7th INTERNATIONAL SCIENTIFIC CONFERENCE
ON DEFENSIVE TECHNOLOGIES
OTEH 2016
Belgrade, Serbia, 6 – 7 October 2016

ISOGEOMETRIC ANALYSIS OF FREE VIBRATION OF ELLIPTICAL LAMINATED COMPOSITE PLATES USING THIRD ORDER SHEAR DEFORMATION THEORY

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Abstract: *In this research paper a isogeometric laminated composite plate finite element formulation based on third order shear deformation theory is presented. Numerical examples illustrate natural frequencies and free vibration mode shapes of elliptical laminated composite plates. Obtained numerical results are presented and then compared to other available numerical results.*

Keywords: *isogeometric analysis, TSDT, elliptical laminated composite plates.*

1. INTRODUCTION

Since the creation of glass fibers in the '30s, first fiberglass boats in the '40s and their introduction in aircraft industry (Boeing 707 in the 1950s had 2% of the structure made from composites) composite materials slowly became ubiquitous in marine, automotive and aerospace industry. Today, Boeing 787 Dreamliner is the first airliner with composite wings and fuselage (50% of all aircraft is composite), and new Airbus A350XWB is 53% made of composites. The main advantages of composites are their strength and lightness which lead to improved fuel efficiency and more cost-effective products.

This increase in composites usage was followed by great research effort by scientific community. Laminated composite plates (laminates) generated particularly large interest because of their industrial applications. In classic book on the subject [1] different plate theories applied to laminated plates and shells as well as appropriate analytical and finite element models and solutions are covered.

Analytical solutions to governing differential equations are generally available only for simple geometries, focus of the scientific community was the development of computational methods that can threat complex geometries with satisfactory accuracy. In this regard, Finite element method (FEM) became standard tool for treatment of stress analysis problems. FEM seeks solution to the weak (integral) form of differential equations through use of low order (mostly linear or quadratic) polynomial basis functions. One of the main shortcomings of FEM is the necessity to build new finite element model (mesh) in order to run analysis. This procedure can take up to 80% of the total time required for analysis [2]. Novel method generally known as Isogeometric analysis (IGA) [2,3] is proposed in order to integrate geometrical design and numerical analysis. Isogeometric finite element method uses NURBS basis functions for approximation of unknown fields, same as almost every CAD or CAM package. NURBS offer general mathematical representation of both analytical geometric objects and freeform geometry. Application of IGA to plate and shell analysis is presented in [4-8] for isotropic plates and shells and [9-15] for composite plate and shell analysis.

In this paper free vibration analysis of elliptical composite plates based on TSDT of Reddy [1] is presented. Natural frequencies and mode shapes are calculated and compared to other solutions.

2. NURBS PRELIMINARIES

In this section only brief recall of Non-uniform rational B-Spline (NURBS) technology is given. Classic textbooks [16,17] provide more details on the subject.

NURBS are mathematical representations of 1D, 2D or 3D objects. They are capable of representing analytical shapes (e.g. conics) as well as free-form shapes with mathematical exactness and offer easy manipulation and control of shape and smoothness to the user.

A p^{th}-degree NURBS curve is defined as

$$\boldsymbol{C}(u)=\frac{\sum_{i=0}^{n}N_{i,p}(u)w_i\boldsymbol{P}_i}{\sum_{i=0}^{n}N_{i,p}(u)w_i},\quad a\le u\le b \tag{1}$$

where the $\{\boldsymbol{P}_i\}$ are the control points, the $\{w_i\}$ are the weights and the $\{N_{i,p}(u)\}$ are the p^{th}-degree B-spline basis functions defined as

$$N_{i,0}(u)=\begin{cases}1 & u_i\le u<u_{i+1}\\ 0 & \text{otherwise}\end{cases} \tag{2}$$

$$N_{i,p}(u)=\frac{u-u_i}{u_{i+1}-u_i}N_{i,p-1}(u)+\frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}}N_{i+1,p-1}(u) \tag{3}$$

on the non-uniform knot vector

$$U=\left\{\underbrace{a,\dots,a}_{p+1},u_{p+1},\dots,u_{m-p-1},\underbrace{b,\dots,b}_{p+1}\right\} \tag{4}$$

