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SAFFMAN – TAYLOR INSTABILITY – HISTORY AND APPLICATION

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Extended Abstract

If one injects a viscous liquid into a narrow gap between two horizontal parallel plates (i.e. a Hele-Shaw cell), it will stay stable while expanding into an air-filled gap, i.e. in a planar channel geometry the gas-liquid interface will be flat, whereas in a radial geometry it will be circular. If the reverse experiment is carried out, i.e. a less viscous fluid, such as air, is injected into a more viscous fluid occupying the region between the walls, the interface might become linearly unstable and grow rapidly into large-amplitude finger-like structures (Fig. 1).

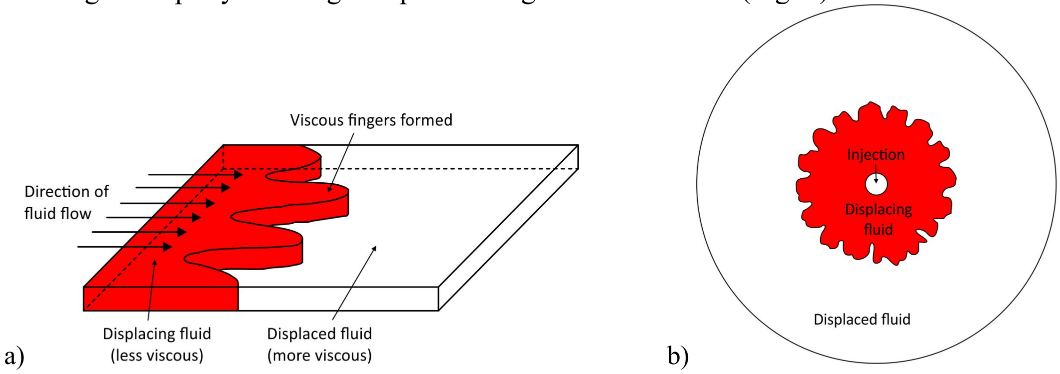

Fig. 1. Viscous fingering in Hele-Shaw cell with a (a) planar channel geometry and (b) radial geometry.

This phenomenon is called Saffman – Taylor instability, or viscous fingering. It is observed when a less viscous fluid displaces a more viscous fluid in confined geometries, such as illustrated in Fig. 1, in which the gap size is much smaller compared to a characteristic length scale in the flow direction. Such boundary conditions are commonly encountered in porous media, for example, during oil recovery. In fact, the interfacial instability was named after Philip Geoffrey Saffman (1931-2008) and Sir Geoffrey Ingram Taylor (1886-1975), who studied it in the paper 'The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid' from 1958. The research was envisioned by Sir G. I. Taylor in 1956 after his visit to the Humble Oil Company.

The continuing interest in viscous fingering arises from its close relation to a wide range of phenomena, and that fact that the problem is tractable using both experimental and theoretical approaches. The geometry of the system renders the flow quasi-two-dimensional, and allows one to simplify its description from the (Navier-)Stokes equations to the Darcy's law:

$$u = -\frac{b^2}{12\mu}\nabla p \tag{1}$$

where b represents the gap thickness, u and p the depth-averaged velocity and pressure, respectively, and μ dynamic viscosity of the fluid [3]. This means that the nonlinearity in the problem is concentrated entirely to the interface, which is subjected to the Young-Laplace equation. This model allows one to understand the physical mechanism for the instability onset in simple terms: any linear perturbations to the stable flow get amplified due to the viscous effects captured by the equation (1), and are counterbalanced by the surface tension effects, which work against the interfacial deformations [1, 2].

As already mentioned, viscous fingering has applications in secondary and tertiary oil recovery, hydrology and filtration [2]. It is responsible for water flooding of oil wells, so is of significant importance to oil reservoir engineering. However, similar morphological pattern growth is reported it other fields, e.g. metastatic tumors often invade healthy nearby tissues by forming multicellular finger-like protrusions emerging from the cancer mass [4]. Therefore, viscous fingering is also one of the archetypical problems of pattern formation. Furthermore, it is similar in nature to other interfacial instabilities, such as the Printer's instability, that occur during coating processes when a solid surface is covered uniformly by a thin layer of fluid [3].

Key words: viscous fingering, Hele-Shaw cell, Saffman – Taylor instability, Reynolds number, oil recovery

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