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OPTIMIZACIJA TANKOZIDNE KONZOLE PRI OGRANIČENOM UVIJANJU

OPTIMIZATION OF A THIN-WALL CANTILEVER BEAM AT CONSTRAINED TORSION

Originalni naučni rad / Original scientific paper
UDK /UDC: 624.014.072.21.04
Rad primljen / Paper received: 15.5.2006.

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Ključne reči
- optimizacija
- konzola
- tankozidni štap
- U profil
- optimalne dimenzije
- ograničena deformacija

Izvod

Razmatran je pristup optimizaciji preseka U profila tankozidne konzole izložene ograničenom uvijanju. Za zadata opterećenja, materijal i geometrijske karakteristike, problem je redukovan na određivanje minimalne mase, tj. određivanje minimalne površine poprečnog preseka tankozidnog U profila konzole. Zbog toga je za funkciju cilja odabrana površina poprečnog preseka nosača. Izabrani kriterijum je ograničena deformacija, odnosno dozvoljeni ugao uvijanja i ugao uvijanja za jedinicu dužine. Primenom metode Lagranžovog množitelja izvedena je jednačina četvrtog stepena čija rešenja predstavljaju optimalne odnose dimenzija poprečnog preseka izabranog profila. Dobijeni rezultati su iskorišćeni pri numeričkom proračunu primenom metode konačnih elemenata.

UVOD

Uticaj ograničenog uvijanja se može naći u mnogim konstrukcijama. Njene posledice su naročito izražene u slučaju tankozidnih profila, u zavisnosti od vrste opterećenja i od načina njihovog dejstva. Ponašanje tankozidnih greda i štapova je specifično i zbog toga je njihova optimizacija poseban problem. U toku procesa projektovanja konstrukcije, osim zahtevanih dimenzija koje je se moraju obezbediti za pojedine delove konstrukcije da bi ona nosila zadato opterećenje, veoma je važno odrediti i optimalne veličine dimenzija poprečnog preseka. Jedan od često korišćenih oblika poprečnih preseka, naročito u čeličnim konstrukcijama, je poprečni presek oblika U profila.

Keywords
- optimization
- cantilever beam
- thin-wall bar
- channel section
- optimal dimensions
- strain constraint

Abstract

An approach to the optimization of a channel section in a thin-wall cantilever beam subjected to constrained torsion is considered. For given loads, material and geometry the problem is reduced to determination of minimal mass, i.e. minimal cross-section area of the thin-wall channel section of cantilever beam. That is why the cross sectional area is chosen to be the objective function. The selected criterion is strain constraint, i.e. the allowable angle of twist and the allowable angle of twist per unit length. By applying the Lagrange multiplier method, an equation of the fourth order is derived with solutions representing optimal ratios of cross section dimensions of chosen profile. The obtained results are used for numerical calculations applying the finite element method.

INTRODUCTION

The effect of constrained torsion can be found in many structures. Its consequences are particularly evident in the case of thin-wall sections, depending on loading cases and on the way they are introduced. The behaviour of thin-wall beams and bars is specific, such that their optimization is a particular problem. During the process of structural design, in addition to the required dimensions which must be met for particular structural parts in order to carry the applied loads, it is also very important to find optimal dimensions of cross sections. An example of regularly used types of cross sections, particularly in steel structures, is the channel section (U profile).

DEFINICIJA PROBLEMA

Razmatrana konzola (sl. 1a) izložena je ograničenom uvijanju jer je jedan njen kraj uklešten, a drugi, slobodan kraj, opterećen koncentrisanim momentom uvijanja M^*. Pretpostavlja se da poprečni presek (sl. 1b) ima pojaseve jednakih širina $b_1 = b_3$ i debljina $t_1 = t_3$. Cilj ovog rada je određivanje minimalne mase nosača ili, drugim rečima, određivanje minimalne površine poprečnog preseka nosača,

$$A = A_{\min} \quad (1)$$

za zadato opterećenje, materijal i geometrijske karakteristike.

U razmatranom problemu se površina poprečnog preseka smatra funkcijom cilja. Sa sl. 1b je jasno da je površina posmatranog tankozidnog profila

$$A = \sum b_i t_i, \quad i = 1, 2, 3 \quad (2)$$

gde je b_i širina, a t_i debljina delova posmatranog poprečnog preseka.

DEFINITION OF THE PROBLEM

The considered cantilever beam (Fig. 1a) is subjected to constrained torsion since its one end is fixed and the other, free end, is loaded by concentrated torque M^*. The cross section (Fig. 1b) is supposed to have flanges of equal widths and thicknesses $b_1 = b_3$, $t_1 = t_3$. The aim of the paper is to determine the minimal mass of the beam or, in other words, to find the minimal cross-section area,

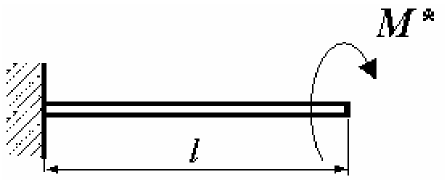
$$A = A_{\min} \quad (1)$$

for applied load, material and geometrical properties.