Multivariate NURBS basis functions are defined through tensor product. A NURBS surface of degree p in the u direction and degree q in the v direction is a bivariate vector-valued piecewise rational function of the form

$$\boldsymbol{S}(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,p}(u)N_{j,q}(v)w_{i,j}\boldsymbol{P}_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,p}(u)N_{j,q}(v)w_{i,j}},\quad 0\le u,v<1 \tag{5}$$

where the $\{\boldsymbol{P}_{i,j}\}$ are the control points, the $\{w_{i,j}\}$ are the weights and $\{N_{i,p}(u)\}$ and $\{N_{j,q}(u)\}$ are the p^{th}-degree and q^{th}-degree B-spline basis functions defined on the nonuniform knot vectors

$$\boldsymbol{U}=\left\{\underbrace{a,\dots,a}_{p+1},u_{p+1},\dots,u_{r-p-1},\underbrace{b,\dots,b}_{p+1}\right\} \tag{6}$$

$$\boldsymbol{V}=\left\{\underbrace{c,\dots,c}_{q+1},u_{q+1},\dots,u_{s-q+1},\underbrace{d,\dots,d}_{q+1}\right\} \tag{7}$$

where $r=n+p+1$ and $s=m+q+1$.

3. EQUATIONS OF MOTION

In third order shear deformation theory (TSDT) of Reddy displacement field is defined as:

$$\begin{aligned}u(x,y,z)&=u_0(x,y)+z\psi_x-\frac{4}{3h^2}z^3\left(\psi_x+\frac{\partial w_0}{\partial x}\right)\\ v(x,y,z)&=v_0(x,y)+z\psi_y-\frac{4}{3h^2}z^3\left(\psi_y+\frac{\partial w_0}{\partial y}\right)\\ w(x,y,z)&=w_0(x,y)\end{aligned} \tag{8}$$

where u_0, v_0, w_0 represent linear displacements of the midplane, ψ_x, ψ_y, are the rotations of normals to the midplane about the y and x-axes respectively and h denotes the total thickness of the laminate.

In-plane strains $\{\varepsilon_{xx}\ \varepsilon_{yy}\ \gamma_{xy}\}^T$ are given as

$$\boldsymbol{\varepsilon}_p=\boldsymbol{\varepsilon}_0+z\boldsymbol{\varepsilon}_1+z^3\boldsymbol{\varepsilon}_3 \tag{9}$$

with

$$\boldsymbol{\varepsilon}_0=\left\{\varepsilon_{xx}^0\ \ \varepsilon_{yy}^0\ \ \gamma_{xy}^0\right\}^T=\left\{\frac{\partial u_0}{\partial x}\ \ \frac{\partial v_0}{\partial y}\ \ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right\}^T \tag{10}$$

$$\boldsymbol{\varepsilon}_1=\begin{Bmatrix}\varepsilon_{xx}^1\\ \varepsilon_{yy}^1\\ \gamma_{xy}^1\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial\psi_x}{\partial x}\\ \dfrac{\partial\psi_y}{\partial y}\\ \dfrac{\partial\psi_x}{\partial y}+\dfrac{\partial\psi_y}{\partial x}\end{Bmatrix} \tag{11}$$

$$\boldsymbol{\varepsilon}_3=\begin{Bmatrix}\varepsilon_{xx}^3\\ \varepsilon_{yy}^3\\ \gamma_{xy}^3\end{Bmatrix}=\left(-\frac{4}{3h^2}\right)\begin{Bmatrix}\dfrac{\partial\psi_x}{\partial x}+\dfrac{\partial^2 w_0}{\partial x^2}\\ \dfrac{\partial\psi_y}{\partial y}+\dfrac{\partial^2 w_0}{\partial y^2}\\ \dfrac{\partial\psi_x}{\partial y}+\dfrac{\partial\psi_y}{\partial x}+2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix} \tag{12}$$

