

# Four-bar Linkage Mechanism Optimization for Linkage Driven Underactuated Robotic Finger

Lazar Matijašević, *PhD Student*, Petar B. Petrović, *Full Professor*

**Abstract** — When designing linkage driven underactuated robotic finger, many parameters need to be satisfied in order to produce robust robotic hand capable of withstanding industrial environment and capable of fulfilling all needs of robotic assembly in terms of precision and dexterity. For this study, four-bar linkage mechanisms are used to drive underactuated robotic finger and design parameter that will be addressed is transmission performance. Optimization method used for obtaining length of links of four-bar mechanism, based on transmission performance is shown. Freudenstein's analytic method for four-bar linkage function generation, is chosen, and calculated link lengths are to be used for acquiring parameter called transmission defect, parameter that is objective function to be minimized in this optimization process. Maximizing transmission performance, leads to increase of the transmitted torque from the actuated joints to the underactuated joints through transmission mechanism. This paper presents design and kinematic analysis of three degrees of freedom (3-DoF) underactuated robotic finger with linkage driven mechanism for CMSysLab robotic hand.

**Index Terms**—Robotic Assembly, Robotic Hand, Design; Transmission performance, Transmission defect;

## I. INTRODUCTION

Concept of underactuation has many advantages and because of these advantages it is used in many branches of industry. In robotics, it has application in many robotic hands that are designed for laboratory and industrial setting. These hands utilize different mechanisms that transmit the actuation torque to the underactuated joints. Two most popular and widely used concepts for underactuated multifingered robot hands, [1], are tendon and linkage based transmission mechanisms, shown on Fig.1.

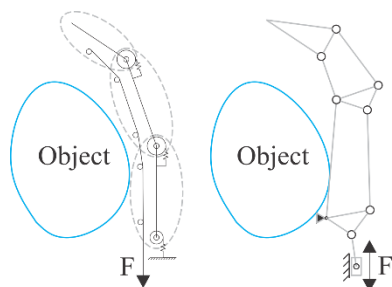


Fig. 1. Tendon and linkage based transmission mechanisms used in underactuated robotic finger. [2].

For our study we have chosen linkage based mechanism because of its rigidity which makes these mechanisms more predictable, more accurate and more controllable.

PhD student Lazar Matijašević is with the Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11120 Belgrade, Serbia (e-mail: [lmatic@mas.bg.ac.rs](mailto:lmatic@mas.bg.ac.rs)).

Full Professor Petar B. Petrović is with the Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11120 Belgrade, Serbia (e-mail: [pbpetrovic@mas.bg.ac.rs](mailto:pbpetrovic@mas.bg.ac.rs)).

These, linkage based mechanisms, are suitable for bigger grasping forces, which is mandatory in assembly operations in industrial setting.

Simplified sketch of underactuated 3-DOF finger with linkage driven mechanism is shown on Fig. 2.

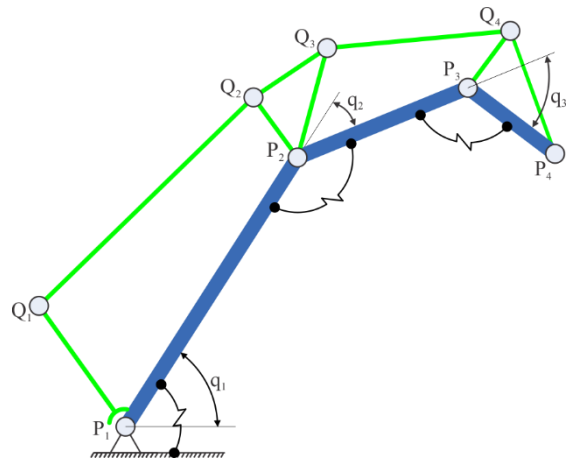


Fig. 2. Representation of 3-DOF finger with linkage driven mechanism.

Transmission mechanism of 3-DOF underactuated finger shown on Fig. 2 consists of two four-linkage mechanisms,  $O_3-O_2-P_2-P_3$  and  $O_2-O_1-P_1-P_2'$ , interconnected with rigid triangular shaped rocker,  $O_2-P_2-P_2'$ .