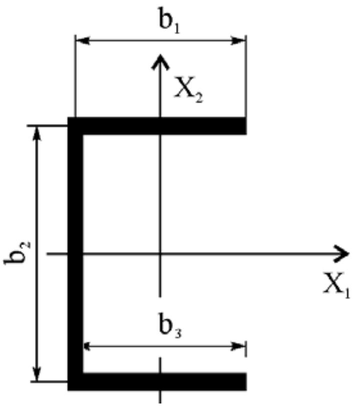
In the considered problem the cross section area will be treated as an objective function. It is obvious from Fig. 1b that the area of the considered thin-wall channel is

$$A = \sum b_i t_i, \quad i = 1, 2, 3 \quad (2)$$

where b_i are the widths and t_i the thicknesses of parts of the considered cross section.



Slika 1a. Prosta greda izložena ograničenom uvijanju
Figure 1a. Cantilever beam subjected to constrained torsion.



Slika 1b. Poprečni presek U profila
Figure 1b. Channel cross section.

OGRANIČENJA

U izvedenom proračunu je uzeta u obzir samo ograničena deformacija. Odnos

$$z = b_2/b_1 \quad (3)$$

predstavlja optimalni odnos dimenzija posmatranog poprečnog preseka.

Ograničenja deformacija

Razmatrana ograničena deformacija je iskazana kao dozvoljeni ugao uvijanja i dozvoljeni ugao uvijanja za jedinicu dužine (relativni), koji su označeni sa θ_0 i θ_0'.

Savojno-uvojna karakteristika tankozidnog poprečnog preseka /2, 4/ je data izrazom

$$k = \sqrt{GI_t/EI_\omega} \quad (4)$$

gde su:
– I_t - torziona konstanta,
– I_ω - sektorski moment inercije,
– E - modul elastičnosti, i
– G - modul klizanja.

CONSTRAINTS

Only strain constraints are taken into account in the performed calculations. The ratio

$$z = b_2/b_1 \quad (3)$$

represents the optimal ratio of dimensions for the considered cross section.

Strain constraints

The considered strain constraint is expressed as the allowable angle of twist and allowable angle of twist per unit length, denoted by θ_0 and θ_0', respectively.

The flexural-torsional cross section characteristic /2, 4/ is given by the expression

$$k = \sqrt{GI_t/EI_\omega} \quad (4)$$

where:
– I_t - torsion constant,
– I_ω - sectorial moment of inertia,
– E - modulus of elasticity, and
– G - shear modulus.

Dozvoljeni ugao uvijanja

U slučaju da se dozvoljeni ugao uvijanja θ_O ($\leq \theta_{max}$) smatra ograničenjem, funkcija ograničenja $\theta(l)$ se može napisati u obliku /2, 4/

$$\theta_{max} = \theta(l) = \frac{M^* l}{GI_t}\left(1 - \frac{\tanh kl}{kl}\right) \leq \theta_0 \tag{5}$$

ili u obliku

$$\varphi_1 = kl - \tanh kl - \frac{GI_t kl}{M^* l}\theta_0 \leq 0 \tag{6}$$

gde su:
l – raspon (sl. 1a),
M^* – moment uvijanja,
k – savojno-uvojna karakteristika.

Dozvoljeni relativni ugao uvijanja

U slučaju kada se ograničenjem smatra dozvoljeni relativni ugao uvijanja θ_o', funkcija ograničenja se može napisati u obliku /2, 4/

$$\theta'_{max} = \theta'(l) = \frac{M^*}{GI_t}\left(1 - \frac{1}{\cosh kl}\right) \leq \theta'_0 \tag{7}$$

ili

$$\varphi_2 = \cosh kl\left(1 - \theta'_0 \frac{GI_t}{M^*}\right) - 1 \leq 0 \tag{8}$$

METODA LAGRANŽOVOG MNOŽITELJA

Metoda Lagranžovog množitelja /1, 3, 5–9/ predstavlja klasičan pristup optimizaciji ograničenja. Lagranžov množitelj, označen sa λ, predstavlja meru promene funkcije cilja s obzirom na ograničenje. Primenom ove metode na vektor koji zavisi od dva parametra b_i, ($i = 1, 2$), izveden je sistem jednačina u obliku $\varphi(b_i) = 0$ ($i = 1, 2$),

$$\frac{\partial}{\partial b_i}(A + \lambda\varphi) = 0, \quad i = 1,2 \tag{9}$$

Nakon eliminacije množitelja λ, dobija se

$$\frac{\partial A}{\partial b_i}\frac{\partial \varphi}{\partial b_j} = \frac{\partial A}{\partial b_j}\frac{\partial \varphi}{\partial b_i}, \quad (i \neq j,\, i = 1,\, j = 2). \tag{10}$$