and cross plane components $\boldsymbol{\gamma}_p=\{\gamma_{yz}\ \gamma_{xz}\}^T$ as

$$\boldsymbol{\gamma}_p=\boldsymbol{\gamma}_0+z^2\boldsymbol{\gamma}_2 \tag{13}$$

with

$$\boldsymbol{\gamma}_0=\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}\psi_y+\dfrac{\partial w_0}{\partial y}\\ \psi_x+\dfrac{\partial w_0}{\partial x}\end{Bmatrix} \tag{14}$$

$$\boldsymbol{\gamma}_2=\begin{Bmatrix}\gamma_{yz}^2\\ \gamma_{xz}^2\end{Bmatrix}=\left(-\frac{4}{h^2}\right)\begin{Bmatrix}\psi_y+\dfrac{\partial w_0}{\partial y}\\ \psi_x+\dfrac{\partial w_0}{\partial x}\end{Bmatrix} \tag{15}$$

Constitutive relations between stresses and strains in the k^{th} lamina in the case of plane stress state, for local coordinate system of the principle material coordinates (x_1,x_2,x_3) where x_1 is fibre direction, x_2 in-plane normal to fibre and x_3 normal to lamina plane, are given by

$$\begin{Bmatrix}\sigma_1^{(k)}\\ \sigma_2^{(k)}\\ \tau_{12}^{(k)}\\ \tau_{23}^{(k)}\\ \tau_{13}^{(k)}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\ Q_{12}&Q_{22}&0&0&0\\ 0&0&Q_{66}&0&0\\ 0&0&0&Q_{44}&Q_{45}\\ 0&0&0&Q_{45}&Q_{55}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_1^{(k)}\\ \varepsilon_2^{(k)}\\ \gamma_{12}^{(k)}\\ \gamma_{23}^{(k)}\\ \gamma_{13}^{(k)}\end{Bmatrix} \tag{16}$$

The quantities Q_{ij} are called the plane reduced stiffness components and are given in terms of material properties of each layer as

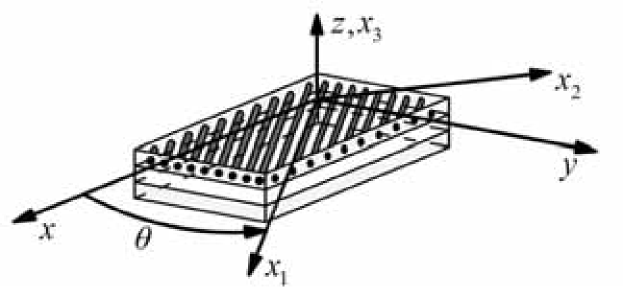
$$Q_{11}^{(k)}=\frac{E_1^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}},\quad Q_{22}^{(k)}=\frac{E_2^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}},\quad Q_{66}^{(k)}=G_{12}^{(k)},$$
$$Q_{12}^{(k)}=\frac{\nu_{12}^{(k)}E_2^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}},\qquad Q_{44}^{(k)}=G_{23}^{(k)},\qquad Q_{55}^{(k)}=G_{13}^{(k)}. \tag{17}$$

$E_1^{(k)}$, $E_2^{(k)}$ are Young moduli, $\nu_{12}^{(k)}$, $\nu_{21}^{(k)}$ are Poisson coefficients and $G_{12}^{(k)}$, $G_{13}^{(k)}$, $G_{32}^{(k)}$ are shear moduli of the lamina.

