A NEURAL NETWORK-BASED CONTROL ALGORITHM FOR A HYDRAULIC HYBRID POWERTRAIN SYSTEM

Marko Kitanović [1]*, Slobodan Popović [2], Nenad Miljić [3], Predrag Mrđa[4]

ABSTRACT: Significant research efforts are invested in the quest for solutions that will increase the fuel economy and reduce the environmental impacts of ICE-powered vehicles. The main objective of the study presented in this paper has been to analyze and assess the performance of a control methodology for a parallel hydraulic hybrid powertrain system of a transit bus. A simulation model of the vehicle has been calibrated by analyzing data obtained during an experiment conducted in real-world traffic conditions aboard a Belgrade transit bus. A Dynamic Programming optimization procedure has been applied on the calibrated powertrain model and an optimal configuration that minimizes the fuel consumption has been selected. A Neural Network-based, implementable control algorithm has then been formed through a machine learning process involving data from the optimal, non-implementable Dynamic Programming-based control. Several Neural Network configurations have been tested to obtain the best fuel economy for the range of conditions encountered during normal transit bus operation. It has been shown that a considerable fuel consumption reduction on the order of 30% could be achieved by implementing such a system and calibration method.

KEY WORDS: *hydraulic hybrid, internal combustion engines, machine learning, dynamic programming, transit bus*

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UPRAVLJAČKI ALGORITAM ZA HIDRAULIČNI HIBRIDNI POGONSKI SISTEM ZASNOVAN NA NEURONSKOJ MREŽI

REZIME: Značajni istraživački napori ulažu se u potragu za rešenjima koja će povećati ekonomičnost goriva i smanjiti uticaj vozila na ICE pogon na životnu sredinu. Glavni cilj studije predstavljene u ovom radu je analiza i procena performansi metodologije upravljanja za paralelni hidraulični hibridni pogonski sistem tranzitnog autobusa. Simulacioni model vozila je kalibrisan analizom podataka dobijenih tokom eksperimenta sprovedenog u realnim saobraćajnim uslovima u beogradskom tranzitnom autobusu. Procedura optimizacije dinamičkog programiranja je primenjena na kalibrisani model pogonskog sklopa i izabrana je optimalna konfiguracija koja minimizira potrošnju goriva. Algoritam upravljanja koji se može primeniti, zasnovan na neuronskoj mreži, je zatim formiran kroz proces mašinskog učenja koji uključuje podatke iz optimalne, nesprovodljive kontrole zasnovane na dinamičkom programiranju. Nekoliko konfiguracija neuronske mreže je testirano da bi se postigla najbolja ekonomičnost goriva za niz uslova koji se javljaju tokom normalnog tranzitnog rada. Pokazalo se da se primenom ovakvog sistema i metode kalibracije može postići značajno smanjenje potrošnje goriva od oko 30%.

KLJUČNE REČI: *hidraulični hibrid, motori sa unutrašnjim sagorevanjem, mašinsko učenje, dinamičko programiranje, tranzitni autobus*

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INTRODUCTION

Rising fuel prices and increasing awareness of environmental issues place greater emphasis on the quest for solutions that improve vehicle fuel economy and reduce harmful emissions. One of the many possible directions in that regard, and perhaps the most promising, is powertrain hybridization. Achieving improved fuel economy, lower emissions and a relatively low price without incurring penalties in performance, safety, reliability, and other vehicle-related aspects represents a great challenge for the automotive industry. For accommodating the hybrid powertrain demands of heavy vehicles, particularly those undergoing frequent deceleration and acceleration phases, the best solutions are those that can sustain very high power levels, such as the hydraulic hybrid or the ultracapacitors-based hybrid electric systems.

The main objective of the study presented in this paper is to analyze and assess the performance of a control methodology for a parallel hydraulic hybrid powertrain system. An experiment has been conducted on a transit bus circulating in real traffic and occupancy conditions in Belgrade, Serbia to assess the circumstances encountered in this particular type of transportation and in order to obtain the real driving cycle and the vehicle powertrain parameters necessary for conducting virtual analyses involving hybrid solutions. Data acquired during this experiment has been of crucial importance; effectively allowing us to conduct identification procedures on a set of powertrain parameters in order to calibrate the vehicle model used in the simulation. By successfully transferring the real-world physical conditions into computer code, a practically infinite number of numerical study possibilities has been opened. In the following section of this paper, methods applied during this research are presented, including an overview of the calibrated hybrid powertrain system simulation model used in this study. The methodology section also includes an overview of the Dynamic Programming method used to derive the optimal control law and to assess the ultimate fuel economy improvement potential of the hybrid powertrain system. Next, the details on the Artificial Neural Network (ANN) configurations used in this study to derive an implementable control algorithm are laid out.