ANALITIČKI PRISTUP

Uvojna konstanta i sektorski moment inercije za razmatrani poprečni presek dati su izrazima /2, 4/

$$I_t = \frac{1}{3} b_1 t_1^3 \left(2 + \psi^3 z\right) \tag{11}$$

$$I_\omega = \frac{1}{12} b_1^3 b_2^2 t_1 \frac{3 + 2\psi z}{6 + \psi z} \tag{12}$$

gde je

$$\psi = t_2/t_1 \tag{13}$$

odnos debljina rebra i pojasa poprečnog preseka.

Za razmatrani slučaj, gde je presek U profila predmet optimizacije, jedn. [10] se redukuje na jedn. [14]. Dobijena je jednačina četvrtog stepena i njena rešenja predstavljaju optimalne odnose dimenzija poprečnog preseka:

Allowable angle of twist

In the case when the allowable angle of twist θ_O ($\leq \theta_{max}$) is taken as the constraint, the constraint function $\theta(l)$ can be written in the form /2, 4/

$$\theta_{max} = \theta(l) = \frac{M^* l}{GI_t}\left(1 - \frac{\tanh kl}{kl}\right) \leq \theta_0 \tag{5}$$

or in the form

$$\varphi_1 = kl - \tanh kl - \frac{GI_t kl}{M^* l}\theta_0 \leq 0 \tag{6}$$

where:
l – beam length,
M^* – torque,
k – flexural-torsional characteristic.

Allowable angle of twist per unit length

If the allowable angle of twist per unit length θ_o' is taken as the constraint, the constraint function can be written in the form /2, 4/

$$\theta'_{max} = \theta'(l) = \frac{M^*}{GI_t}\left(1 - \frac{1}{\cosh kl}\right) \leq \theta'_0 \tag{7}$$

or

$$\varphi_2 = \cosh kl\left(1 - \theta'_0 \frac{GI_t}{M^*}\right) - 1 \leq 0 \tag{8}$$

LAGRANGE MULTIPLIER METHOD

Lagrange Multiplier Method /1, 3, 5–9/ is the classical approach to constraint optimization. The Lagrange multiplier designated as λ, measures the change of objective function with respect to constraint. By applying this method to the vector depending on two parameters b_i, ($i = 1, 2$), the system of equations is derived in the form $\varphi(b_i) = 0$ ($i = 1, 2$),

$$\frac{\partial}{\partial b_i}(A + \lambda\varphi) = 0, \quad i = 1,2 \tag{9}$$

By eliminating the multiplier λ, the form is obtained

$$\frac{\partial A}{\partial b_i}\frac{\partial \varphi}{\partial b_j} = \frac{\partial A}{\partial b_j}\frac{\partial \varphi}{\partial b_i}, \quad (i \neq j,\, i = 1,\, j = 2). \tag{10}$$

ANALYTICAL APPROACH

The torsion constant and sectorial moment of inertia for the considered channel section are given by equations /2, 4/

$$I_t = \frac{1}{3} b_1 t_1^3 \left(2 + \psi^3 z\right) \tag{11}$$

$$I_\omega = \frac{1}{12} b_1^3 b_2^2 t_1 \frac{3 + 2\psi z}{6 + \psi z} \tag{12}$$

where

$$\psi = t_2/t_1 \tag{13}$$

is the ratio of web and flange thicknesses in the cross section.

In the considered case, when the channel section is the object of optimization, Eq. [10] is reduced to Eq. [14]. The equation of the fourth order is formed with solutions that represent optimal ratios of cross sectional dimensions:

$$\sum_{i=0}^{4} c_i z^i = 0 \qquad (14)$$

gde su koeficijenti c_i dati izrazima [15] ako je ograničenje dozvoljeni ugao uvijanja:

$$c_0 = 72,$$

$$c_1 = 6\psi\left[7 + 3\psi^2 - 6\frac{\psi^2 - 1}{1 - \dfrac{kl\tanh^2 kl}{kl - \tanh kl}}\right],$$

$$c_2 = -\psi^2\left[13 + 3\psi^2 + 30\frac{\psi^2 - 1}{1 - \dfrac{kl\tanh^2 kl}{kl - \tanh kl}}\right], \qquad (15)$$

$$c_3 = -4\psi^3\left[1 + 4\psi^2 + \frac{\psi^2 - 1}{1 - \dfrac{kl\tanh^2 kl}{kl - \tanh kl}}\right],$$