Composite laminates are usually made of several orthotropic layers of different orientation. In order to express constitutive relations in referent laminate (x,y,z) coordinate system (Pic.3.) lamina constitutive relations are transformed as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11}&\bar{Q}_{12}&\bar{Q}_{16}&0&0\\ \bar{Q}_{12}&\bar{Q}_{22}&\bar{Q}_{26}&0&0\\ \bar{Q}_{16}&\bar{Q}_{26}&\bar{Q}_{66}&0&0\\ 0&0&0&\bar{Q}_{44}&\bar{Q}_{45}\\ 0&0&0&\bar{Q}_{45}&\bar{Q}_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{18}$$

where elements of the matrix in eq.(18) are layer plane-stress-reduced stiffnesses in the laminate coordinate system[1].



Picture 1. Local and global coordinate systems of a laminate

The dynamic form of the principle of virtual work in matrix form is given by

$$\int_\Omega \delta\boldsymbol{\varepsilon}_p^T \boldsymbol{D}\boldsymbol{\varepsilon}_p d\Omega+\int_\Omega \delta\boldsymbol{\varepsilon}_s^T \boldsymbol{D}^s\boldsymbol{\varepsilon}_s d\Omega=\int_\Omega \delta\boldsymbol{u}^T\boldsymbol{m}\ddot{\boldsymbol{u}}d\Omega \tag{19}$$

where $\boldsymbol{m}$ is defined as

$$\boldsymbol{m}=\begin{bmatrix}I_0&0&0&J_1&0&-c_1I_3&0\\ 0&I_0&0&0&J_1&0&-c_1I_3\\ 0&0&I_0&0&0&0&0\\ J_1&0&0&K_2&0&-c_1I_4&0\\ 0&J_1&0&0&K_2&0&-c_1I_4\\ -c_1I_3&0&0&-c_1I_4&0&c_1^2I_6&0\\ 0&-c_1I_3&0&0&-c_1I_4&0&c_1^2I_6\end{bmatrix} \tag{20}$$

With

$$\left(I_0,I_1,I_2,I_3,I_4,I_6\right)=\sum_{k=1}^{N}\int_{-h/2}^{h/2}\rho^{(k)}\left(1,z,z^2,z^3,z^4,z^6\right)dz,$$

$$J_1=I_1-c_1I_3 \text{ and } K_2=I_2-2c_1I_4+c_1^2I_6.$$

Matrices that relate the stress resultants to the strains are given as

$$\boldsymbol{D}=\begin{bmatrix}\boldsymbol{A}&\boldsymbol{B}&\boldsymbol{E}\\ \boldsymbol{B}&\boldsymbol{D}&\boldsymbol{F}\\ \boldsymbol{E}&\boldsymbol{F}&\boldsymbol{H}\end{bmatrix},\quad \boldsymbol{D}^s=\begin{bmatrix}\boldsymbol{A}^s&\boldsymbol{D}^s\\ \boldsymbol{D}^s&\boldsymbol{F}^s\end{bmatrix} \tag{21}$$

with

$$\left(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}\right)=$$
$$=\sum_{k=1}^{N}\int_{-h/2}^{h/2}\bar{Q}_{ij}^{(k)}\left(1,z,z^2,z^3,z^4,z^6\right)dz,$$

$$\left(A_{ij}^s,D_{ij}^s,F_{ij}^s\right)=\sum_{k=1}^{N}\int_{-h/2}^{h/2}\bar{Q}_{ij}^{(k)}\left(1,z^2,z^4\right)dz;\ i,j=4,5$$

and

$$\boldsymbol{u}^T=\left\{u_0\quad v_0\quad w_0\quad \psi_x\quad \psi_y\quad \frac{\partial w_0}{\partial x}\quad \frac{\partial w_0}{\partial y}\right\}^T.$$

4. ISOGEOMETRIC FINITE ELEMENT MODEL OF TSDT PLATE

In isogeometric formulation of TSDT plates, the field variables are inplane displacements, transverse displacements and rotations at control points.

$$\bar{\boldsymbol{u}}=\left\{u_0\quad v_0\quad w_0\quad \psi_x\quad \psi_y\right\}^T \tag{22}$$

