blue lines show the limit values of the ratio of the number of bolts, depending on the bolt size and the limits of the clamping constant C , defined earlier (Eq. 5). The orange dots are the possible solution combinations of size and a number of bolts when $S_f = S_c$. For bolts larger than M24, other combinations of bolt numbers would be possible, e.g. 4/9, 5/12 etc, but larger bolts are not considered in this paper. The cases, when the number of bolts is greater than 12, which would also increase the area of possible solutions (below the level $\xi_n = 0.25$, up to the lower limit value for the clamping constant $C = 36$), were not considered. Depending on the clamping parameters S_μ , μ and ξ_c , i.e. constant C , group bolted joints with the following combinations of size Md and numbers of bolts n_f/n_c have equal load capacity (Figure 9):

- 6 ... M24: 3/9, 3/10, 3/11, 3/12 and $\approx 4/12$; M
- 12 ... M24: 4/11; M
- 14 ... M24: 3/8; M
- 22: 4/10. M

Since the equal load-carrying capacity is caused by the values of the clamping parameters, an expression for the clamping constant, which corresponds to that condition should be derived (Eq. 19):

$$C = \frac{1}{0.7} \left(\xi_n \frac{D_2}{d_3} \right)^{-2} \quad (20)$$

The values of the clamping constant C for which the load-carrying capacity of the fitted and clamped bolted joints are the same are given in Table 3. The value of this constant is within the limits (6 ... 15) for given conditions of bolt size and number, as well as equality of FOS's. The pitch diameters of the appropriate bolted joints with the same load-carrying capacity are given in Table 4. These values are obtained from Eq. 14 for the appropriate values of width across flats s and quantity $i = 3$, which is the minimum defined value. This means that the diameters in Table 4 are the minimum possible diameters for the given conditions. Decimal numbers are for simplicity rounded off to whole numbers, and rounding-off errors are negligible. An illustration of several randomly selected combinations from Table 4 is shown in Figure 9.

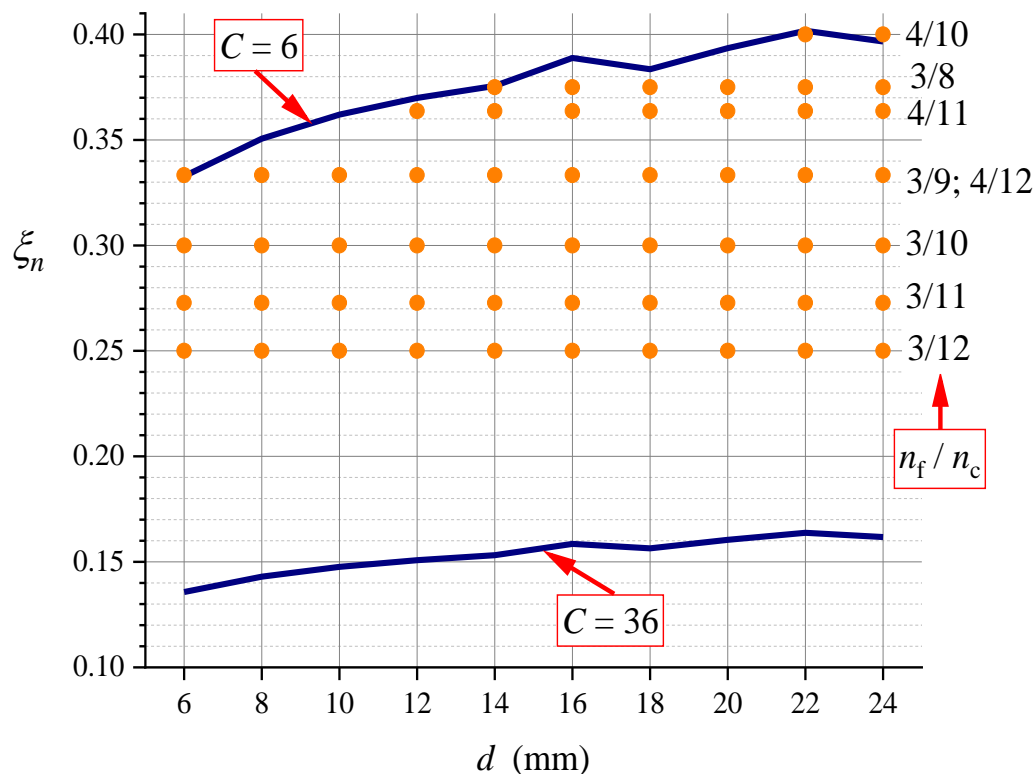


Figure 9. Possible combinations of size and number of fitted and clamped bolts in group bolted joint with equal load-carrying capacity (equal FOS's).

