7th International Congress of Serbian Society of Mechanics
Sremski Karlovci, Serbia, June 24-26, 2019

PROBABILISTIC APPROACH IN THE DYNAMIC REANALYSIS

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Abstract

The methods of structural dynamic modification, especially those with their roots in finite element models, have often been described as reanalysis. The paper will discuss the introducing of a probabilistic treatment of important problem parameters. Based on the investigation performed in the [26], further research included the execution of simulations kinetic and potential energy, growth rates *Ek* and *Ep*, differences in growth rates, first frequency in a cantilever beam and a modified beam for 1000 values of Young's modulus of elasticity according to the Gaussian distribution.

Key words: reanalysis, eigenvalues, design variables, uncertainty quantification, simulation

1. Introduction (section title, 11 pt, bold)

Dynamic response of mechanical systems depends on structural parameters. The objective is to evaluate the structural response for successive modifications in the design avoiding the difficult solution of the modified equations. The structural modifications may be caused by external factors or by the designer in order to improve the characteristic of the response (eigenvalues and eigenvectors). Modification of dynamic characteristics means change of corresponding design variables to get desired dynamic behavior of structure. The design variables depend on the type of optimization problem. In the design of structural components, such as stiffened panels and cylinders, the design parameters represent the spacing of the stiffeners, the size and shape of the stiffeners, and the thickness of the skin.

1.2 Probabilistic approach - Literature review

Uncertainty quantification in structures is a very important field of investigation, due to its influence on subjects such as structures reliability and model validation amongst others. Uncertainty quantification in structures can be used where the uncertainties introduced by random forces were applied to the structure Lin (1969). Conducted research Stanojevic at.all (2013), Tadic (2012, 2011), Dragojlovic (2012), Milanovic (2008) gives retrospective in uncertainty assessment in different research area.

Pascual (2012) cited that he followed the study of the case where uncertainty is introduced by random variables or by random fields modeling material properties (e.g., Young's modulus, mass density, Poisson's ratio, damping coefficient) or geometric parameters.

Cacciola at all (2005) did research on the procedure for the dynamic reanalysis of linear systems subjected to deterministic or stochastic loads. The structural modifications may be imposed by external factors (e.g. design alterations for operational reasons, or discrepancies between the predicted and measured properties of the structures) or by the designer in order to improve the characteristic of the response (e.g. layout optimization). Joints and members could be eventually added or deleted during the design procedure so that the topology of the structures may be modified. Reanalysis techniques are commonly devoted to efficiently determine the structural response produced by the following events:

1. modification in the geometry with no further change in the number of degrees of freedom (DOFs);
2. alteration of dynamic characteristics of structural components (mass, damping and stiffness);
3. variation of the number of DOFs due to addition or deletion of joints and members;
4. alteration of loads due to both modification of the original number and position of joints and for changing in the intensity of external excitations.

Cacciola at all (2005) stated that most reanalysis methods are not able to deal with the last two modifications, which are usually named topological modifications as they imply a change in the dimension of the system due to addition or deletion of DOFs.

Kirsch and Liu (1997) focused a static reanalysis method by researching the characteristic of a modified initial design for the case of layout modification (no changes in the number of degrees of freedom).

Lecomte (2013) investigated the response of uncertain vibro-acustic and structural dynamic systems. In this paper, it is shown the comparison of the exact means, variances, covariances, as well as the exact stochastic and covariance coefficients, with their estimates obtained through Monte-Carlo simulations that confirmed the advantages of the analytical approach.

Voormeeren (2010) dealt with the problem of small random errors in substructure measurements in experimental dynamic substructuring using the frequency response functions (FRF). An uncertainty propagation method is derived, which allows the quantification of the uncertainty of the coupled system's FRFs propagated from uncertainties in measured substructure FRFs. A numerical example was used to verify the proposed method [26], the verification was performed through comparison with a Monte Carlo simulation.

2. Case Study

Using the example of a cantilever beam, the application of the reanalysis formula [26] has been demonstrated in determining the zones of the construction that are most sensitive to changes. Two models are observed: original and arbitrarily modified. The condition is that the modification be small, otherwise the linearization of modification equations, presented in the previous section, would not hold. Note that during the reanalysis, instead of eigencalculations for the changed construction, the corresponding reanalysis formula can be applied, where it is necessary to calculate the *coefficients of modification* α and β, as well as *relative modification ratios* ψ and ζ. It is thus possible to considerably save calculation time, and it will be particularly demonstrated that by the line finite elements the reanalysis formula generates entirely reliable results. The type of modification is determined by the type of finite elements, type of boundary conditions, model geometry, and the like.