Same NURBS basis functions that are used to describe plate geometry are used for the interpolation of field variables

$$\bar{\boldsymbol{u}}=\sum_{I=1}^{nxm}\boldsymbol{N}_I\boldsymbol{q}_I \tag{23}$$

where $n\times m$ is the number of control points (basis functions), N_I are the rational basis functions and q_I the degrees of freedom associated with the control point I

$$\boldsymbol{N}_I=\begin{bmatrix}N_I&0&0&0&0\\ 0&N_I&0&0&0\\ 0&0&N_I&0&0\\ 0&0&0&N_I&0\\ 0&0&0&0&N_I\end{bmatrix} \tag{24}$$

$$\boldsymbol{q}_I=\left\{u_{0I}\quad v_{0I}\quad w_{0I}\quad \psi_{xI}\quad \psi_{yI}\right\}^T \tag{25}$$

The in-plane strains and shear strains are obtained using eq.(3.2),(3.3) and (4.1) as

$$\boldsymbol{\varepsilon}_p=\sum_I\left[\boldsymbol{B}_I^0+z\boldsymbol{B}_I^1-c_1z^3\boldsymbol{B}_I^3\right]\boldsymbol{q}_I \tag{26}$$

$$\boldsymbol{\varepsilon}_s=\sum_I\left[\boldsymbol{B}_I^{S0}-c_2z^2\boldsymbol{B}_I^{S2}\right]\boldsymbol{q}_I \tag{27}$$

where

$$\boldsymbol{B}^0 = \begin{bmatrix} N,x & 0 & 0 & 0 & 0 \\ 0 & N,y & 0 & 0 & 0 \\ N,y & N,x & 0 & 0 & 0 \end{bmatrix} \tag{28}$$

$$\boldsymbol{B}^1 = \begin{bmatrix} 0 & 0 & 0 & N,x & 0 \\ 0 & 0 & 0 & 0 & N,y \\ 0 & 0 & 0 & N,y & N,x \end{bmatrix} \tag{29}$$

$$\boldsymbol{B}^3 = \begin{bmatrix} 0 & 0 & N,xx & N,x & 0 \\ 0 & 0 & N,yy & 0 & N,y \\ 0 & 0 & 2N,xy & N,y & N,x \end{bmatrix} \tag{30}$$

and

$$\boldsymbol{B}^{S0} = \boldsymbol{B}^{S2} = \begin{bmatrix} 0 & 0 & N,y & 0 & N \\ 0 & 0 & N,x & N & 0 \end{bmatrix} \tag{31}$$

$N_{,x}$ and $N_{,y}$ denote the first and $N_{,xx}$, $N_{,yy}$, $N_{,xy}$ second derivatives of N with respect to x and y.

For the free vibration analysis dynamic form of the principle of virtual work reduces to

$$\left(\boldsymbol{K} - \omega^2 \boldsymbol{M}\right)\boldsymbol{q} = 0 \tag{32}$$

$\boldsymbol{K}$ is the global stiffness matrix defined as

$$\boldsymbol{K} = \int_\Omega \begin{bmatrix} \boldsymbol{B}^0 \\ \boldsymbol{B}^1 \\ \boldsymbol{B}^3 \end{bmatrix}^T \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{E} \\ \boldsymbol{B} & \boldsymbol{D} & \boldsymbol{F} \\ \boldsymbol{E} & \boldsymbol{F} & \boldsymbol{H} \end{bmatrix} \begin{bmatrix} \boldsymbol{B}^0 \\ \boldsymbol{B}^1 \\ \boldsymbol{B}^3 \end{bmatrix} d\Omega + \begin{bmatrix} \boldsymbol{B}^{s0} \\ \boldsymbol{B}^{s2} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{A}^s & \boldsymbol{D}^s \\ \boldsymbol{D}^s & \boldsymbol{F}^s \end{bmatrix} \begin{bmatrix} \boldsymbol{B}^{s0} \\ \boldsymbol{B}^{s2} \end{bmatrix} d\Omega \tag{33}$$