Table 4 gives very useful information on how to form a group shear-loaded bolted joint, fitted or clamped, without affecting the load-carrying capacity, i.e. when the FOS of the fitted and clamped joint is the same.

Table 3. Clamping constant C , depending on bolt size and number of bolts in fitted and clamped group bolted joints with equal FOS's.

Md	D_2/d_3	$\xi_n = n_f/n_c$							
		3/8	3/9	3/10	3/11	3/12	4/10	4/11	4/12
M6	1.467	/	5.97	7.38	8.92	10.62	/	/	5.98 (≈ 6)
M8	1.392	/	6.64	8.19	9.91	11.80	/	/	6.64
M10	1.348	/	7.08	8.74	10.57	12.58	/	/	7.08
M12	1.319	/	7.39	9.12	11.04	13.14	/	6.21	7.39
M14	1.299	6.02	7.62	9.41	11.38	13.55	/	6.40	7.62
M16	1.255	6.45	8.16	10.08	12.19	14.51	/	6.86	8.16
M18	1.272	6.28	7.95	9.81	11.87	14.13	/	6.68	7.95
M20	1.240	6.61	8.36	10.32	12.49	14.87	/	7.03	8.36
M22	1.215	6.88	8.71	10.75	13.01	15.48	6.05	7.32	8.71
M24	1.230	6.72	8.50	10.49	12.70	15.11	/	7.14	8.50

Table 4. Pitch diameters D_f and D_c (mm) of fitted and clamped group bolted joints with equal FOS's, depending on bolt size and number of bolts (for $i = 3$)

Md	s mm	fitted clamped	$\xi_n = n_f/n_c$							
			3/8	3/9	3/10	3/11	3/12	4/10	4/11	4/12
M6	10	f	/	24				/	/	32
		c	/	72	80	88	96	/	/	96
M8	13	f	/	31				/	/	42
		c	/	93	104	114	124	/	/	124
M10	17	f	/	41				/	/	54
		c	/	122	135	149	162	/	/	162
M12	19	f	/	46				/	61	
		c	/	136	151	166	182	/	166	182
M14	22	f	53				/	70		
		c	140	158	175	193	210	/	193	210
M16	24	f	58				/	77		
		c	153	172	191	210	229	/	210	229
M18	27	f	65				/	86		
		c	172	194	215	236	258	/	236	258
M20	30	f	72				/	96		
		c	191	215	239	263	287	/	263	287
M22	32	f	77				102			
		c	204	229	255	280	306	255	280	306
M24	36	f	87				115			
		c	229	258	287	315	344	/	315	344

