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Deformation and Stress Fields in Thin Metallic Partially Fixed Plate Induced by Electromagnet with Constant Flux Obtained by Analytical Method and FEM

Vesna Milošević Mitić*, Taško Maneski*, Nina Anđelić*

** Faculty of Mechanical Engineering, University of Belgrade*
Kraljice Marije 16, 11120 Belgrade 35

e-mail correspondence: vmilosevic@mas.bg.ac.yu

Abstract: In the paper the behavior of thin elastic metallic partially fixed plate produced by electromagnet with constant flux is considered. Plate is simply supported along three edges and fixed along the fourth one. As nonstationary electromagnetic field in one part of the plate varies with time, conducting currents appear. Distribution of the power of eddy-current losses is obtained by using complex calculation. The power of the volume heat source (Joule's heat) depends of permeability and conductivity of the plate and frequency of the alternative-current in electromagnet. Characteristics of the electromagnet are described by using Pulsation, Heaviside and similar functions. Differential equation governing temperature field is solved using by integral transform technique. The mathematical form of the transversal vibrations is obtained in analytical way using only simple finite Fourier transforms because the differential equation describing transverse vibrations is adapted to form which enabling very easy simulation of the bending moments along the fixed edges. Vibrations and stress in the middle surface of the plate are calculated by using program package KOMIPS, based on the finite element method.

Key words: electromagnet, temperature field, stress, deformation, finite element method

1. Introduction

Vibrations of the thin metallic plates from soft ferromagnetic materials are described using four coupled systems of differential equations. The first system is a system of Maxwell's equations (with the relations for slowly moving media and modified Ohm's low) [5]:

$$\mathrm{rot}\,\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \qquad \mathrm{rot}\,\mathbf{K} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad div\,\mathbf{D} = 0, \qquad div\,\mathbf{B} = 0. \tag{1.1}$$

where the following notation is applied: **H**-magnetic intensity vector, **K**-electric intensity vector, **B**-vector of magnetic induction, **D**- vector of electric induction and **J**-current density vector.

Temperature field in thin plates can be described using two values: τ_0 - temperature in the middle surface and τ_1 – the rate of temperature across the plate thickness [7]. So, second system of equations is consisted of two partial differential equations (1.2), where κ is coefficient of thermal intensity, η^* is representing the coupling between the temperature and the deformation fields, ε' is deformation in the middle surface of the plate, h is the plate thickness, λ_0 is heat conduction coefficient, σ is electric conductivity and ∇_1^2 is Laplace operator [2]:

$$\left(\nabla_1^2 - \beta_k - \frac{1}{\kappa}\frac{\partial}{\partial t}\right)\tau_k + \frac{\beta_k^k}{h}\left[x_3^k \frac{\partial \theta}{\partial x_3}\right]_{-\frac{h}{2}}^{\frac{h}{2}} - \eta^*\varepsilon_k = -\frac{\beta_k^k \int_{-\frac{h}{2}}^{\frac{h}{2}} W(x_1,x_2,x_3,t)\,x_3^k\,dx_3}{h\lambda_0},$$

$$\beta_k = \begin{cases} 0, & k=0 \\ \dfrac{12}{h^3}, & k=1 \end{cases}, \qquad \varepsilon_k = \begin{cases} \varepsilon', & k=0 \\ \nabla_1^2\left(\partial_t^2 w\right), & k=1 \end{cases}, \qquad W = W_S + W_H + \frac{J^2}{\sigma}, \qquad (k=0,1). \tag{1.2}$$

Losses in a plate $W(x_1, x_2, x_3, t)$ are consisted of three factors: volume heat source intensity, hysterisis losses and Joule's heat (eddy-current losses).

In the consideration of the vibrations of the plate, the assumption that the longitudinal vibrations are independent of the transverse vibrations is taken. Transverse vibrations can be obtained using by next differential equation [2]

$$D\nabla_1^4 w + D(1+\nu)\alpha_t \nabla_1^2 \tau_1 + \rho h(\partial_t^2 w) - \frac{\rho h^3}{12}\nabla_1^2(\partial_t^2 w) = (\sigma_{33}^+ - \sigma_{33}^-) + (T_{33}^+ - T_{33}^-) + \tag{1.3}$$
$$+\frac{h}{2}\frac{\partial}{\partial x_i}(\sigma_{i3}^+ + \sigma_{i3}^-) + \frac{h}{2}\frac{\partial}{\partial x_i}(T_{i3}^+ + T_{i3}^-) + \int_{-\frac{h}{2}}^{\frac{h}{2}}(X_{i,i} + f_{i,i})x_3 dx_3 + \int_{-\frac{h}{2}}^{\frac{h}{2}} X_3 dx_3 \,,$$

where w denotes deflection of the plate in x_3-direction, ν is Poisson ratio, α_t is coefficient of thermal expansion, D is flexural rigidity of the plate, E is modulus of elasticity, X_i and f_i are the components of mechanical force and Lorenz force, σ_{ij} and T_{ij} are mechanical and magnetic stress tensors (+ on the upper and – on the lower side of the plate) and ρ is plate density.