Requirements for the design of linkage based robotic finger must be met in order to ensure desired motion. Many different optimization algorithms are devised in order to obtain better characteristics of transmission mechanisms. Some of those characteristics are reduced power consumption, reduced structure errors for different mechanisms, force transmission, weight, size and many more. According to aforementioned characteristics there are many design parameters to be met in order to design robust and dexterous robotic hand that can fulfill all requirements of robotic assembly in industrial setting. Focus of this paper will be on transmission performance optimization. Proposed method of optimization focuses on introducing and minimizing parameter named transmission defect and, based on its minimal value, determining optimal value of lengths of four-bar mechanism links.

## II. GENERAL CONCEPT OF ANALYSIS

Variety of useful mechanisms can be formed from a four-link mechanism, shown on Fig. 3, through slight variations, such as changing the character of the pairs, proportions of links, etc. In this paper, two four-bar linkage mechanisms [3], connected with rocker are used for movement and force transmission on phalanx of the underactuated robotic finger.

### A. Freudenstein's equation

Analytical method of kinematic synthesis for four-bar linkage mechanism used in this paper is Freudenstein's

method [4]. Using this method, it is possible to calculate link lengths that accommodate generated function. Also this method is useful in calculating link lengths of four-bar linkage mechanisms because we can use only three positions of mechanism to solve Freudenstein's equation by solving system of three linear equations as will be shown. On Fig 3., a typical four-bar linkage mechanism is shown.

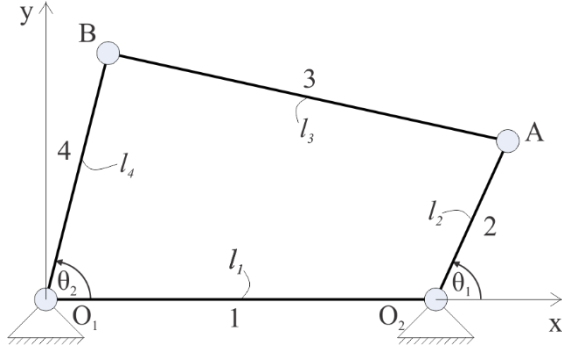


Fig. 3. Representation of four-bar linkage mechanism.

Following parameters are presented on Fig. 3 of  $O_1O_2AB$  four-bar linkage mechanism:

- Fixed link  $O_1O_2$ , ( $l_1$ ), also a phalange of robotic finger,
- Input link  $O_2A$ , ( $l_2$ ),
- Coupler link  $AB$ , ( $l_3$ ),
- Output link  $BO_1$ , ( $l_4$ ),
- Orientation of the input and output link is defined by the angles they construct with x axis and those are input angle  $\theta_1$  and output angle  $\theta_2$ .

Using three position synthesis method allows us to solve equation by generating two pairs of prescribed coordinated movement, shown in (1), of the input and output link.

$$(\theta_1^{12}, \theta_2^{12}), (\theta_1^{23}, \theta_2^{23}). \quad (1)$$

From position 1 to position 2 change in the angle  $\theta_i$  is given by  $\theta_i^{12}$  and corresponding change in the angle  $\theta_2$  is given by  $\theta_2^{12}$ . Same stands for movement from position 2 to position 3. These are two pairs of prescribed movement.

In three position synthesis using Freudenstein's method, we assume the values of  $\theta_i$  corresponding to the first configuration ( $\theta_i^1$ ) and the value of  $\theta_2$  corresponding to first configuration ( $\theta_2^1$ ). Using inputs from (1) we can write:

$$\begin{aligned} \theta_1^2 &= \theta_1^1 + \theta_1^{12}; \\ \theta_2^2 &= \theta_2^1 + \theta_2^{12}, \end{aligned} \quad (2)$$

and:

$$\begin{aligned} \theta_1^3 &= \theta_1^2 + \theta_1^{23}; \\ \theta_2^3 &= \theta_2^2 + \theta_2^{23}. \end{aligned} \quad (3)$$

Freudenstein's equation, which is displacement equation for this planar mechanism, must be valid for all three sets of aforementioned input and output angles. Coordinates of points A and B are:

$$\begin{aligned} x_A &= l_1 + l_2 \cos \theta_1; & x_B &= l_4 \cos \theta_2; \\ y_A &= l_2 \sin \theta_1, & y_B &= l_4 \sin \theta_2. \end{aligned} \quad (4)$$