$$c_4 = -3\psi^6,$$

i, ako je ograničenje relativni dozvoljeni ugao uvijanja, izrazima [16]:

$$c_0 = -72,$$

$$c_1 = -6\psi\left[7 + 3\psi^2 - 6\frac{\psi^2 - 1}{\dfrac{kl\tanh kl}{1 - \cosh kl}}\right],$$

$$c_2 = \psi^2\left[13 + 3\psi^2 + 30\frac{\psi^2 - 1}{\dfrac{kl\tanh kl}{1 - \cosh kl}}\right], \qquad (16)$$

$$c_3 = 4\psi^3\left[1 + 4\psi^2 + \frac{\psi^2 - 1}{\dfrac{kl\tanh kl}{1 - \cosh kl}}\right],$$

$$c_4 = 3\psi^6.$$

Rezultati dobijeni analitičkim pristupom

U daljem tekstu su uvedene sledeće oznake

$$D = \frac{\psi^2 - 1}{1 - \dfrac{kl\tanh^2 kl}{kl - \tanh kl}}, \quad D_1 = \frac{\psi^2 - 1}{\dfrac{kl\tanh kl}{1 - \cosh kl}} \qquad (17)$$

Proračun je izveden za konzolnu gredu izabranog preseka, dužine $0,25 \le l \le 200$ cm. Vrednosti kl su izračunate korišćenjem podataka za standardne profile, za odnos [13] $\psi = 0,5; 0,75; 1$. Rezultati za odnos [3], $z = b_2/b_1$, dobijeni korišćenjem jedn. [10] su dati u tabelama 1 i 2. Rezultati su grafički prikazani na sl. 2a i 2b.

$$\sum_{i=0}^{4} c_i z^i = 0, \qquad (14)$$

where coefficients c_i are given in the form [15] for the constraint in the form of allowable angle of twist:

$$c_0 = 72,$$

$$c_1 = 6\psi\left[7 + 3\psi^2 - 6\frac{\psi^2 - 1}{1 - \dfrac{kl\tanh^2 kl}{kl - \tanh kl}}\right],$$

$$c_2 = -\psi^2\left[13 + 3\psi^2 + 30\frac{\psi^2 - 1}{1 - \dfrac{kl\tanh^2 kl}{kl - \tanh kl}}\right], \qquad (15)$$

$$c_3 = -4\psi^3\left[1 + 4\psi^2 + \frac{\psi^2 - 1}{1 - \dfrac{kl\tanh^2 kl}{kl - \tanh kl}}\right],$$

$$c_4 = -3\psi^6$$

and if the allowable angle of twist per unit length is the constraint, in the form [16]:

$$c_0 = -72,$$

$$c_1 = -6\psi\left[7 + 3\psi^2 - 6\frac{\psi^2 - 1}{\dfrac{kl\tanh kl}{1 - \cosh kl}}\right],$$

$$c_2 = \psi^2\left[13 + 3\psi^2 + 30\frac{\psi^2 - 1}{\dfrac{kl\tanh kl}{1 - \cosh kl}}\right], \qquad (16)$$

$$c_3 = 4\psi^3\left[1 + 4\psi^2 + \frac{\psi^2 - 1}{\dfrac{kl\tanh kl}{1 - \cosh kl}}\right],$$

$$c_4 = 3\psi^6.$$

Results obtained by analytical approach

In the following text new expressions are introduced

$$D = \frac{\psi^2 - 1}{1 - \dfrac{kl\tanh^2 kl}{kl - \tanh kl}}, \quad D_1 = \frac{\psi^2 - 1}{\dfrac{kl\tanh kl}{1 - \cosh kl}} \qquad (17)$$

The calculation is made for cantilever beam of chosen section of length $0.25 \le l \le 200$ cm. Values kl are calculated using data for standard sections and the ratio [13] is taken as $\psi = 0.5; 0.75; 1$. Results for ratios [3], $z = b_2/b_1$,obtained from Eqs. [10] are given in Tables 1 and 2. The results are presented graphically in Figs. 2a and 2b.