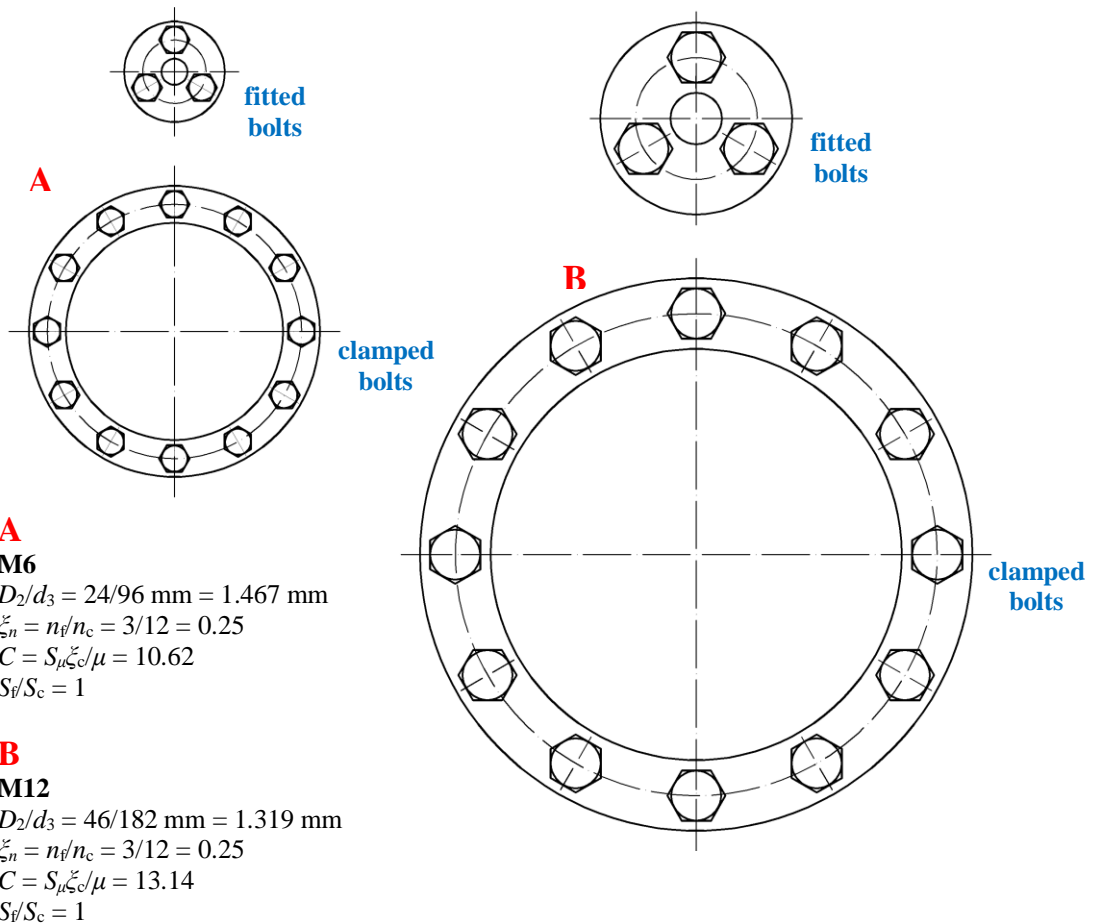


Figure 10. Examples of fitted and clamped group bolted joints with the same bolt size, FOS, and i value.

3. CONCLUSION

The initial idea for this paper topic was published in paper [16], where it was established that in the case of the same load conditions (operational torque of a group shear-loaded bolted joint) and geometry (the same size and number of bolts, the same pitch diameter on which they are installed) the load-carrying capacity of fitted bolted joints is greater than the load-carrying capacity of clamped joints from 6 to 54 times, depending on the tightening conditions of the clamped bolts. This is due to the objectively higher load-carrying capacity of the fitted bolts that "act" as pins so that their shank (with a diameter greater than the thread major diameter) is loaded by surface pressure and shear force. Clamped bolts generally have a lower load-carrying capacity, because they are loaded in tension in shear-loaded joints, where the tensioning force is carried by the bolt core with a diameter equal to the minor thread diameter. In this paper, functions have been derived that enable the reduction of the difference in the load-carrying capacity of fitted and clamped bolted joints by varying their number in the group bolted joint and the pitch diameter on which they are mounted in the torque transmission. In this way, it was achieved that with certain combinations of these geometric characteristics and the tightening characteristics of clamped joints, the fitted and clamped group joints are close in terms of load capacity, i.e. that the FOS ratio of fitted and clamped bolts should be reduced and be from 0.4 to 9. It is interesting that with certain combinations of geometrical parameters of the group bolted joint and clamping parameters when transmitting the torque, a higher load-carrying capacity of

clamped bolted joints can be achieved, which was not the case before [16]. Cases of equal load-carrying capacity were also analyzed, i.e. when the FOS ratio of the fitted and clamped bolted joint is equal to 1. Useful practical results were obtained that can be used by constructors to make an appropriate choice of bolted joint type (fitted or clamped) by combining various influence parameters of group shear-loaded bolted joints, and at the same time achieve the same load-carrying capacity or, in the case of different load-carrying capacities, to know exact FOS ratio.

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