The results and concluding remarks are presented in their respective sections, following the methodology section.

1. METHODOLOGY

The methods applied in this study are presented in the following subsections.

1.1 Hybrid Powertrain System Model

In order to calibrate the hybrid powertrain system model used in the study, an experiment was conducted on an Ikarbus IK206 transit bus circulating in real occupancy and traffic conditions. It was equipped with a MAN D2066 LUH 11 engine (10.5 dm^3, 6-cylinder, turbocharged diesel engine) and a Voith 864.5 automatic transmission. An autonomous data acquisition system based on National Instrument's CompactRIO hardware platform and LabVIEW software has been designed for this purpose. The powertrain parameters were

acquired by accessing the vehicle's J1939 CAN bus by means of a high-speed NI 9853 CAN module. The raw network stream has been logged and afterwards processed according to the SAE J1939 standard [1]. In order to obtain the GPS coordinates of the driving cycle, which are needed for determining the road slope, a Garmin GPS 18x 5 Hz receiver streaming NMEA messages was used. Suspension system pressure sensors have also been installed in order to log the vehicle mass during the experiment.

This experiment has been conducted for the duration of several weeks, during which a vast amount of highly valuable data has been collected. It has allowed the calibration of a MATLAB model of a conventional transit bus powertrain that has served afterwards as an input for Dynamic Programming (DP) optimization runs involving various hydraulic hybrid configurations. Rolling friction coefficients, aerodynamic friction coefficients, brake torque limits maps (Figure 1) and the engine BSFC (Figure 2), along with data concerning the gearbox, hydrodynamic torque converter and various drivetrain components among others, have been implemented into the base model. Detailed procedures and values can be found in [2, 3, 4].
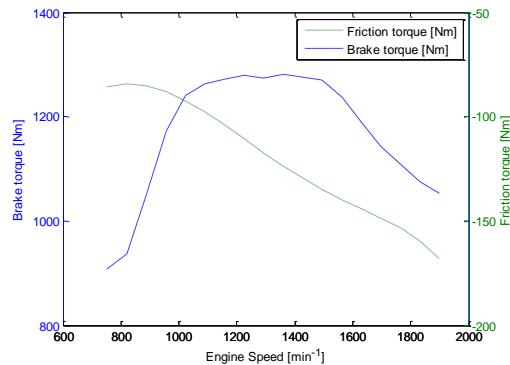

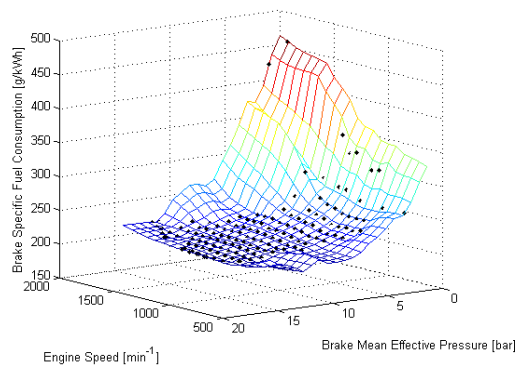*Figure 1 Max. engine brake torque and friction torque*

Figure 2 Brake Specific Fuel Consumption (BSFC) map

A model of a 250 cm^3, variable displacement swashplate axial piston pump (Rexroth A4VSO) has been used as the main hydraulic unit for recuperating the regenerative braking energy and subsequently providing traction to the vehicle during acceleration phases. A fixed ratio (1.2) gearbox is positioned between the pump and the vehicle drivetrain in order to match the operational range of the hydraulic unit with the engine speed range.

Compared to electrical batteries, hydro-pneumatic accumulators are characterized by a higher specific power and a lower specific energy. Its high specific power renders it suitable for heavy vehicles with frequent acceleration/deceleration phases. On the other hand, the low specific energy represents a disadvantage due to the limited braking recuperation potential and is a challenge that must be overcome in order to maximize the fuel economy benefits of the hydraulic hybrid system.