Tabela 1. Ograničenje – dozvoljeni ugao uvijanja θ_o

ψ	1	0,75				0,5			
D	0	0,22	0,35	0,58	1,33	0,38	0,6	1	2,27
z	1,72	2,29	2,14	1,90	1,35	3,44	2,99	2,39	1,41

Tabela 2. Ograničenje – dozvoljeni relativni ugao uvijanja θ_o'

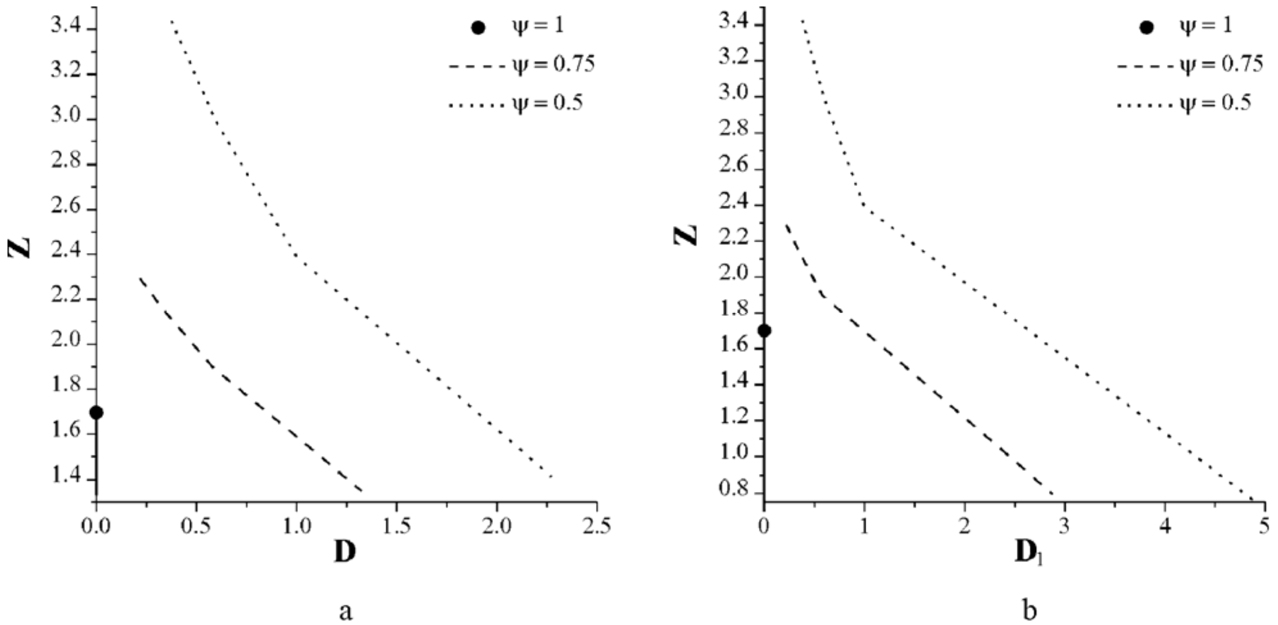
ψ	1	0,75				0,5			
D_l	0	0,22	0,35	0,58	2,88	0,38	0,6	1	4,93
z	1,72	2,28	2,14	1,90	0,79	3,43	2,99	2,39	0,74

Table 1. Constraint – allowable angle of twist θ_o.

ψ	1	0.75				0.5			
D	0	0.22	0.35	0.58	1.33	0.38	0.6	1	2.27
z	1.72	2.29	2.14	1.90	1.35	3.44	2.99	2.39	1.41

Table 2. Constraint – allowable angle of twist for unit length θ_o'.

ψ	1	0.75				0.5			
D_l	0	0.22	0.35	0.58	2.88	0.38	0.6	1	4.93
z	1.72	2.28	2.14	1.90	0.79	3.43	2.99	2.39	0.74



Slika 2. Optimalni odnos z za: a) θ_o; b) θ_o'
Figure 2. Optimal ratios z for: a) θ_o; b) θ_o'.

Na osnovu tab. 2 i 3 i sl. 2 može se zaključiti da ako vrednosti ψ opadaju (odnosno, vrednosti D i D_1 rastu), vrednosti z opadaju. Na osnovu izvedenog proračuna definisana su područja optimalnih vrednosti dimenzija razmatranog profila konzole:

- za ograničenje – dozvoljeni ugao uvijanja θ_o:
  - $\psi = 1 \Rightarrow D = 0 \Rightarrow z =$ const. = 1,72
  - $\psi = 0{,}75 \Rightarrow 0{,}22 \le D \le 1{,}33 \Rightarrow 2{,}29 \ge z \ge 1{,}35$
  - $\psi = 0{,}5 \Rightarrow 0{,}38 \le D \le 2{,}27 \Rightarrow 3{,}44 \ge z \ge 1{,}41$.
- za ograničenje –dozvoljeni relativni ugao uvijanja θ_o':
  - $\psi = 1 \Rightarrow D_1 = 0 \Rightarrow z =$ const. = 1,72
  - $\psi = 0{,}75 \Rightarrow 0{,}22 \le D_1 \le 2{,}88 \Rightarrow 2{,}28 \ge z \ge 0{,}79$
  - $\psi = 0{,}5 \Rightarrow 0{,}38 \le D_1 \le 4{,}93 \Rightarrow 3{,}43 \ge z \ge 0{,}74$.