The global mass matrix $\boldsymbol{M}$ is given by

$$\boldsymbol{M} = \int_\Omega \boldsymbol{N}_m^T \boldsymbol{m} \boldsymbol{N}_m d\Omega \tag{34}$$

with

$$\boldsymbol{N}_m = \begin{bmatrix} N_I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_I & 0 & 0 & N_I,x & N_I,y \\ 0 & 0 & 0 & N_I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_I & 0 & 0 \end{bmatrix}^T \tag{35}$$

5. NUMERICAL EXAMPLE

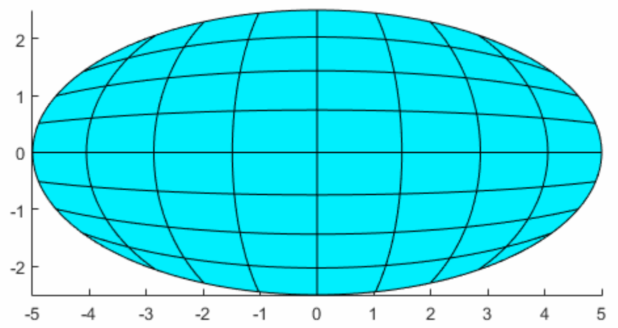
In this section, the performance of the proposed isogeometric method is considered.

Dynamic response of elliptical plate with major radius equal to 5 and minor radius equal to 2.5 was considered (Pic.2.). We used cubic basis functions in all examples and 11×11 control point net (Pic.3.). Boundary conditions are set to fully clamped. Plate is made of [0°/90°/0°] composite laminate with following material properties:

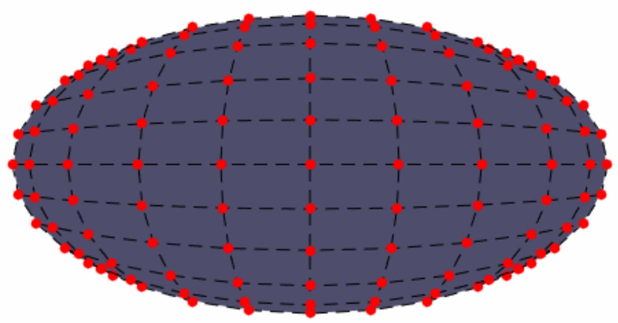
$E_{11}=2.45\cdot E_{22}$ $G_{12}=G_{13}=0.48\cdot E_{22}$ $G_{23}=0.2\cdot E_{22}$

$\nu_{12}=0.23$ $\rho=1.$

This problem was also solved by Chen et al.[18] using element free Galerkin method (EFG) and classical plate theory (CPT). For thicker plates (*a/h<100*) results are compared with Thai et al. who used plate elements based on isogeometric formulation of layerwise deformation theory (LDT) [12] and inverse trigonometric shear deformation theory (ITSDT) [13]. Obtained results are presented in table 1. and they are in good agreement with other ones. Pic. 4 illustrate first six mode shapes of a moderately thick plate (a/h=10).