Vibrations in the middle surface of the plate are defined by the equation

$$\Delta_2^2\left(\Delta_1^2 F + Eh\alpha_t \tau_0\right) = 0\,, \tag{1.4}$$
$$\Delta_i^2 \equiv \nabla_1^2 - \frac{1}{C_i^2}\partial_t^2 \qquad (i=1,2), \qquad C_1^2 = \frac{D_z}{\rho h}\,, \quad C_2^2 = \frac{E}{2\rho(1+\nu)}\,,$$

where F is Airy-s Stress function. Forces N_{ij} can be expressed as [7]

$$N_{ij} = -F_{,ij} + \delta_{ij}\left(\nabla_1^2 - \frac{1}{2C_2^2}\partial_t^2\right)F \qquad (i,j=1,2).$$

Presented system of equations has to be added with an appropriate boundary and initial conditions.

2. Electromagnet with constant flux

As a result of time-changing magnetic induction, in one part of the plate ($d\times c\times h$) conducting currents appear. The calculation of the Joule's heat intensity is done by using local coordinate system (x,y,z) with next approximations (Fig. 2.1): component of the magnetic induction B_y is zero; component B_z is minor compared to the component B_x. So, H_x and K_y components can describe electromagnetic field in the plate, which are simplified Maxwell's equations (1.1). As the changes in x and y directions are minor, differential equation is [6]

$$\frac{\partial^2 \underline{H}_x}{\partial z^2} - \gamma\frac{\partial \underline{H}_x}{\partial t} = 0, \quad \gamma^2 = j\sigma\mu\omega, \quad \gamma = (1+j)\sqrt{\frac{\sigma\mu\omega}{2}} = (1+j)k\,, \tag{2.1}$$

where μ is magnetic permeability of the plate material and ω is angular frequency of the magnetic field.

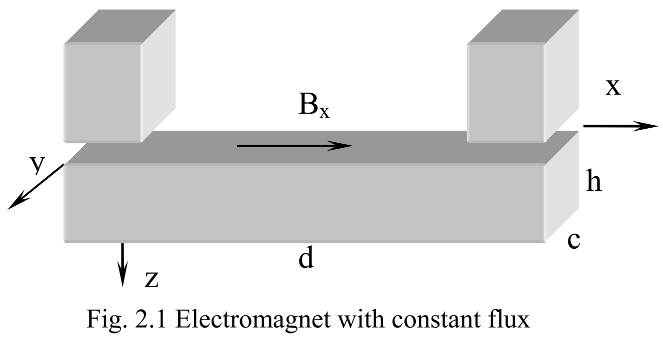

Fig. 2.1 Electromagnet with constant flux

The solution can be represented in the form $\underline{H}_x = \underline{C}_1 e^{\gamma z} + \underline{C}_2 e^{-\gamma z}$, where coefficients $\underline{C}_1, \underline{C}_2$ can be obtained from the boundary conditions ($\underline{H}_0 = \underline{H}_x(z=0)$) and they are

$$\underline{C}_1 = -\frac{H_0 e^{-\gamma h}}{2Sh\gamma h}, \qquad \underline{C}_2 = \frac{H_0 e^{\gamma h}}{2Sh\gamma h}. \tag{2.2}$$

Conducted currents and Joule's heat are defined by [2]

$$\underline{J}_x = \frac{\partial \underline{H}_x}{\partial z} = -\underline{H}_0 \gamma \frac{Ch\gamma(z-h)}{Sh\gamma h}, \tag{2.3}$$

$$W = \frac{1}{2}\frac{\left|J^2\right|}{\sigma} = \frac{k^2 H_0^2}{\sigma}\frac{Ch2k(z-h)+\cos 2k(z-h)}{Ch2kh - \cos 2kh}. \tag{2.4}$$

3. Temperature field

Let the initial and the boundary conditions are

$$\theta\big|_{t=0} = 0, \quad \theta\big|_{x_1=0,a} = 0, \quad \theta\big|_{x_2=0,b} = 0, \quad \frac{\partial\theta}{\partial x_3}\bigg|_{x_3=\pm\frac{h}{2}} = 0. \tag{3.1}$$