PRIMENA METODE KONAČNIH ELEMENATA

Dobijene optimalne veličine z primenjene su za numerički proračun metodom konačnih elemenata (MKE) korišćenjem programa KOMIPS, /10, 11/.

Metoda konačnih elemenata je primenjena na konzolu poprečnog preseka U profila, dužine $l = 100$ cm, uklještenu na jednom, a na drugom, slobodnom kraju, opterećenu koncentrisanim momentom uvijanja $M = 10$ kNcm, (sl. 3).

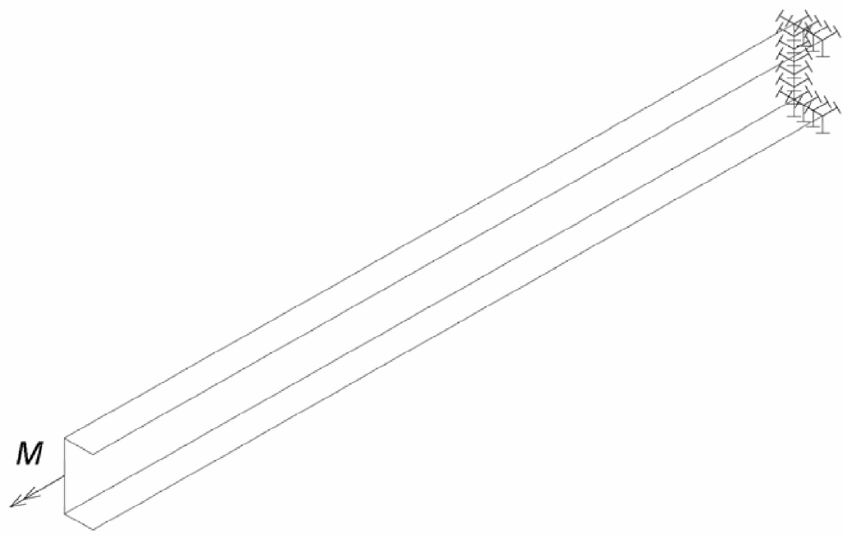
It is possible to conclude based on Tables 2 and 3, and Fig. 2, that when values of ψ decrease (i.e. values of D i D_1 increase) the value of z decreases. Based on the performed calculation, the regions of optimal dimensional values of the considered cantilever beam profile are defined:

- for constraint – allowable angle of twist θ_o:
  - $\psi = 1 \Rightarrow D = 0 \Rightarrow z =$ const. = 1.72
  - $\psi = 0.75 \Rightarrow 0.22 \le D \le 1.33 \Rightarrow 2.29 \ge z \ge 1.35$
  - $\psi = 0.5 \Rightarrow 0.38 \le D \le 2.27 \Rightarrow 3.44 \ge z \ge 1.41$.
- for constraint – allowable angle of twist for unit length θ_o':
  - $\psi = 1 \Rightarrow D_1 = 0 \Rightarrow z =$ const. = 1.72
  - $\psi = 0.75 \Rightarrow 0.22 \le D_1 \le 2.88 \Rightarrow 2.28 \ge z \ge 0.79$
  - $\psi = 0.5 \Rightarrow 0.38 \le D_1 \le 4.93 \Rightarrow 3.43 \ge z \ge 0.74$.

APPLICATION OF FINITE ELEMENT METHOD

The obtained optimal values for z are applied in the numerical calculation performed by finite element method (FEM) using the software KOMIPS, /10, 11/.

The finite element method is applied on the channel cross section of cantilever beam of length $l = 100$ cm, fixed on one end, and on the other free end loaded by concentrated torsion moment $M = 10$ kNcm, (Fig. 3).



Slika 3. Konzola poprečnog preseka U profila – srednja površ, opterećenje, oslonci
Figure 3. Cantilever beam of channel cross-section – mid surface, load, supports.

Rezultati dobijeni primenom MKE

Kao primer numeričkog proračuna razmatran je poprečni presek standardnog profila U 10 (JUS C.B3.131). Problem je razmatran na četiri načina:

a) Početne dimenzije zadatog poprečnog preseka profila U 10 su: $b_{1početno}$ = 4,7 cm, $b_{2početno}$ = 9,15 cm, t_1 = 0,85 cm, t_2 = 0,6 cm. Ove vrednosti daju početni odnos $z_{početno}$ = 1,95. Za početne vrednosti t_1 i t_2, na osnovu izraza izvedenih u ovom radu, dobijen je optimalan odnos $z_{optimalno}$ = 2,34.

b) Optimalne dimenzije poprečnog preseka $b_{1optimalno}$ i $b_{2optimalno}$ su dobijene izjednačavanjem početnih i optimalnih površina ($A_{početno} = A_{optimalno}$) i korišćenjem izračunatih optimalnih vrednosti $z_{optimalno}$ = 2,34 (optimalni model br. 1).