Length of the coupler link 3 is:

$$l_3^2 = (x_A - x_B)^2 + (y_A - y_B)^2. \quad (5)$$

Substitution of (4) in (5) yields:

$$\cos(\theta_1 - \theta_2) = \frac{l_1}{l_4} \cos \theta_1 - \frac{l_1}{l_2} \cos \theta_2 + \frac{l_1^2 + l_2^2 - l_3^2 + l_4^2}{2l_2l_4}. \quad (6)$$

We now introduce term design parameter  $D_i$ ,  $i=1,2,3$ , described as follows:

$$D_1 = \frac{l_1}{l_4}; D_2 = \frac{l_1}{l_2}; D_3 = \frac{l_1^2 + l_2^2 - l_3^2 + l_4^2}{2l_2l_4}, \quad (7)$$

and substitute it in (6), which in return leads to Freudenstein's equation for four-bar linkage mechanism shown on Fig. 3:

$$\cos(\theta_1 - \theta_2) = D_1 \cos \theta_1 - D_2 \cos \theta_2 + D_3. \quad (8)$$

General form of Freudenstein's equation is:

$$\cos(\theta_1^i - \theta_2^i) = D_1 \cos \theta_1^i - D_2 \cos \theta_2^i + D_3; \quad (9)$$

$$i = 1, 2, 3.$$

From there we can solve three linear equations using angle values determined in (2) and (3) and calculate design parameters  $D_i$ ,  $i=1,2,3$ . These design parameters are used to calculate length values  $l_2$ ,  $l_3$ , and  $l_4$ . Length of link  $l_1$  is chosen for example, based on characteristics of an index human finger or any other assumption. These ratios are important because they allow us to scale robotic finger but to keep same relative movement between various links.

### B. Transmission angle

Now, concept of transmission angle [5], needs to be introduced as well, because that is another parameter, besides aforementioned design parameters  $D_i$ ,  $i=1, 2, 3$ , that are used in optimization method described in this paper. Let's assume that input torque  $T_1$  is acting on link  $O_2A$ . That link is now called input link.

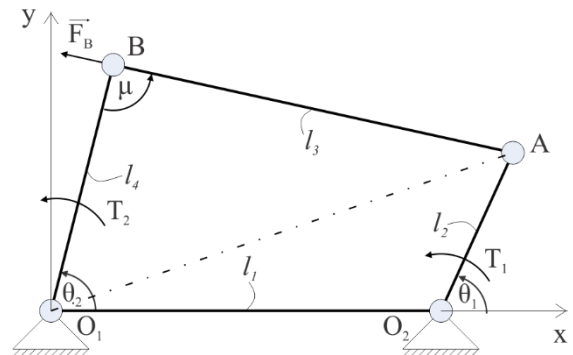


Fig. 4. Representation of transmission angle in four-bar linkage mechanism.

In Fig. 4, if  $O_2A$  is the input link, torque  $T_1$  to the input link is causing force  $\vec{F}_B$ , which is transmitted through the coupler link  $AB$ . For sufficiently slow motions (where we can neglect inertia forces), the force in the coupler link is pure tension or compression and is directed along  $AB$ . For a given force in the coupler link, the torque transmitted to the output link,  $T_2$  (about point  $O_1$ ), is maximum when the angle  $\mu$  between

coupler link  $AB$  and output link  $O_1B$  is  $90^\circ$ . Therefore, angle  $ABO_1$  is called **transmission angle**.

It varies throughout the range of operation from acute angle  $\mu$  to obtuse angle  $\pi - \mu$ .

According to Fig. 4 transmission angle can be calculated by writing the cosine theorem for  $O_1A$  using the triangles  $O_1AB$  and  $O_1AO_2$  and equating the length  $O_1A$ . It is expressed in following form:

$$\mu = \cos^{-1} \frac{l_1^2 + l_2^2 - l_3^2 - l_4^2 - 2l_2l_1 \cos(\pi - \theta_1)}{-2l_3l_4}. \quad (10)$$

Most favorable value of transmission angle is  $90^\circ$  and recommended variations of transmission angle [5] is  $90^\circ \pm 50^\circ$ . When transmission angle is  $0^\circ$  or  $180^\circ$  transmission of motion is impossible. No torque can be realized on output link if transmission angle is  $0^\circ$  and mechanism is at its dead center position.