In this study, a two state simulation model of a hydro-pneumatic accumulator has been used. The first state variable - gas temperature T is derived from the gas energy equation [5]:

$$\left(\tau + \frac{m_t c_t}{hA_w}\right)\frac{dT}{dt} + T = T_w - \frac{T\tau}{c_v}\left(\frac{\partial p_g}{\partial T}\right)_v \frac{dv}{dt} \tag{1}$$

where

$$\tau = \frac{m_g c_v}{hA_w} \tag{2}$$

is the thermal time constant. Due to high pressures encountered in the accumulator, the ideal gas law cannot be used with sufficient accuracy. Instead, a Benedict-Webb-Rubin equation of state has been used for modelling the state of the nitrogen gas:

$$p_g = \frac{RT}{v} + \frac{1}{v^2}\left(B_0 RT - A_0 - \frac{C_0}{T^2}\right) + \frac{bRT - a}{v^3} + \frac{a\alpha}{v^6} + \frac{c}{v^3 T^2}\left(1 + \frac{\gamma}{v^2}\right)\exp^{-\gamma/v^2} \tag{3}$$

where the corresponding coefficients for nitrogen are taken from [6].

The second state variable, specific volume v, is derived from the continuity equation

$$\frac{dv}{dt} = \frac{Q_a}{m_g} \tag{4}$$

where Q_a is the pump/motor actual flow rate and m_g is the accumulator gas mass.

1.2 Dynamic Programming

Dynamic programming is a technique for solving optimal control problems. It has been used in this study to derive the optimal load distribution between the hydraulic pump/motor and the internal combustion engine, subject to various constraints and conditions, in order to minimize the fuel consumption.

Dynamic programming relies on the principle of optimality, which states that [7] "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

By decomposing a control problem into segments or sub-problems, an optimal decision can be discovered at each stage, starting from the end and moving toward the initial instant. By defining the allowable final system state constraints, a DP algorithm starts with evaluating the optimal decision at the stage preceding the final stage that will result in the system reaching this final state at minimal cost. This is done by discretizing the state space, which results in a time-state space grid with nodes at which the cost is evaluated by sweeping the admissible control values, subject to state constraints. By proceeding backwards, an optimal control decision can be stated for each stage-state combination that will bring the system

from the current stage-state point to the desired final state at minimal cost. By ultimately reaching the initial time stage, the cost-to-go and optimal control matrices are obtained, representing respectively the cost and optimal control decisions for each admissible stage-state combination. Mathematically, this can be stated through a recurrence relation [8]:

$$J^*_{N-K,N}\left(\vec{x}(N-K)\right) = \min_{u(N-K)} \left\{ \begin{array}{l} g_D\left(\vec{x}(N-K),\vec{u}(N-K)\right) + \\ + J^*_{N-(K-1),N}\left(\vec{a}_D\left(\vec{x}(N-K),\vec{u}(N-K)\right)\right) \end{array} \right\} \tag{5}$$

By knowing $J^*_{N-(K-1),N}$, the optimal cost at the K-1 stage, the optimal cost for the stage K, $J^*_{N-K,N}$ can be determined, along with its corresponding control. Only one DP control variable has been used in this study: the load distribution u. In control phases, this control variable represents the engine to hydraulic motor torque ratio. During deceleration phases, it is defined as the ratio of hydraulic pump to friction brakes torque ratio. A generic MATLAB implementation of the DP algorithm has been used in this study [9].

1.3 Machine Learning

The DP-derived optimal control law is not implementable due to its dependency on future system states and conditions. This is why a machine learning algorithm based on ANN has been considered with the goal of trying to derive an implementable control algorithm that will yield near-optimal performance in a multitude of conditions observed in transit bus operation. An ANN was to be configured and trained using the optimal hybrid powertrain system load distribution obtained during the DP optimization runs for different representative driving cycles.