Appropriate values of the power of the heat source from equation (1.2) are

$$W_0^* = \int_0^h W(z)dz = \frac{kH_0^2}{2\sigma}\frac{Sh2kh + \sin 2kh}{Ch2kh - \cos 2kh}, \tag{3.2a}$$

$$W_1^* = \int_0^h W(z)\left(\frac{h}{2} - z\right)dz = \frac{hkH_0^2}{4\sigma}\left(\frac{Sh2kh + \sin 2kh}{Ch2kh - \cos 2kh} - \frac{1}{kh}\right). \tag{3.2b}$$

The position of electromagnet is presented on figure 3.1 and described by the relation $\Pi\left(\frac{x_1 - e}{c}\right)\Pi\left(\frac{x_2 - f}{d}\right)$.

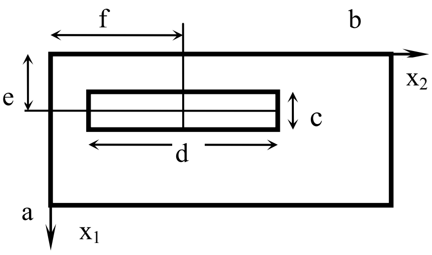

Fig. 3.1 Position of electromagnet

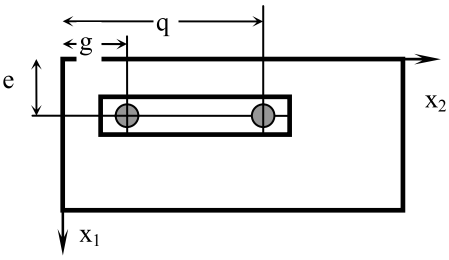

Fig 3.2 Position of mechanical forces

Differential equations describing temperature field have the next form

$$\left(\nabla_1^2 - \beta_k - \frac{1}{\kappa}\partial_t\right)\tau_k = -\frac{\beta_k^k}{\lambda_0 h} W_k \Pi\left(\frac{x_1 - e}{c}\right)\Pi\left(\frac{x_2 - f}{d}\right)H(t), \quad \beta_k = \begin{cases} 0, & k = 0 \\ \frac{12}{h^3}, & k = 1 \end{cases}, \quad (k=0,1). \tag{3.3}$$

Subjected to the boundary conditions (3.1) the equation (3.3) can be solved by using the integral-transform technique. Applying double Fourier finite-sine transform and Laplace transform we arrive to the next solution

$$\tau_k = \frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{4\beta_k^k W_k}{\lambda_0 h}\frac{\sin\alpha_n e \sin\alpha_m f}{\beta_k + \Delta_{mn}}\frac{\sin\alpha_n \frac{c}{2}}{\alpha_n}\frac{\sin\alpha_m \frac{d}{2}}{\alpha_m}\left[1 - e^{-\kappa(\beta_k + \Delta_{mn})t}\right]H(t)\sin\alpha_n x_1 \sin\alpha_m x_2 \tag{3.4}$$

$$\Delta_{mn} = \alpha_n^2 + \alpha_m^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 .$$

Numerical example is given for the steel rectangular plate dimension 45×24×0.1cm; position of electromagnet is defined with parameters: e=15cm, f=7.5cm, c=6cm, d=9cm, S=18cm^2 (Figure 4.1). Material constants are: λ_o=0.5W/cmK, $\sigma=10^7$S/m and μ_r=1000. Electromagnetic field parameters in the plate are: H_0=2000A/m (B_0=2.5T) and f=100Hz.

On diagram in Figure 3.3 the maximal value of the temperature τ_0 in the middle surface of the plate is presented as a function of time *t*. Figure 3.4 shows distribution of the isotherm lines in the middle surface for the stationary state.

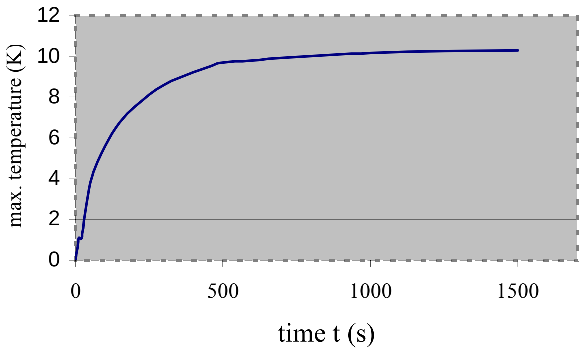

Fig. 3.3. Maximal temperature in the plate τ_0 as a function of time *t*

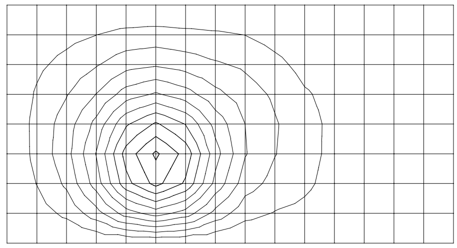

Fig. 3.4. Isotherm lines in the middle surface of the plate, τ_0 = 0÷10 (step 1°C)