### C. Transmission quality

The term transmission index [6] was introduced as a generalization of the concept of transmission angle. It is one of many performance indexes of one mechanism. For planar four-bar linkages, the transmission index is shown to be the sine of the transmission angle. Just like the transmission angle, which varies with the configuration of the linkage, the transmission index is also dependent on configuration. As a global performance parameter that measure the force and motion transmission of the linkage, transmission quality [7] is used, which is defined in terms transmission index. Transmission quality is suitable to evaluate mechanism performance throughout its whole range of motion and is not defined only to a local evaluation at a single configuration.

Transmission quality of four-bar linkage mechanism from Fig. 4, is defined in following equation:

$$z = \sqrt{\frac{1}{\theta_1^1 - \theta_1^3} \int_{\theta_1^1}^{\theta_1^3} \sin^2 \mu d\theta_1}. \quad (11)$$

The complement [15] of the transmission quality, is defined as:

$$z' = \sqrt{\frac{1}{\theta_1^1 - \theta_1^3} \int_{\theta_1^1}^{\theta_1^3} \cos^2 \mu d\theta_1}, \quad (12)$$

and is called **transmission defect** which is an objective function to be minimized, and conforms to:

$$z + z' = 1 ; 0 < z' < 1. \quad (13)$$

In this paper, link lengths, which yields minimum transmission defect will be defined. Note that this parameter is only one of many parameters that need to be taken into account when designing underactuated robotic finger, and despite the fact that best transmission of force and movement is desired, lengths of links may differ from values calculated here. It is important to keep in mind that these robotic hands should be as light as possible, robust and in order to be industry acceptable they need to be cost efficient.

It is also stated that with mechanisms that have reversal motion, as robotic hand has (opening and closing of hand)

transmission angle must be investigated for both cases of motion transmission but that will be scope of future research because in this research only pinch and form grips are taken into account, and in those, direction of forces acting upon object of manipulation are only in direction of closing of robotic hand and that direction will be examined.

In next chapter, analysis described above, will be used on optimization of design of CMSysLab underactuated robotic hand.

## III. SYNTHESIS OF OPTIMAL PLANAR FOUR-BAR LINKAGE MECHANISM FOR CMSysLAB UNDERACTUATED ROBOTIC FINGER

When designing transmission mechanism, the crucial issue is selection of mechanism and dimensioning of chosen mechanism. Optimization of planar four-bar linkage [8] mechanism is carried out, as mentioned in previous chapter, with regard to three positions of input and output links which yields three linear equations. These are solved using Freudenstein's equation. Later, optimization of force transmission is carried out using transmission defect as objective function to be minimized. The linkages connected in series are synthesized by starting from the four-bar linkage, which moves the distal phalanx.

### A. Synthesis of the four-bar linkage $O_3, O_2, P_2, P_3$

For synthesis of the function generating four-bar linkage  $O_3, O_2, P_2, P_3$ , shown in Fig. 5, we used method described in section II.

Freudenstein's equation for shown mechanism can be expressed in the form:

$$\cos(\theta_3^i - \theta_4^i) = D_1 \cos \theta_3^i - D_2 \cos \theta_4^i + D_3, \quad i = 1, 2, 3. \quad (14)$$

Design parameters  $D_i, i=1, 2, 3$ , based on (7) are calculated as:

$$D_1 = \frac{L_2}{c_2}; D_2 = \frac{L_2}{a_2}; D_3 = \frac{L_2^2 + a_2^2 - b_2^2 + c_2^2}{2a_2c_2}. \quad (15)$$

The four-bar linkage shown in Fig. 5, consists of links  $a_2, b_2, c_2$  of four-bar linkage:  $O_2P_2, P_2P_3$  and  $P_3O_3$  and medial and distal phalanges  $L_2$  and  $L_3$  of underactuated robotic finger. As described in [1], during motion phalanges move until they come in contact with object that needs to be manipulated. Phalanx that comes in contact with object stops moving, in our case medial phalanx  $L_2$ , which is fixed on Fig. 5, and distal phalanx  $L_3$  can move until it reaches object or mechanical hardstop which constricts its movement.