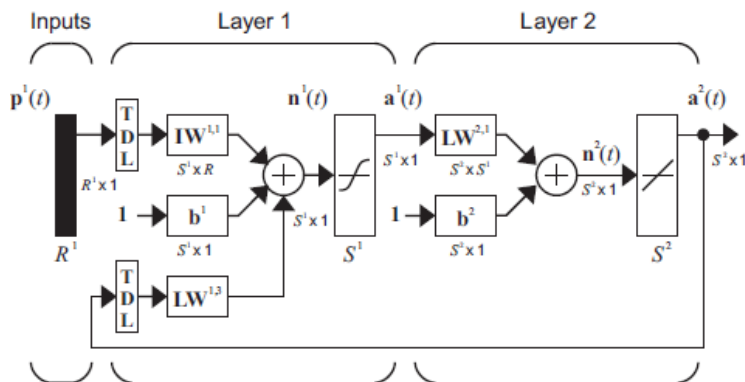A NARX network (Figure 3) has been used in this research.

Figure 3 Nonlinear Autoregressive Network with eXogenous inputs (NARX) [10]

For exogenous inputs, a vector of 4 variables in total have been used – the instantaneous vehicle speed, the driveshaft torque as a representative of the actual powertrain load, the hydraulic machine normalized load and the hydro-pneumatic accumulator gas pressure. The motivation behind this choice lies in the necessity to give the ANN a sufficient amount of information regarding the state of the hybrid powertrain system with the intention to allow for the training process to be successfully accomplished, while providing parameters that would be relatively easy to acquire in a real-world scenario.

Optimal control data from the DP optimization routines that had been obtained for driving cycles in different traffic and vehicle occupancy conditions have been relied upon for the ANN training process. The characteristics of the driving cycles are presented in Tables 1

and 2. This set of data has then further been divided into the training dataset (comprising 70% of the set), the validation and test datasets (each containing 15% of the original data). The training set is used for weights and biases adjustment, while the validation set serves the purpose of stopping the training procedure before overfitting occurs. Specifically, the training process is set to be interrupted when no improvement in the validation MSE occurs for 6 consecutive iterations.

The Levenberg-Marquardt backpropagation technique has been employed for the training process, while the weights and biases of the network have been initialized using the Nguyen-Widrow initialization method.

Table 1 *Characteristics of driving cycles in direction 1 that have been selected for the ANN training process*

	Mean negative accel.	Mean positive accel.	Cycle duration	Mean vehicle mass	Total fuel mass consumed	Vehicle stationary periods fraction	Total stationary vehicle duration	Mean moving velocity
cycle name	aneg	apos	Δt	mveh	mf	xtstat	Δtstat	vpos
[-]	[m/s^2]	[m/s^2]	[s]	[t]	[kg]	[%]	[s]	[m/s]
330001_05_1	-0.518	0.453	2769	18.85	6.54	33.2	919	6.748
370001_01_1	-0.407	0.414	3271	17.97	6.21	35.5	1160	5.924
290001_07_1	-0.521	0.488	3446	18.73	7.93	38.5	1327	5.902

Table 2 *Characteristics of driving cycles in direction 2 that have been selected for the ANN training process*

	Mean negative accel.	Mean positive accel.	Cycle duration	Mean vehicle mass	Total fuel mass consumed	Vehicle stationary periods fraction	Total stationary vehicle duration	Mean moving velocity
cycle name	aneg	apos	Δt	mveh	mf	xtstat	Δtstat	vpos
[-]	[m/s2]	[m/s2]	[s]	[t]	[kg]	[%]	[s]	[m/s]
330001_06_2	-0.487	0.457	2882	18.14	5.63	30.7	885	6.400
270001_11_2	-0.472	0.433	3377	19.40	5.60	38.4	1297	6.079
360001_09_2	-0.487	0.435	3755	17.64	6.41	40.7	1530	5.759

Different configurations of the network have been tested in order to find the one that will yield the closest performance to the reference control law obtained using the Dynamic Programming method.

2. RESULTS

The results of the research are presented in the following subsections. First, an analysis of the trained networks performance is provided, after which the selected ANN is applied on a set of driving cycles not used during the training process in order to analyze its fuel economy improvement results compared to the optimal solution calculated using DP.

