

## NONLINEAR FORCED VIBRATION OF A FUNCTIONALLY GRADED NONLOCAL NANOBEM EMBEDDED IN A FRACTIONAL VISCOELASTIC MEDIUM

Danilo Z. Karličić<sup>1</sup>, Milan S. Cajić<sup>2</sup>, Predrag Kozić<sup>1</sup>, Mihailo Lazarević<sup>3</sup>

<sup>1</sup>Faculty of Mechanical Engineering,  
University of Niš, Aleksandra Medvedeva 14, Niš, Serbia  
e-mail: [danilo.karlicic@masfak.ni.ac.rs](mailto:danilo.karlicic@masfak.ni.ac.rs)

<sup>2</sup>Department of Mechanics  
Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, 11000, Serbia  
e-mail: [mcajic@mi.sanu.ac.rs](mailto:mcajic@mi.sanu.ac.rs), web page: <http://www.mi.sanu.ac.rs/~mcajic/>

<sup>3</sup>Department of Mechanics  
University of Belgrade, Faculty of Mechanical Engineering, Belgrade, 11000, Serbia  
e-mail: [mlazarevic@mas.bg.ac.rs](mailto:mlazarevic@mas.bg.ac.rs), web page: <http://www.mlazarevic-fracmeh.com/>

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### ABSTRACT

In recent years, nonlinear and damping effects have become more important in the study of the dynamic behavior of micro- and nano- systems and devices. Therefore, investigators direct special attention to the mathematical modeling of the dynamic behavior of nano-structures such as carbon nanotubes, ZnO nanotubes and functionally graded beams.

The functionally graded materials (FGM) are types of structures that are composed of at least two-phase inhomogeneous particulate composite and synthesized in such manner that the volume fractions of constituents vary continuously along any desired spatial direction. This results in smooth variation of mechanical properties along desired direction. Nazemnezhad et al. [1] have analyzed the free nonlinear vibration of FG nanobeam based on the von Karman deformation, Euler-Bernoulli beam theory and nonlocal elasticity. They obtained approximated analytical solution for the nonlinear natural frequency by applying the multiple scales perturbation method. Ansari et al. [2] proposed nonlinear dynamic model to analyze the nonlinear forced vibration of FG nanobeam in thermal environment based on the surface elasticity theory.

Some authors describe dissipation effects in viscoelastic structures and nanostructures using fractional derivative models [3]. Ansari et al. [4] investigated the nonlinear vibration of a nonlocal fractional viscoelastic nanobeam using numerical methods.

By browsing the literature, the authors found a small number of studies focused on the vibration analysis of FG nanobeams embedded in certain type of medium. In this report, we investigated the dynamical model of a functionally graded (FG) beam modeled as a nanobeam with geometric nonlinearity embedded in a fractional Kelvin-Voigt viscoelastic medium by using the nonlocal continuum theory. The material properties of FG nanobeam vary continuously through thickness direction, which is based on the power-law distribution. We assume that the FG nanobeam has simply-supported boundary conditions and vibrates under the influence of the transversal periodic load. Based on the nonlocal Euler-Bernoulli beam theory, von Karman nonlinear strain-displacements relation, we obtain the nonlinear fractional partial differential equations of transversal motion of the embedded FG nanobeam. By using the assumption of small fractional damping we employed the perturbation method of multiple-scales to obtain the approximated analytical solution of the governing equation of motion. The relationships between frequency-amplitude and force-amplitude in the presence of fractional damping are derived by using the multiple scales method. It is shown that the nonlocal parameter, fractional damping and material property gradient index have significant effects on the vibration behavior of FG nanobeam and therefore receive substantial attention.

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