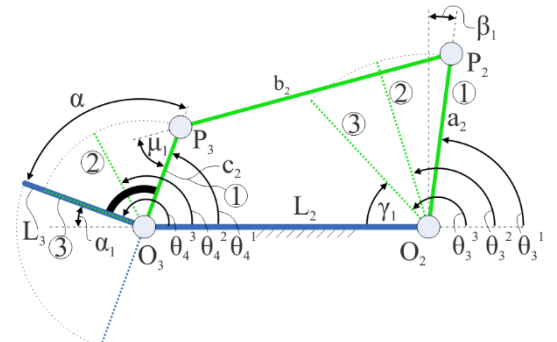


Fig. 5. Representation of four-bar linkage  $O_3, O_2, P_2, P_3$ .

Angles  $\theta_3^i$  and  $\theta_4^i$  for  $i=1, 2, 3$  are input and output angles of mechanism, respectively.

Many of design parameters on Fig. 5 are empirically acquired, based on actual human hand proportions. According to the proposed mechanical design of the finger, the design parameters are  $\alpha = 90^\circ$ ,  $\alpha_1 = 20^\circ$ ,  $\beta_1 = 7.5^\circ$  and  $\gamma_1 = 47^\circ$ . With that in mind, pairs of angles for the starting and final position of links  $a_1$  and  $c_1$  are easily acquired. Values of those angles are  $(\theta_3^1, \theta_4^1) = (82.5^\circ, 70^\circ)$  for starting position and for final position  $(\theta_3^3, \theta_4^3) = (133^\circ, 160^\circ)$ .

Freudenstein's equation (14) can be solved when three positions ①, ② and ③ of the input link  $a_2$ , and corresponding three positions ①, ② and ③ of the output link  $c_2$  are known. Angle pairs, determined above, are to be substituted in (14), and by solving three linear equations we obtain design parameters  $D_i$ ,  $i=1, 2, 3$ , that are used in (15) to calculate link lengths.

Since only two of the three pairs of angles required by the Freudenstein's equations are assigned as design specification of the function generating four-bar linkage  $ABCD$ , an optimization procedure in terms of force transmission has been developed by varying values of  $(\theta_3^2, \theta_4^2)$  during process of optimization.

These values of link lengths are then substituted in transmission angle equation for four-bar linkage mechanism shown on Fig. 5:

$$\mu_1 = \cos^{-1} \frac{L_2^2 + a_2^2 - b_2^2 - c_2^2 - 2L_2 a_2 \cos(\pi - \theta_3)}{2b_2 c_2}. \quad (16)$$

Transmission defect for mechanism on Fig. 7 is defined as:

$$z' = \sqrt{\frac{1}{\theta_3^3 - \theta_3^1} \int_{\theta_3^1}^{\theta_3^3} \cos^2 \mu_1 d\theta_3}. \quad (17)$$

Substituting (16) in (17) gives the value of  $z'$ . If it is minimum value, calculated lengths of links will be optimal, if not, then angle values  $(\theta_3^2, \theta_4^2)$  should be varied in a specific range until minimum value of  $z'$  is found.

### Proposed algorithm

**Require:** Defining the base phalanx length  $L_2$  and pairs of minimum and maximum values of the input and output angles  $(\theta_3^1, \theta_4^1)$  and  $(\theta_3^3, \theta_4^3)$  of four bar linkage

**Step 1:** Substitute angle pairs into Freudenstein's equation

**Step 2: FOR** (defined increment)

Symbolic solving the system of three equations, resulting in equations for lengths  $a_2$ ,  $b_2$  and  $c_2$ , which are function of angles  $(\theta_3^2, \theta_4^2)$ ;

**end FOR**

**Step 3: FOR** (defined increment)

$(\theta_3^1, \theta_4^1) < (\theta_3^2, \theta_4^2) < (\theta_3^3, \theta_4^3)$

Solve:

$\cos(\theta_3^i - \theta_4^i) = D_1 \cos \theta_3^i - D_2 \cos \theta_4^i + D_3, i = 1, 2, 3.$

Calculated values store in predefined matrices A, B and C

Solve:

$$z' = \sqrt{\frac{1}{\theta_3^3 - \theta_3^1} \int_{\theta_3^1}^{\theta_3^3} \cos^2 \mu_1 d\theta_3}.$$

Calculated values store in predefined matrix D

**end FOR**

**Step 4: WHILE**  $z' \sim \min(z')$

Go through matrices A, B, C and D

**end WHILE**

**Step 5:** Obtain values of  $a_2$ ,  $b_2$  and  $c_2$  for  $z' = \min(z')$

Values of link lengths obtained from optimization process are shown in Table III and Fig. 6.

