INFLUENCE OF NONSTATIONARITY CAUSED BY THE INITIAL CONDITIONS OF THE SYSTEM TO ITS CHARACTERISTICS

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INTRODUCTION

Nonstationarity within Markov model of queuing theory can be the consequence of:

1. Convenient initial conditions through system analysis

2. Application of variability of intensity of unit arrival and/or variability of intensity of unit serving within the system. Initial conditions have, in certain cases, very special importance to this nonstationary model, which is the subject that will be discussed later.

In the literature (for example Newell (1971) [4], Cooper (1981) [1]), presence of the nonstationarity caused by the initial conditions is often stated and the analytical solution of changes of condition probabilities in the due of time for the most simple model for birth-and-death process (M/M/1) is given as well, while the solutions for more complicated models have been stated rarely, because of complexity of the mathematical apparatus and obtained solutions. The example of the analytical solution for variability of condition probability in the due of time for the model M/M/1/∞ can be found with Kleinrock (1975) [3], and the model with accumulated units with Vukadinović (1988) [6].

The influence of the nonstationarity caused by variable intensities of unit arrivals, i.e. intensity of serving, to the system behaviour can be found in papers written by Green, Kolesar and Svoronos (1990) [2], as well as by Zrnić and Bugarić (1993) [7].

The initial conditions during the analysis of models of the queuing theory are limited to setting up of certain initial values to the corresponding condition-probabilities. Regarding that, following conditions can be defined:

1. Defined (one) probability of the system condition has initial value 1, while all other probabilities have value 0. In practice, during the system analysis, it means that if, at the beginning, probability of zero condition has value 1, and all other condition probabilities value 0, the system, at the starting point, is empty. If there is a possibility that, at the beginning of the analysis of system probabilities, system could be in 1 condition (*equal to one*), and all other probabilities are equal to zero, that means that there is *one unit* at the starting point, and that system begins to work when the first *unit* comes to it. Generally speaking, when at the beginning of the analysis, the probability of condition n of the system equals to one, and all other probabilities equal to zero, there will be n unit at the starting point.

2. Two or more condition probabilities (in general case all condition probabilities) has the appropriate values at the beginning. The special case originates when the initial values of the system probabilities are equal to the stationary probabilities of the system condition. In that case, this form of the nonstationarity disappears, i.e. from the very beginning of the analysis the system works in the stationary regime.

Nonstationarity caused by the different initial conditions lasts for a certain period of time, and then disappears when the system enters the stationary working regime. In theory, the system enters stationary working regime when working time tends to eternity. However, for the practical application of Markov models of queuing theory (precise enough for engineering calculations), the system enters stationary working regime after determinate final time.

In this paper, the criterion for determination of the duration time of the nonstationarity caused by initial conditions during the system analysis, i.e. interval of time after which the system enters stationary working regime, as well as the influence of this form of nonstationarity to the characteristics of the system, will be shown.

Change of time intensity of the unit arrival to the system, i.e. intensity of unit serving in general case can be shown as a continual function of the time or as a gradual function through convenient histogram. From the aspect of the influence of the initial conditions to the achievement of the stationary working order of the system, it is interesting to discuss graphic survey (presented by the bars) of the changes of intensity of arrival, i.e. serving of the units. By the transition from one time interval where stands determinate intensity of arrival or serving, all probabilities of condition have certain values, which represent initial conditions for the following time interval, where stands some other intensity of serving or arrival of the units. The suggested criterion for the determination of duration time of nonstationarity caused by initial conditions, in this case can be used for determination whether the system in the interval in which stands the given constant intensity would enter the stationary working order or not.

The determination of the time period of duration of nonstationarity caused by initial conditions would be shown on multiserver model of the queuing theory with complete help between the servers, batch arrival of the units to the

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system, limited waiting queue and cancellation regarding the whole group. The above mentioned model of the queuing theory has originated as the synthesis of the following model of the queuing theory:

- multiserver system with batch arrival of the units to the system and infinite waiting queue (see Kleinrock (1975) [3], Cooper (1981) [1]).

- multiserver system with complete help between the servers and finite waiting queue (see Vukadinović (1988) [6]).

The shown model of the queuing theory, with slight changes, has been used for project of a river terminal for the bulk cargo unloading, made for the Metallurgical complex in Republic of Serbia.