2.1 ANN Training Performance

A total of 4 artificial neural network configurations using the NARX architecture have been considered in this investigation. The values for the input and feedback delays have been varied from 10 to 20 and 30 to 60, respectively, while 2 different layer size values of 4 and 8

neurons have been applied. For each set of applied configuration parameters, 10 training sessions have been attempted for the purpose of minimizing the influence of the random weights and biases initialization on the ANN performance. The actual number of valid NNs per configuration depends on the number of early training stop occurrences caused when breaching the upper limit on the *mu* parameter (regulating the training gain). The results shown in Figures 4 to 7 are obtained for a section of the original driving cycles that belongs to the test set data, i.e. not having been used in the training procedure. The load distribution from three different sources is plotted against time: the optimal hybrid powertrain load distribution (obtained through the DP calculation) is shown, along with the output of the trained individual ANN with the lowest testing Mean Square Error (MSE) and the combined average response of all the valid ANNs obtained by repeating the training process.
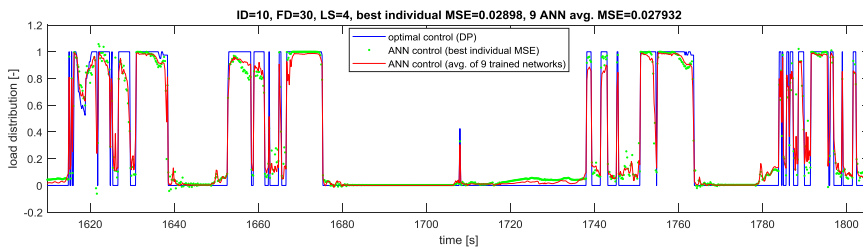

Figure 4 ANN control response for the best individual network, the average of the set of trained networks, compared to the optimal control (Input Delay of 10, Feedback Delay of 30, Layer Size of 4 neurons)
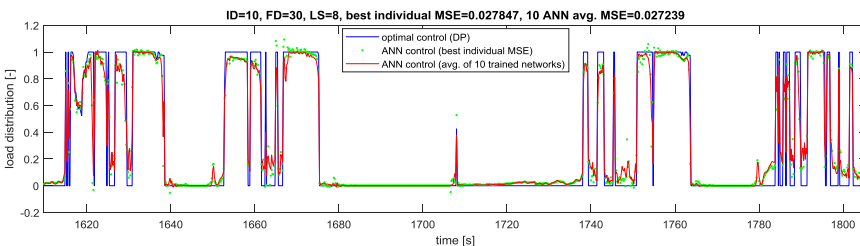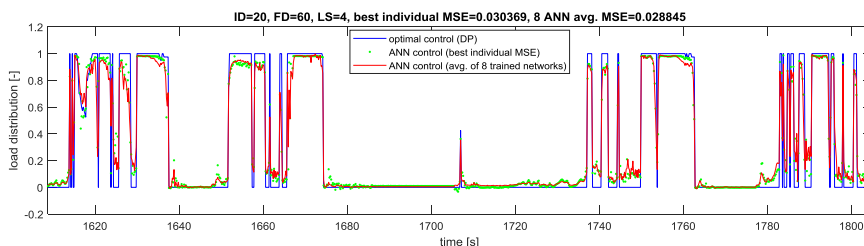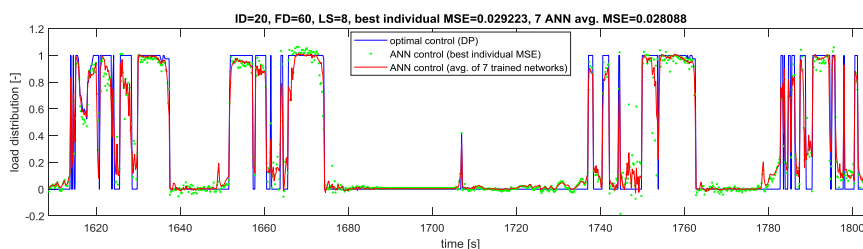

Figure 5 ANN control response for the best individual network, the average of the set of trained networks, compared to the optimal control (Input Delay of 10, Feedback Delay of 30, Layer Size of 8 neurons)

Figure 6 ANN control response for the best individual network, the average of the set of trained networks, compared to the optimal control (Input Delay of 20, Feedback Delay of 60, Layer Size of 4 neurons)

Figure 7 ANN control response for the best individual network, the average of the set of trained networks, compared to the optimal control (Input Delay of 20, Feedback Delay of 60, Layer Size of 8 neurons)