TABLE I  
INPUT PARAMETERS FOR OPTIMIZATION ALGORITHM USED FOR  
OPTIMIZATION OF THE FOUR-BAR LINKAGE O<sub>3</sub>, O<sub>2</sub>, P<sub>2</sub>, P<sub>3</sub>

Minimum and maximum values of input and output angles			
$\theta_3^1$	82.5°	$\theta_3^3$	133°
$\theta_4^1$	70°	$\theta_4^3$	160°

TABLE II  
STARTING VALUES OF PARAMETERS FOR OPTIMIZATION ALGORITHM  
USED FOR OPTIMIZATION OF THE FOUR-BAR LINKAGE O<sub>3</sub>, O<sub>2</sub>, P<sub>2</sub>, P<sub>3</sub>

Starting values of parameters to be optimized with limits of values	
$\theta_3^2$	107.75° ± 25°
$\theta_4^2$	115° ± 45°
$L_2$	37.5 (fixed value)
$a_2$	23mm ± 10mm
$b_2$	36mm ± 10mm
$c_2$	14mm ± 5mm
$z'$	0.103

TABLE III  
OBTAINED PARAMETERS FROM OPTIMIZATION

$z' = 0.02526$			
$L_2$	37.5	$a_2$	28.361
$b_2$	36.993	$c_2$	15.82

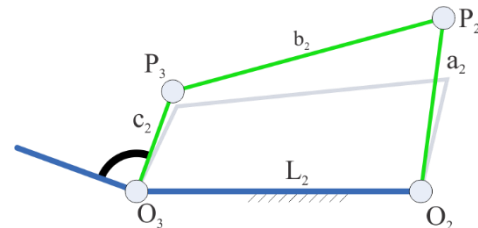


Fig. 6. Optimized parameters of four-bar linkage O<sub>3</sub>, O<sub>2</sub>, P<sub>2</sub>, P<sub>3</sub>.

On Fig. 6, four-bar mechanism of grey color is proportional to starting dimensions of link lengths. Green four-bar mechanism is shown with calculated link lengths. It is obvious that there is difference between calculated values after optimization and starting values. It is because starting values are empirically acquired based on human finger proportions and many more parameters like space limitations, etc. All those parameters need to be taken into account when designing underactuated robotic finger and not just transmission quality as described in this paper. It is obvious that this is iterative process and it needs to accommodate many criteria in order to produce optimal result.

Now dimensions of links of second four-bar linkage will be addressed.



### B. Synthesis of the four-bar linkage $O_2, O_1, P_1, P_2'$

After optimization of four-bar transmission mechanism of medial phalanx it is important to do the same with four-bar transmission mechanism of proximal phalanx.

For synthesis of the function generating four-bar linkage  $O_2, O_1, P_1, P_2'$ , shown in Fig. 7, we used same method as for aforementioned four-bar linkage. Freudenstein's equation for shown mechanism can be expressed in the form:

$$\cos(\theta_1^i - \theta_2^i) = D_1 \cos \theta_1^i - D_2 \cos \theta_2^i + D_3, \quad (18)$$

$$i = 1, 2, 3.$$

Design parameters  $D_i, i=1, 2, 3$ , based on (7) are calculated as:

$$D_1 = \frac{L_1}{c_1}; D_2 = \frac{L_1}{a_1}; D_3 = \frac{L_1^2 + a_1^2 - b_1^2 + c_1^2}{2a_1c_1}. \quad (19)$$

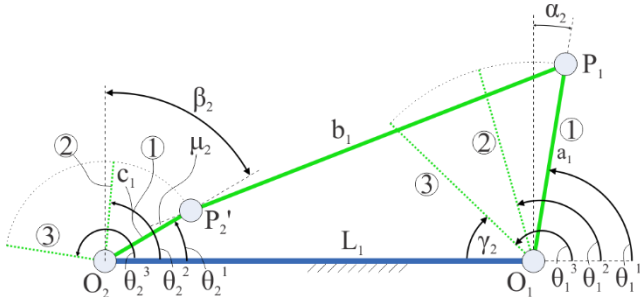


Fig. 7. Representation of four-bar linkage  $O_2, O_1, P_1, P_2'$ .