c) Optimalne dimenzije poprečnog preseka $b_{1optimalno}$ i $b_{2optimalno}$ dobijene su uz pretpostavku da je $b_{1optimalno} = b_{1početno}$ i korišćenjem izračunatog optimalnog odnosa $z_{optimalno}$ = 2,34 (optimalni model br. 2).

d) Optimalne dimenzije poprečnog preseka $b_{1optimalno}$ i $b_{2optimalno}$ dobijene su uz pretpostavku da je $b_{2optimalno} = b_{2početno}$ i korišćenjem izračunatog optimalnog odnosa $z_{optimalno}$ = 2,34 (što predstavlja optimalni model br. 3).

Za svaki model izračunate su veličine površina poprečnog preseka, a rezultati su dati u tab. 3.

Results obtained by FEM

As an example of numerical calculation, a standard channel cross-section U 10 (JUS C.B3.131) is considered. The problem is analyzed in four different ways:

a) Initial dimensions of channel cross-section U 10 are: $b_{1initial}$ = 4.7 cm, $b_{2initial}$ = 9.15 cm, t_1 = 0.85 cm, t_2 = 0.6 cm. These values produce initial ratio of $z_{initial}$ = 1.95. For initial values t_1 and t_2 the optimal relation $z_{optimal}$ = 2.34 is obtained from expressions derived in this paper.

b) Optimal dimensions of the cross section $b_{1optimal}$ and $b_{2optimal}$ are obtained by equalizing initial and optimal areas ($A_{initial} = A_{optimal}$) and by using calculated optimal values $z_{optimal}$ = 2.34 (optimal model No. 1).

c) Optimal dimensions of cross section $b_{1optimal}$ and $b_{2optimal}$ are obtained assuming $b_{1optimal} = b_{1initial}$ and by using calculated optimal ratio $z_{optimal}$ = 2.34 (optimal model No. 2).

d) Optimal dimensions of cross section $b_{1optimal}$ and $b_{2optimal}$ are obtained assuming $b_{2optimal} = b_{2initial}$ and by using the calculated optimal ratio $z_{optimal}$ = 2.34 (optimal model No. 3).

For each model the cross section area is calculated and results are given in Table 3.

Tabela 3. Površine poprečnog preseka
Table 3. Cross section areas.

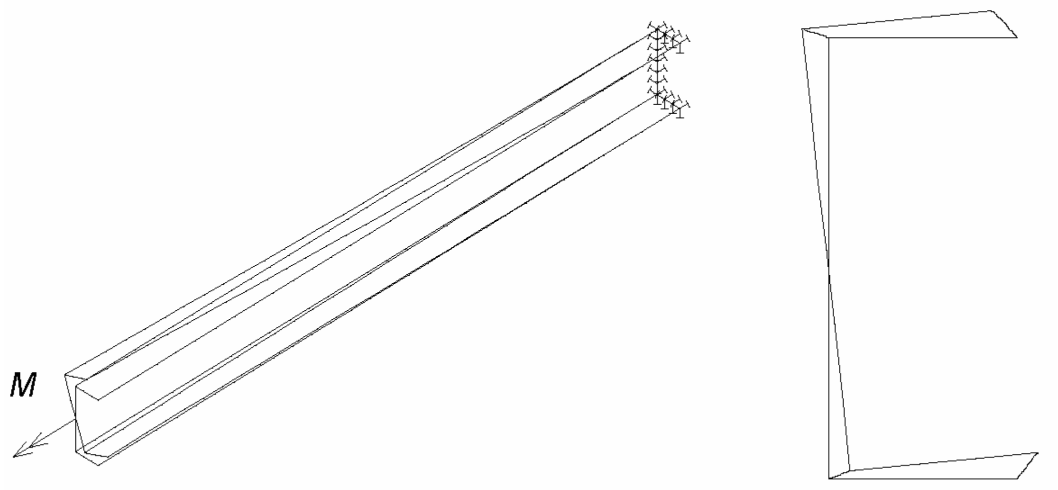
Model	Model	z	A (cm^2)
Početni	Initial	1.95	13.49
Optimalni br. 1	Optimal No. 1	2.34	13.49
Optimalni br. 2	Optimal No. 2	2.34	14.59
Optimalni br. 3	Optimal No. 3	2.34	12.14

Primenjujući MKE, izračunata su pomeranja na slobodnom kraju konzole razmatranog poprečnog preseka, dužine 70 cm (sl. 4), i sračunat je relativni ugao uvijanja θ_o'. Dobijeni rezultati dati su u tab. 4.

Radi provere dobijenih rezultata izračunata je vrednost relativnog ugla uvijanja i primenom analitičkog pristupa.