Figures 4 and 5 show data that has been obtained for the lower input and feedback delay sizes (10 and 30, respectively). It can be said that even for the simplest NARX configuration considered in this research, the ANN control yields acceptable results. The individual ANN (from this configuration variation batch) with the best achieved testing performance (i.e. the lowest MSE) tracks the optimal load distribution on the driving cycle section shown quite well, even though the control parameter overshoots the upper bound or does not quite reach the required optimal level repeatedly. The lowest individual MSE obtained by taking into account the testing set data (representing 15% of the whole dataset) is on the order of 0.029. By combining the outputs of 9 trained ANN with the simplest configuration, the resulting MSE drops by approximately 3.6%, with visibly better tracking of the optimal load distribution on the test data section shown in Figure 4. Overshoots are dampened and the oscillations where the optimal control value is constant are moderated.

By increasing the number of neurons in the hidden layer of the NARX from 4 to 8, the individual ANN with the best performance gets its MSE lowered by 3.9%. In this case, overshoots occur at different instants of time but the ANN control yields better tracking at extreme reference values. By combining the outputs of 10 trained networks and calculating the average, the MSE is reduced by only 2.2% compared to the best individual ANN MSE and by 2.5% compared to the corresponding MSE of the NARX with 4 neurons.

By increasing the input and feedback delay sizes by a factor of 2, it can be seen that the best individual and combined ANN performance drops compared to the NARX with input/feedback delays of 10/30 and the layer size of 8 neurons. Specifically, the best individual ANN yields an MSE that is 9% higher than the corresponding MSE of the NARX with the best configuration. The effect of the increase in delay sizes is an increase in best individual ANN MSE of 4.8% for the NARX with 4 neurons. By combining the outputs of the 8 valid ANN of this case and calculating the average control value, a reduction in the MSE of 5% is achieved. By increasing the number of neurons from 4 to 8, a marginal improvement of approximately 3.8% is achieved in best individual MSE and an improvement of 2.6% in combined ANN MSE.

2.2 ANN Control Performance

In this subsection of the article, the results of the application of the ANN control on a hydraulic hybrid transit bus powertrain system simulation are shown and compared against the optimal control results obtained using the dynamic programming algorithm. The individual ANN with the best control variable matching (Input/Feedback Delay of 10/30 and with layer size of 8) has been chosen to perform the analysis.

In total, 6 different driving cycles have been considered for evaluating the performance of the selected ANN control, three for each route direction. All six cycles have been obtained in real traffic and occupancy conditions. For each route direction, one cycle per congestion state has been chosen in order to analyze the impact of the ANN control on the fuel consumption improvement potential for different driving conditions. Indeed, all driving cycles acquired during the experiment have been divided into three categories based on the total duration of the runs. These categories have been further divided into three subcategories according to the values of the mean vehicle positive velocity. For each total run duration category, one cycle with moderate positive vehicle speed value has been chosen. In the end, a cycle representative of low, moderate and high congestion states has been selected for both route directions. It should be noted that the cycles used for the validation of the control methodology have not been part of the ANN training selection.
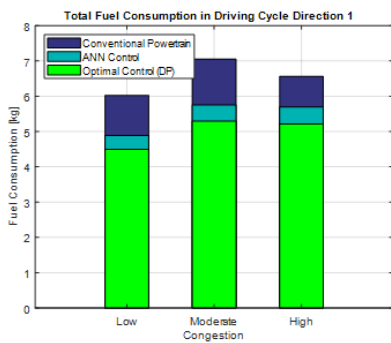

Figure 8 Absolute fuel consumption comparison for the driving cycles in direction 1
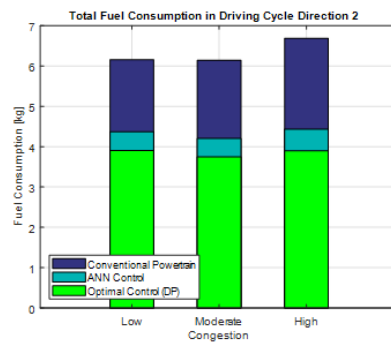

Figure 9 Absolute fuel consumption comparison for the driving cycles in direction 2