Parameters shown on Fig. 7:  $L_1$  is the length of proximal phalanx of the finger,  $a_1, b_1$  and  $c_1$  are lengths of links of four-bar linkage:  $O_1P_1, P_1P_2'$  and  $P_2'O_2$ , angles  $\theta_1^i$  and  $\theta_2^i$  for  $i=1, 2, 3$  are input and output angles of mechanism, respectively.

As already shown, (18) can be solved when three positions (1), (2) and (3) of the input link  $a_1$ , and corresponding three positions (1), (2) and (3) of the output link  $c_1$  are predefined.

Many of design parameters on Fig. 7 are empirically acquired, based on actual human hand proportions. According to the proposed mechanical design of the finger, the design parameters are  $\alpha_2 = 10^\circ, \beta_2 = 60^\circ$  and  $\gamma_2 = 43^\circ$ . With that in mind, pairs of angles for the starting and final position of links  $a_1$  and  $c_1$  are easily acquired. Values of those angles are  $(\theta_1^1, \theta_2^1) = (80^\circ, 30^\circ)$  for starting position and for final position  $(\theta_1^3, \theta_2^3) = (137^\circ, 171^\circ)$ . These are to be substituted in (18), and by solving three linear equations we obtain design parameters  $D_i, i=1, 2, 3$ , that are used in (19) to calculate link lengths. These values are then substituted in transmission angle equation:

$$\mu_2 = \cos^{-1} \frac{L_1^2 + a_1^2 - b_1^2 - c_1^2 - 2L_1a_1 \cos(\pi - \theta_1)}{2b_1c_1}. \quad (20)$$

Transmission defect for mechanism on Fig. 7 is defined as

$$z' = \sqrt{\frac{1}{\theta_1^3 - \theta_1^1} \int_{\theta_1^1}^{\theta_1^3} \cos^2 \mu_2 d\theta_1}. \quad (21)$$

Substituting (20) in (21) gives the value of  $z'$ . If it is minimum value, calculated lengths of links will be optimal, if not, then angle values  $(\theta_1, \theta_2)$  should be varied in a specific range until minimum value of  $z'$  is found, the same way as with previous four-bar linkage. Values of link lengths

obtained from optimization process are shown in Table VI and Fig. 8.

TABLE IV  
INPUT PARAMETERS FOR OPTIMIZATION ALGORITHM USED FOR OPTIMIZATION OF THE FOUR-BAR LINKAGE  $O_2, O_1, P_1, P_2'$

Minimum and maximum values of input and output angles			
$\theta_1^1$	80	$\theta_1^3$	137°
$\theta_2^1$	30°	$\theta_2^3$	171°

TABLE V  
STARTING VALUES OF PARAMETERS FOR OPTIMIZATION ALGORITHM USED FOR OPTIMIZATION OF THE FOUR-BAR LINKAGE  $O_2, O_1, P_1, P_2'$

Starting values of parameters to be optimized with limits of values	
$\theta_1^2$	$108.5^\circ \pm 28^\circ$
$\theta_2^2$	$100.5^\circ \pm 70.5^\circ$
$L_1$	37.5 (fixed value)
$a_1$	$32\text{mm} \pm 10\text{mm}$
$b_1$	$60\text{mm} \pm 10\text{mm}$
$c_1$	$15\text{mm} \pm 5\text{mm}$
$z'$	0.081

TABLE VI  
OBTAINED PARAMETERS FROM OPTIMIZATION

$z'=0.02993$			
$L_1$	64.5	$a_1$	41.83
$b_1$	60.554	$c_1$	16.39

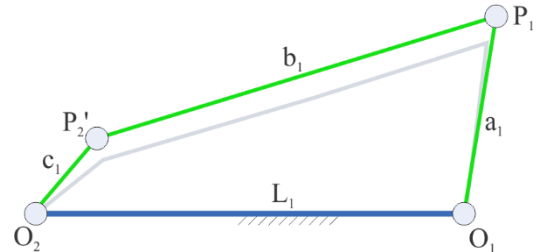


Fig. 8. Optimized parameters of four-bar linkage  $O_2, O_1, P_1, P_2'$ .