2.1 Deterministic input

Consider a cantilever beam of length $1\,m$, rectangular cross-section, $b \times h = 100mm \times 50mm$, divided into 5 finite elements [26]. In designations, in the tables and diagrams, this cantilever beam is referred to as the original cantilever beam. For the analysis of sensitivity to changes, the *original cantilever beam* is modified across the entire length, with small modifications[1]. That cantilever beam is called a *modified cantilever beam*. In this case, the chosen construction variable is the height of the rectangular cross-section *h*. Calculations are performed with the software package MatLab that possesses the function for calculating eigenvalues and eigenvectors. The lowest frequencies are always of the utmost interest for analysis. The table below (table 1) shows a few initial eigenvalues for the original cantilever beam and the modified one, where the height, as a construction variable, is increased by 10%.

Table 1. Few initial eigenvalues for the original cantilever beam and the modified one, where the height, as a construction variable, is increased by 10%

Original cantilever beam		Height increased by 10% across the entire length		Modified shape, I, II,III,IV,V Δh[%], Mat Lab: 8.6, +4, +0.97, -0.98, -2.59	
Frequencies, f_{0i}[Hz]	Eigenvalues, λ_i	Frequencies f_{0i}[Hz]	Eigenvalues, λ_i	Frequencies f_{0i}[Hz]	Eigenvalues, λ_i
260.24	2673654.72	286.26	3235122.21	270.70	2893010.45
41.51	68010.88	45.66	82293.17	45.82	82876.53
		$\Delta\lambda_1$ = +21 %, Δf_{01} = +9.99 %			

Perfect accuracy is noticeable in the example of the first three frequencies. This evidences that a 'small' modification has been really made and that the reanalysis formula holds.

$$\Delta\lambda_1 = \frac{\left\{\underline{Q}_1\right\}^T [\Delta K]\left\{\underline{Q}_1\right\} - \lambda_1 \left\{\underline{Q}_1\right\}^T [\Delta M]\left\{\underline{Q}_1\right\}}{\left\{\underline{Q}_1\right\}^T [M]\left\{\underline{Q}_1\right\}} = 14282.29 \ s^{-2}$$

$$\lambda_{1,\mathrm{mod}} = \lambda_1 + \Delta\lambda_1 = 68010.88 + 14282.29 = 82293.17 \ s^{-2}$$

2.2 Unceratinity quantification and simulation

2.2.1 Stochastic input

Further research included the execution of simulations *Ek, Ep*^{i}, growth rates *Ek* and *Ep*, differences in growth rates, first frequency in a cantilever beam and a modified beam for 1000 values of Young's modulus of elasticity according to the Gaussian distribution.

The figure below (Fig. 1) displays distributions of differences in potential and kinetic energy growth rates on the beam for all five finite elements and for 1000 simulation results.

The biggest difference in potential and kinetic energy growth rates was registered in the first finite element (nearest to the fixed point). In other finite elements the differences in kinetic and potential energy growth rates are decreased respectively, however on the very free end the values of kinetic energy are dominant, so that the difference in growth rate is negative.

The figure below shows the distributions of differences in potential and kinetic energy growth rates on the optimized beam (Fig. 2), for all five finite elements and for 1000 simulation results.

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[1] In the literature dealing with dynamic reanalysis it is stressed that modifications should be small, so that the chosen modification process converges to the desired eigenvalues of the pairs, however it is not easy to determine what is 'small';

It is noticeable that the first element is the most sensitive to any change because the growth rate difference declines or rises very fast. The elements located in the middle of the beam length are almost non-sensitive, which means they are not suitable for the reanalysis. In order to increase eigenfrequencies, the free-end element is sensitive, but it is needed to decrease its kinetic energy, which can be achieved by decreasing its mass (reduction of height).

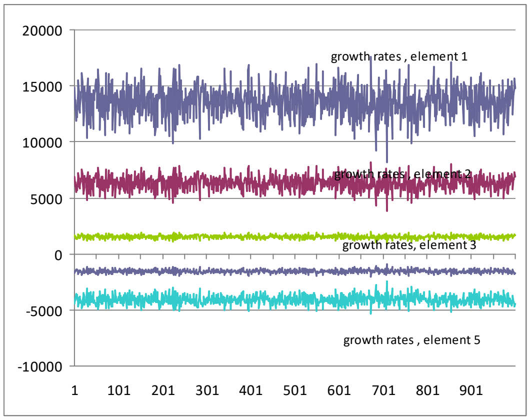

Fig. 1. Difference in potential and kinetic energy growth rates for five finite elements for 1000 simulation results for the cantilever beam