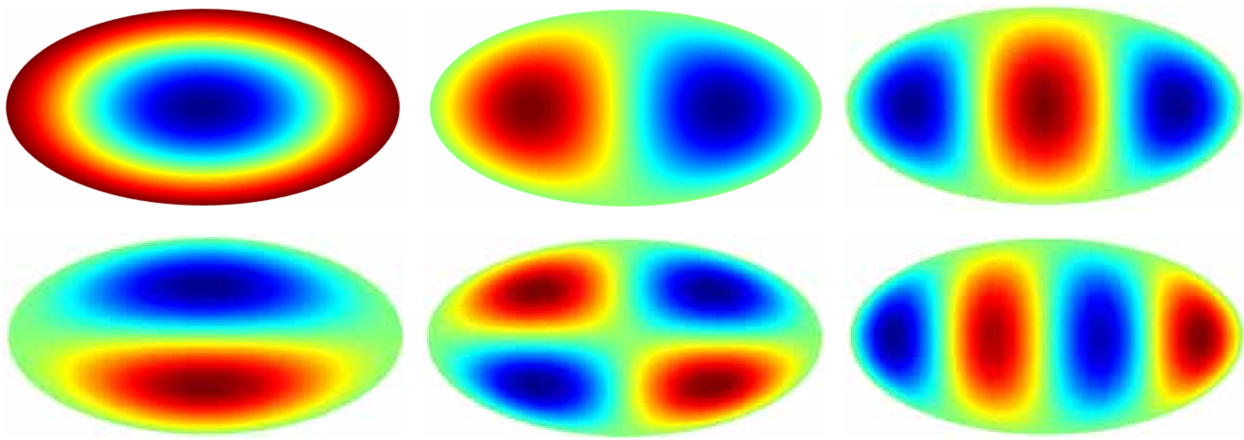
Picture 2. Elliptical plate geometry and assosiated mesh



Picture 3. Control points

Table 1. A non-dimensional frequencies parameter of a [0°/90°/0°] clamped laminated elliptical plate. Frequency parameter is non-dimensionalized as $\omega = \left(\omega a^2\right)\left(\rho h / D_0\right)^{1/2}$ with $D_0 = E_{11}h^3 / 12(1-\nu_{12}\nu_{21})$ [18]

a/h	Method	Modes					
		1	2	3	4	5	6
5	IGA LDT [12]	14.157	19.976	27.143	28.862	34.955	35.162
	IGA ITSDT [13]	14.6407	20.7582	28.1961	30.4532	36.4321	36.8598
	IGA TSDT (present)	**14.4230**	**20.3827**	**27.8591**	**29.6258**	**35.5606**	**36.2486**
10	IGA LDT [12]	17.184	25.714	36.982	39.196	49.148	50.259
	IGA ITSDT [13]	17.4003	26.1718	37.7157	39.9878	50.3411	51.2958
	IGA TSDT (present)	**17.2878**	**25.9383**	**37.5323**	**39.5681**	**49.7803**	**50.8396**
20	IGA LDT [12]	18.329	28.280	42.255	44.321	57.090	59.827
	IGA ITSDT [13]	18.4305	28.5333	42.6563	44.6033	57.6329	60.3551
	IGA TSDT (present)	**18.3787**	**28.4142**	**42.4677**	**44.4904**	**57.3234**	**60.1597**
100	EFG CPT [18]	18.81	29.58	44.99	46.72	61.34	65.14
	IGA CPT [12]	18.793	29.428	44.848	46.642	60.959	64.930
	IGA LDT [12]	18.755	29.332	44.792	46.508	60.792	65.623
	IGA ITSDT [13]	18.8113	29.4718	44.8216	46.5445	60.9286	64.7845
	IGA TSDT (present)	**18.7910**	**29.3921**	**44.8050**	**46.5328**	**60.8958**	**65.0696**



Picture 4. First six mode shapes of a cubic [0°/90°/0°] clamped laminated elliptical plate with *a/h=10*

6. CONCLUSION

This paper presented isogeometric formulation of plate element based on TSDT theory of Reddy. Main focus was on the implementation of presented method for analysis of dynamic response of composite plates. As shown through the example of elliptical plate proposed method can be successfully used for frequency analysis of composite plates.

IGA offers many advantages of which most important one is absence of meshing in classical sense. Since its introduction 10 years ago IGA continues to prove itself as efficient, accurate and robust method but industrial application through integration in commercial CAE packages is absent. This is due to problems such as mesh refinement, treatment of irregular NURBS geometries (e.g. trimmed surfaces). It is our opinion that if the scientific interest in the subject continue to grow, these obstacles will be overcome.

ACKNOWLEDGMENT

This work is supported by the Ministry of Science and Technological Development of Republic of Serbia through Technological Development Project No. 35035.

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