4. Vibrations. Stress field

4.1 Transversal vibrations

The solution of the transversal vibrations is founded in analytical form for the plate simply supported along three edges (x_1=a, x_2=0,b) and fixed along the fourth one (x_1=0) (Fig. 4.1). Boundary conditions have the form [4]

$$w\big|_{x_1=0,a} = 0\,,\ \frac{\partial w}{\partial x_1}\bigg|_{x_1=0} = 0\,,\ M_{11}\big|_{x_1=a} = \left[\frac{\partial^2 w}{\partial x_1^2} + \nu\frac{\partial^2 w}{\partial x_2^2} + (1+\nu)\alpha_t\tau_1\right]D\bigg|_{x_1=a} = 0\,,$$

$$w\big|_{x_2=0,b} = 0\,,\ M_{22}\big|_{x_2=0,b} = \left[\nu\frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2} + (1+\nu)\alpha_t\tau_1\right]D\bigg|_{x_2=0,b} = 0\,. \tag{4.1}$$

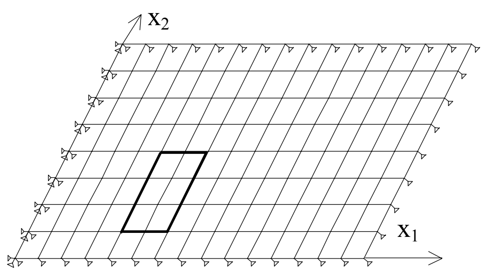

Fig. 4.1 Boundary conditions

Initial conditions are responsible to the natural undeformed state

$$w\big|_{t=0} = 0\ ,\quad \frac{\partial w}{\partial t}\bigg|_{t=0} = 0\,. \tag{4.2}$$

The position of the appropriate mechanical forces is shown on figure 3.2. If B_v and H_v are magnetic induction and magnetic field in air gaps, p_0 Maxwell stress and S surface of one air gap, mechanical forces can be represented (using Impulsive Dirac functions δ) by the following relations

$$p_0 = \frac{1}{2} H_0 B_0 \quad , \qquad p_0 S \delta(x_1 - e)[\delta(x_2 - g) + \delta(x_2 - q)] \ . \tag{4.3}$$

Using differential equation (1.3) we can form the appropriate equation for the stationary problem ($t \to \infty$) by simulated moment along the edge $x_1=0$ throw the stress σ_{13} on the next way

$$\nabla_1^4 w + (1+\nu)\alpha_t \nabla_1^2 \tau_1 = \frac{h}{D}\frac{\partial}{\partial x_1}\left[\sigma_{13}(x_2)\delta(x_1)\right] + \frac{p_0 S}{D}\delta(x_1 - e)[\delta(x_2 - g) + \delta(x_2 - q)]. \tag{4.4}$$

By using relation (3.4), double Fourier finite-sine transform and the relation for the first derivative of δ function

$$\int_0^a \frac{\partial \delta(x_1)}{\partial x_1} \sin(\alpha_n x_1) dx_1 = -\alpha_n \ ,$$

the solution for transversal vibrations can be represented in the form

$$w(x_1, x_2, t) = \frac{4}{ab} \sum_{m=1,3,...}^{\infty} \sum_{n=1}^{\infty} w_{nm} \sin \alpha_n x_1 \sin \alpha_m x_2 \ , \tag{4.5}$$

$$w_{nm} = \frac{\alpha_t (1+\nu)\tau_{1nm}}{\Delta_{mn}} - \frac{h}{D}\frac{\alpha_n}{\Delta_{mn}^2}\sigma_{13m} + \frac{p_0 S}{D}\frac{\sin \alpha_n e}{\Delta_{mn}^2}(\sin \alpha_m g + \sin \alpha_m q).$$

Using the boundary condition for the edge $x_1=0$ we can calculate the "moment stress" $\sigma_{13}(x_2)$ as

$$\sigma_{13m} = \frac{\sum_{n=1}^{\infty} \frac{\alpha_t (1+\nu)\tau_{1nm}\alpha_n}{\Delta_{mn}} + \frac{p_0 S}{D}\frac{\alpha_n \sin \alpha_n e}{\Delta_{mn}^2}(\sin \alpha_m g + \sin \alpha_m q)}{\frac{h}{D}\sum_{n=1}^{\infty}\frac{\alpha_n^2}{\Delta_{mn}^2}} \ ,$$

$$\sigma_{13}(x_2) = \frac{2}{b} \sum_{m=1,3,...}^{\infty} \sigma_{13m} \sin \alpha_m x_2 \ . \tag{4.6}$$

The solution is consisted of two parts: the influence of the rate of temperature across the plate thickness and the influence of the mechanical forces.