By applying FEM, displacements of the free-end cantilever beam of considered cross-section, length l = 70 cm (Fig. 4) and angle of twist per unit length θ_o' are calculated. The obtained results are given in Table 4.

The value of allowable angle of twist per unit length is also calculated by analytical approach.



a) b)

Slika. 4. Pomeranja slobodnog kraja konzole (f_{max} = 0,334 cm): a) pogled u izometriji; b) xy–ravan
Figure 4. Displacements of cantilever beam free-end (f_{max} = 0.334 cm): a) isometric view; b) xy–plane.

Tabela 4. Relativni ugao uvijanja θ_o'
Table 4. Angle of twist per unit length θ_o'.

Model	Model	θ_o' (°/m)	θ_o' (°/m) -u odnosu na masu (with respect to the mass)
Početni	Initial	2.68	2.68
Optimalni br. 1	Optimal No. 1	2.25	2.25
Optimalni br. 2	Optimal No. 2	1.79	1.93
Optimalni br. 3	Optimal No. 3	2.68	2.4

Rezultati dobijeni primenom programa KOMIPS odgovaraju rezultatima dobijenim analitičkim putem za početni model za dužinu 70 cm: $\theta'_{o\ analitički}$ = 2,64 °/m, a $\theta'_{o\ KOMIPS}$ = 2,68 °/m, tab. 4.

ZAKLJUČAK

U ovom radu je dat pristup optimizaciji tankozidnih konzola poprečnog preseka U profila primenom metode Lagranžovog množitelja. Birajući površinu poprečnog preseka za funkciju cilja, a ograničenje deformacije za funkciju ograničenja, određeni su optimalni odnosi pojedinih delova poprečnog preseka (rebra i pojasa). Proračun je prvo izveden analitičkim putem, a zatim su dobijeni rezultati iskorišćeni u proračunu primenom MKE.

Na osnovu dobijenih rezultata [15, 16] može se videti da postoje izvesne razlike u koeficijentima c_i izračunatim korišćenjem kriterijuma θ_o ili θ_o', i uočeno je minimalno neslaganje rezultata dobijenih za veličinu z. Optimalne veličine z dobijene korišćenjem kriterijuma θ_o su nešto veće nego vrednosti dobijene preko kriterijuma θ_o' (tab. 1 i 2).

Rezultati dobijeni primenom MKE pokazuju da početni i optimalni model br. 3 (koji ima minimalnu masu) – tab. 4, imaju isti relativni ugao uvijanja θ_o'. Početni i optimalni model br. 1 imaju iste mase, ali je optimalni model br. 1 bolji, jer je vrednost njegovog relativnog ugla uvijanja θ_o' manja. Optimalni model br. 2 ima najmanju vrednost θ_o', ali je to model sa najvećom masom. Ovo je najbolji model s obzirom na ograničenje deformacije, ali je isto tako i sa najvećom masom. Optimalni model br. 3 ima najmanju masu i isti ugao θ_o' kao i početni model. Prevedeno na indeks mase, svi optimalni modeli su bolji od početnih.

Results obtained by applying the KOMIPS programme correspond to analytically obtained values for the initial model of 70 cm length: $\theta'_{o\ analytical}$ = 2.64 °/m, $\theta'_{o\ KOMIPS}$ = 2.68 °/m, Table 4.

CONCLUSION

This paper presents an approach to the optimization of thin-walled cantilever beams of open channel cross-section using the Lagrange multiplier method. Choosing the cross section area as the objective function and strain constraint as the constraint function, optimal ratios of individual cross section parts (webs and flanges) are determined. Analytical calculation is made first and the obtained results are used for calculations applying FEM.

Based on the obtained results [15, 16] it can be seen that some differences exist between coefficients c_i, calculated using criteria θ_o or θ_o', and minimum disagreement is observed between obtained values for z. Optimal values z obtained by using criterion θ_o are slightly higher than values obtained by the θ_o' criterion (Tables 1 and 2).

Results obtained by FEM show that initial and optimal model no. 3 (with minimal mass) – Table 4, have the same angle of twist per unit length θ_o'. Initial and optimal model no. 1 have the same mass, but optimal model no. 1 is better since the angles of twist per unit length θ_o' have lower values. Optimal model No. 2 has the lowest value of θ_o', but this is the optimal model with the highest mass. This model is the best regarding strain constraint, but is also with the highest mass. Optimal model No. 3 has minimal mass and the same θ_o' as the initial model. In terms of mass index, all optimal models are better than initial ones.

Na osnovu predloženog postupka optimizacije moguće je izračunati optimalne odnose pojedinih delova razmatranog tankozidnog profil na jednostavan način.

On the basis of the proposed optimization procedure it is possible to calculate optimal ratios between parts of the considered thin-walled sections in a simple way.

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