On Fig. 8, with grey color, four-bar mechanism with starting link lengths is shown. Green color represents new four-bar mechanism with calculated link lengths.

Previously described method of optimization of four-bar linkage geometry is useful for process of designing linkage based underactuated robotic hand, because calculated values, now can be used as starting values for later designing processes which will use another set of criteria for acquiring optimal geometry of underactuated finger based on many criteria and not just one described in this paper.

## IV. CONCLUSION

In this paper, a method for obtaining optimal lengths of links based on transmission, quality in a four-bar linkage mechanism, was presented. This kind of analysis is important so that researchers and designers of underactuated robotic hands can design proper underactuated finger that can transmit sufficient force onto object that been manipulated with.

As shown, this is only one of many parameters that need to be addressed in order to design proper, robust and dexterous enough robotic hand that is able to accommodate all requirements placed upon it by industrial assembly operations. Some of those requirements are:

**Precision:** assembly process, especially when we take into consideration precise assemblies in auto or aerospace sectors of manufacturing engineering, require high precision and

accuracy of placing of different parts in a assembly.

**Strength of grip:** it is needed in order for robotic hand to be able to counteract contact forces when handling objects. Rigidity and errors that are imposed from lack of rigidity is important factor to take into account. Errors and lack of rigidity are direct consequence of clearances that exist in assemblies.

**Access to the zone of assembly:** access to the zone in which assembly of parts needs to take place in required position and orientation.

#### ACKNOWLEDGMENT

This research work is supported by the Serbian Ministry for Education, Science and Technology Development through the projects titled 'Smart Robotics for Customized Manufacturing', grant No.: TR35007, and 'Development and Experiments of Mobile Collaborative Robot With Dual-Arm', grant No.: 401-00-00589/2018-09, which is jointly realized by Anhui University of Technology, Ma'anshan, Anhui, and Tsinghua University, Beijing, from one side, and Institute Mihajlo Pupin – Belgrade (IMP), Faculty of Mechanical Engineering, University of Belgrade (MFB), and Faculty of technical sciences of Novi Sad, University of Novi Sad (FTN), from the other side, supported by a cluster of

industrial partners from both sides, as a bilateral scientific project of high national relevance with the People's Republic of China.

#### REFERENCES

- [1] Birglen, L., et al. “*Underactuated Robotic Hands*”, Springer, Berlin, 2008.
- [2] Matijasevic, L, Milivojevic, M, Petrovic, P, “Multifingered under-actuated hands in robotic assembly”, *13th International Scientific Conference MMA 2018 Flexible Technologies*, Proceedings, ISBN 978-86-6022-094-5, pp.91-94.
- [3] L. Birglen ; C.M. Gosselin “Kinetostatic Analysis of Underactuated Fingers”, *IEEE Transactions on Robotics and Automation*, vol. 20 Issue: 1, pp.211 -221, april 2004
- [4] F. Freudenstein, “Approximate Synthesis of Four-Bar Linkages”, *ASME Trans.*, Vol.77, No.8, pp.853–861, August 1955.
- [5] S. S. Balli ; S. Chand “Transmission angle in mechanisms (Triangle in mech)”, in *Mechanism and Machine Theory*, Elsevier vol. 37 Issue: 2, pp.175-195, 2002.
- [6] Chen, C., & Angeles, J.: “Generalized transmission index and transmission quality for spatial linkages”. *Mechanism and Machine Theory*, 42(9), 1225–1237, 2007.
- [7] Angeles, J., & Bernier, A.: “The Global Least-Square Optimization of Function-Generating Linkages”, *Journal of Mechanisms Transmissions and Automation in Design*, 109(2), 204, 1987.
- [8] C. Gosselin ; J. Angeles “Optimization of planar and spherical function generators as minimum-defect linkages”, in *Mechanism and Machine Theory*, Elsevier vol. 24 Issue: 4, pp.293-307, 1989.