4.2 Deformation in the middle surface

Vibrations and stress in the middle surface of the plate can be obtained in analytical form by using differential equation (1.4) and appropriate boundary conditions. In this paper it is done for presented numerical example by using finite element method, Program package KOMIPS [1] author T. Maneski. Input file was done on the analytical solution for the temperature in the middle surface of the plate (3.4).

4.3 Numerical example

For numerical values presented in section 3 of this paper, the solutions for the deformation and stress fields are presented on figures 4.2 to 4.5. Total deformation is presented on figure 4.2. Maximal calculated deformation is 0.281mm.

Figures 4.3, 4.4 and 4.5 show appropriate stress field. Maximal temperature in the plate for the stationary state is θ = 10.3°C and maximal stress value in the total case of loading, calculated by finite element method, is 1.354 kN/cm^2.

Table 4.1 Maximal deformation and maximal stress

	Loading		
	Temperature	Mechanical	Total
Maximal deformation f_{max} [mm]	0.014	0.280	0.281
Maximal stress σ_{max} [kN/cm^2]	1.242	0.693	1.354

For the presented numerical example temperature gradient across the plate thickness can be neglected. Analytical results for the transversal vibrations agree with the numerical results.

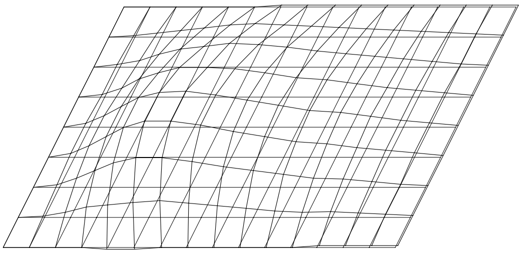

Fig. 4.2. Total deformation

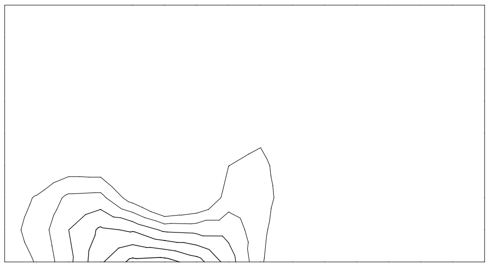

Fig. 4.3. Total Stress field, $\sigma = 0.7 \div 1.3/0.1\ kN/cm^2$

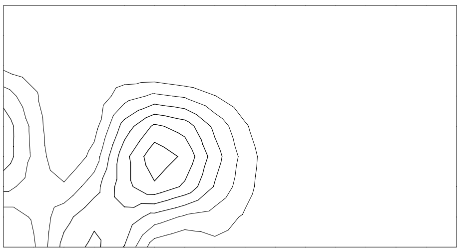

Fig. 4.4. Mechanical loading. Stress field
$\sigma = 0.35 \div 0.65/0.05\ kN/cm^2$

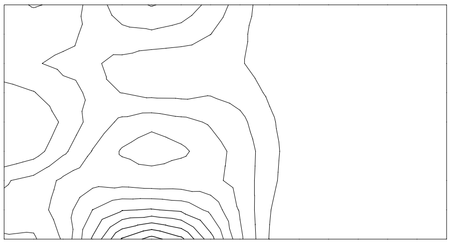

Fig. 4.5. Temperature loading. Stress field
$\sigma = 0.3 \div 1.2/0.1\ kN/cm^2$

5. Conclusion

Magneto-thermoelasticity has received considerable attention because of the possible applications in detection of flaws in ferrous metals, optical acoustics, levitation by superconductors, magnetic fusion and many other electro-mechanical devices.

The problem of the influence of electromagnet with constant flux on the behaviour of thin metallic plate can be described in analytical form throw four systems of differential equation. In that case, the influence on the stress field has two factors: increasing the temperature (appearance of volume heat source in one part of the plate) and mechanical forces (Maxwell stress) placed in air gaps surfaces.

Very suitable method for solving temperature field and transversal vibrations in analytical form, as it was shown in the paper, is integral transform technique. But for membrane case of loading, dynamic and geometrical complicated problems with non homogeneous boundary conditions it is very difficult to find vibrations and stress in analytical form. So, finite element method has been involved in calculation.

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