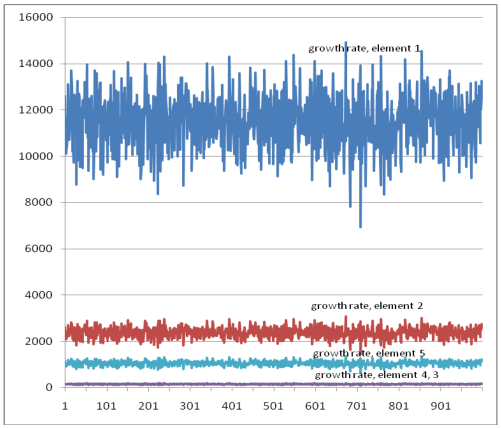

Fig. 2. Difference in potential and kinetic energy growth rates for five finite elements, for 1000 simulation results, for the optimized beam

2.2.2 Comparison of frequency sensitivity between original and modified cantilever beams

Further analysis explored the dependence between the first frequency and Ek and Ep in each element of the original beam and the modified one.

The largest change in kinetic energy was found with respect to the change in the first frequency of the fifth element, and the largest change in potential energy was found with respect to the change in the first frequency of the first element, which is in agreement with the deterministic theory. Identical regularities were established for both the original and the modified beam, however with different values.

2.2.3 Uncertainty in cantilever beam redesign calculations and frequency calculations

Table 2 shows the simulation results for the original beam frequency (frequency) and the modified beam frequency (frequency 1) for different values of Young's modulus of elasticity. Frequency growth rate was calculated as a difference between the modified beam frequency and the original beam frequency.

Tab. 2 The simulation results for the original beam frequency (frequency) and the modified beam frequency (frequency 1) for different values of Young's modulus of elasticity

No	Young's modulus	Frequency	Frequency 1	Frequency growth
1	216008194057,00	43,52331847	47,87565032	4,352331847
2	221020777903,00	40,47158724	44,51874596	4,047158724
…				
1000		43,37335183	47,71068701	4,337335183

A set of 1000 results obtained by the normal distribution (program **R**[2]) was used for further analysis where the Laplace criterion was applied to determine the uncertainty.

The Laplace criterion assumes equal probability for certain states to take place, so that probability represents

$$v\left(s_{ij}\right)=\frac{1}{m}$$

where m is the number of likely states (1000 in this case). Therefore the expected value is:

$$\overline{p_i}=\sum_{j=1}^{m} p_{ij} * v\left(s_{ij}\right)=\frac{1}{m}\sum_{j=1}^{m} p_{ij}$$

In Table 3 a division into the confidence intervals was performed and frequency of the observed quantities occurrence was calculated. The result for the occurrence of the oscillation frequency rounded growth rate indicates normal distribution.

Table 3. Confidence intervals

Frequency rounded growth rate	3,2	3,4	3,5	3,6	3,7	3,8	3,9	4	4,1	4,2	4,3	4,4	4,5	4,6	4,7
Frequency of occurrence	1	1	2	5	18	49	80	135	180	206	163	104	35	19	2

By transforming the frequencies of occurrence of eigenvalues growth rate difference according to the Laplace criterion, we will obtain normal distribution of the occurrence probability, i.e. the degrees of uncertainty of the occurrence of eigenfrequencies growth rate difference.

Tab. 4. Occurrence probability (%)

Frequency growth rate	3,2	3,5	3,7	3,8	4	4,1	4,3	4,5	4,6	4,7
Occurrence probability (%)	0,10	0,20	1,80	4,90	13,50	18,00	16,30	3,50	1,90	0,20

The above text gives the probability of eigenfrequencies differences distribution in the original and the modified beam. Given that the normal distribution of eigenfrequencies growth rate difference was obtained, it is interesting to take a look at the probability distribution of eigenfrequencies in the original and the modified beam.

3. Concluding Remarks

Studying the dynamic behavior of a construction can predict its response to change in shape, changes in size of its elements or change in materials used. Generally, the aim of system modification with respect to improvements in dynamic behavior is to increase eigenfrequencies and widen the distance between two neighboring frequencies. The specific importance lies in lowest frequencies and those close to the system exciting frequencies.