Figures 8 and 9 show the absolute fuel consumption for the conventional powertrain, along two other values obtained by simulating the hybrid system solution – one using the optimal control derived by the dynamic programming algorithm and the other realized using the selected ANN control. Calculations have been carried out for the three traffic conditions mentioned earlier and for both driving cycle directions. Maximum absolute fuel consumption savings occur in the route direction 2, primarily due to the difference in the terrain elevations of the bus terminal stations. Indeed, the starting "Crveni Krst" terminal station is located approximately 60 m above the destination of the driving cycle direction 2 – the "Zemun Bačka" terminal station, allowing greater potential energy of the vehicle to be harnessed by the use of the hybrid powertrain system. Maximum achievable fuel consumption reductions range from 1.3 to 1.7 kg in the route direction 1, while savings on the order of 2.2 to 2.8 kg can be achieved in the direction 2 in the optimal case. The savings achieved by using an implementable control algorithm range from 0.86 to 1.3 kg in direction 1, and 1.8 to 2.2 kg in route direction 2. The fuel consumption improvement relative to the conventional powertrain system is shown in Figures 10 and 11 for all six driving cycles considered in this ANN control validation investigation. The least relative amount of fuel saved is achieved for the most congested cycle in direction 1, where only 20.5% can be optimally achieved. Using the implementable ANN control algorithm, 13.2% of fuel can be saved. Ideally, approximately 25% of the fuel used for powering the conventional powertrain system may be saved in low and moderate congestion states in direction 1. The ANN control can cut back approximately 18.5% of the fuel consumed in a non-hybrid

transit bus. The values achieved in direction 2 show a different trend. Namely, potential fuel savings rise with increasingly congested traffic conditions, with over 40% of fuel savings ideally achieved for the most congested driving cycle. The least amount of fuel consumption improvement in driving cycle direction 2 reaches a figure of over 36%, which is significantly higher than the most favorable case in direction 1. The range of values representing fuel savings achievable using the selected ANN control is from 29.1% for the least, to 33.7% for the most congested state.
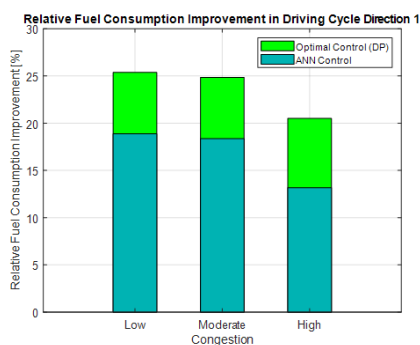
Figure 10 Relative fuel consumption improvement compared to the conventional powertrain system for the driving cycles in direction 1
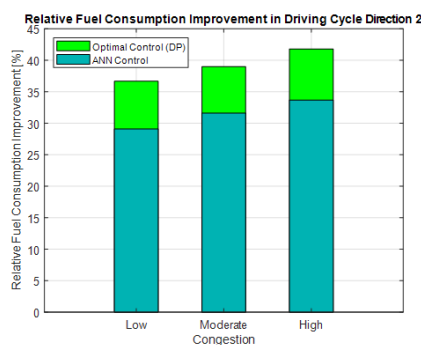
Figure 11 Relative fuel consumption improvement compared to the conventional powertrain system for the driving cycles in direction 2

By analyzing the optimal fuel consumption savings that could ideally be achieved and the values obtained using the ANN control, it can be concluded that the proposed implementable control algorithm yields a good performance. Over 64% of the potential fuel savings can be achieved by using the ANN control in the most congested driving conditions in direction 1. In the most favorable case (in the most congested conditions in direction 2), over 80% of the maximally achievable fuel consumption reduction can be accomplished by using the suboptimal, implementable control algorithm.

3. CONCLUSIONS

An implementable, artificial neural network based control algorithm has been devised to control the load distribution in a parallel, hydraulic hybrid powertrain system for a transit bus. A physical experiment involving the use of a transit bus circulating in real traffic and occupancy conditions as part of the Belgrade's public transportation service has been conducted in order to acquire the data needed to calibrate a simulation model of the powertrain system. This endeavor has also allowed to acquire the driving cycles in differing traffic congestion states, a prerequisite for training and validating the proposed ANN control.

By using a NARX ANN, up to 80% of the ultimate fuel consumption improvement potential obtained using a non-implementable optimization algorithm can be achieved. Further research efforts shall be invested in order to analyze the conditions required for closing the gap to the optimal solution.

ACKNOWLEDGMENT

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