The analysis of uncertainty in the original, modified and optimized beams established for all three cases normal probability distribution in the rate of frequency occurrence. Difference was found in the interval of frequency normal distribution in the original cantilever beam compared to the distribution interval in modified and optimized beams. A broader confidence interval in modified and optimized beams indicates adverse effects of non-ideal material on the procedure of dynamic

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[2] **R** is a free software programming language and a software environment for statistical computing and graphics. The R language is widely used among statisticians and data miners for developing statistical software and data analysis. Polls and surveys of data miners are showing R's popularity has increased substantially in recent years.

modification. A versatile procedure for conducting reanalysis studies in the presence of uncertainty has been developed by. Combining Monte Carlo simulation tools with finite element modelling modules.

Acknowledgement. This research was performed within the TR 35011 and ON 74001, projects supported by Ministry of Science and Technological Development, Republic of Serbia, whose funding is gratefully acknowledged.

REFERENCES

[1] Canfield, R.A., *High-Quality Approximation of Eigenvalues in Structural Optimization*, AIAA Journal, Vol. 28, No. 6, 1990, pp. 1116-1122.

[2] Dragojlovic P., Misita M., Milanovic D.D., Tadic D., Kirin S., Risk management and multicriteria optimization of production program, Revista Metalurgia International, 2012, Vol. 17, No. 6, pp. 35-39. ISSN 1582-2214

[3] Lecomte, C., Exact statistics of szstem with uncertainties: Am analytical theorz of rank-one stohastics dynamics systems, Journal of Sound and Vibration 332 (2013), 2750-2776.

[4] Lin YK. Probabilistic thoery of strcutural dynamics. NY, USA: McGraw-Hill Inc; 1967.

[5] Mateus, H.C., Mota Soares, C.M., and Mota Soares, C.A., *Sensitivity Analysis and Optimal Design of Thin Laminated Composite Structures*, Computers and Structures, Vol. 41, No. 3, 1991, pp. 501-508.

[6] Milanović D.D., Misita M, Decision support and management support systems, FME, Belgrade, 2008. ISBN 978-86-7083-642-6.

[7] Pascual, B. Adhikari S., Combined parametric–nonparametric uncertainty quantification using random matrix theory and polynomial chaos expansion, Computers and Structures 112–113 (2012) 364–379.

[8] Pierfrancesco Cacciola, Nicola Impollonia, Giuseppe Muscolino, A dynamic reanalysis technique for general structural modifications under deterministic or stochastic input, Computers and Structures 83 (2005) pp. 1076–1085.

[9] Rao, S.S., and Reddy, C.P., *Optimum Design of Stiffened Conical Shells with Natural Frequency Constraints*, Computers and Structures, Vol. 14, Nos.1-2, 1981, pp 103-110. Conference, 2001.

[10] Rao, S.S., and Reddy, C.P., *Optimum Design of Stiffened Cylindrical Shells with Natural Frequency Constraints*, Computers and Structures, Vol. 12, Aug. 1980, pp 211-219.

[11] Sergeyev, O., and Mroz., Z., *Sensitivity Analysis and Optimal Design of 3D Frame Structures for Stress and Frequency Constraints, Computers and Structures*, Vol. 75, No. 2, 2000. pp. 167-185

[12] Stanojevic Petar Orlic Branislav Misita Mirjana Z Tatalovic Nada Lenkey Gyoengyver Online monitoring and assessment of emerging risk in conventional industrial plants: possible way to implement integrated risk management approach and KPI's (Article J, JOURNAL OF RISK RESEARCH, (2013), vol. 16 br. 3-4, str. 501-512)

[13] Tadic D., Djapan M., Misita M., Stefanovic M., Milanovic D.D. A Fuzzy Model for Assessing Risk of Occupational Safety in Processing Industry, The International Journal of Occupational Safety and Ergonomics, 2012, Vol.18, No.2, 115-126.

[14] Tadic D., Milanovic D.D, Misita M., Tadic B., A new integrated approach to the problem of ranking and supplier selection under uncertainties, Proceedings of the Institution of Mechanical Engineers, Part B, Journal of Engineering Manufacture, London, UK, 2011. Vol. 225, No. B9, oo. 1713-1724, ISSN 0954-4089.

[15] Trišović, N., Maneski, T., Kozak, D. „Developed procedure for dynamic reanalysis of structures“, Strojarstvo - Journal for Theory and Application in Mechanical Engineering, Vol. 52, Number. 2, pp: 147-158, 2010; ISSN 0562-1887.

[16] Trisovic, R., N., Modification of the Dynamics Characteristics in the Structural Dynamic Reanalysis, PhD, Thessis, University of Belgrade, Serbia, 2007

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[i] Kinetic and potential energy