Frequent stops, i.e. pauses during working period of the terminal (impossibility of working of crane - server) throughout the winter caused by bad weather conditions (rain, ice, snow), as well as frequent discharges of the system because of bad scheduling for arrival of composition of barges (batch arrival), has originated the need for analysis of the nonstationarity caused by initial conditions of the system (Zrnić (1986) [8]).

Our findings are:

1) Suggestion of criterion for determination of duration time of the nonstationary regime of the system caused by determinate initial conditions;

2) Determination of influence of the nonstationarity caused by initial conditions to the characteristics of the system, as well as determination of the time interval of the continuous work of the system after which disappears the influence of this form of the nonstationarity to the characteristics of the system.

3) Presentation of the new model of queuing theory, in which the novelty is in cancellation for the whole group of units, if the whole group can not step in the waiting queue.

1. METHODOLOGY

1.1. Determination of duration time of the nonstationarity caused by initial conditions

Physicality of the suggested criterion for determination of duration time of nonstationarity caused by initial conditions, because of possibility of obtaining of relatively simple analytical expressions for changes of condition probabilities in the unit of time, will be shown on the most simple model of birth-and-death process, whose graph of the condition is shown on figure 1.

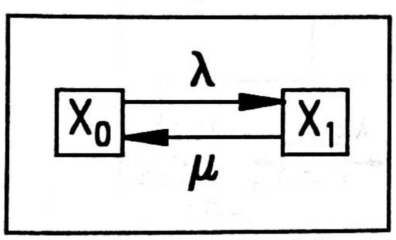

Figure 1 M/M/1 model

Differential equations that describe condition changes of this system has the following form (Newell (1971) [4], Cooper (1981) [1]): (1)

$$p_0'(t) = -\lambda \cdot p_0(t) + \mu \cdot p_1(t); \quad p_1'(t) = \lambda \cdot p_0(t) - \mu \cdot p_1(t);$$

where: λ -arrival rate, μ -service rate, $p_i(t)$ -probability of i-units in the system at time t (i=0,1).

Since, in general case, the probabilities of the system conditions are exponential functions of time $p_i(t)=f(t, A\cdot e^{-t})$ (where: A=const, i=0,1), at the moment when $t\to\infty$ and when λ=const and μ=const, system of the differential equations will transit to the system of algebraic equations (because $p_i'(t)\to 0$, when $t\to\infty$), i.e. the system will transit to the stationary working regime. The algebraic equations have the following form: (2)

$$0 = -\lambda \cdot p_0 + \mu \cdot p_1; \quad 0 = \lambda \cdot p_0 - \mu \cdot p_1;$$

By the solution of system of the differential equations, with the convenient initial conditions, changes of condition probabilities in the time unit are obtained, while by the solution of algebraic equations are obtained the probabilities of system conditions in the stationary working regime. As was stated before, after long enough period of work, the system transits from the nonstationary to the stationary working regime. The change of condition probabilities during the time is asymptotic. The condition probabilities, obtained by solution of the system of the differential equations (1) during the time, are asymptotically coming close to the stationary values of the probabilities obtained by solution of the system of algebraic equations (2).

The exact criterion for determination of moment of the transition of the system from the nonstationary to the stationary working regime does not exist. It is possible only to determine approximate time interval in which system transits from the nonstationary to the stationary working regime. The approximate time moment, up from which we can expect that the system is in the stationary working regime, precise enough for engineering calculations, can be obtained on the basis of difference of the condition probabilities in nonstationary and stationary working regime. The moment when that difference is less then some other given value can be taken for the moment of entering of the system to the stationary working regime.

The suggested criterion is based on the relative error of probability differences. The moment when the absolute values of the relative error of the all probability differences are less then for example 5% is the moment of the entering of the system to the stationary working regime, and the time interval from the beginning of the work of the system until that moment represents duration time of the nonstationarity caused by initial conditions during the system analysis. The criterion has the following form: (3)

$$\delta_i = \frac{|p_i(t) - p_i|}{p_i} \cdot 100 \le 5\%; \quad \text{za } \forall \;\; i \in (0,1)$$

where: $p_i(t)$ system condition probability at time t obtained by solution of system of differential equations (1), p_i i-probability of stationary system condition obtained by solution of system of the algebraic equations (2).

Solution of the system (1), depending to the initial conditions have the following form:

$$p_0(t) = \frac{1}{1+\rho} + \frac{\rho \cdot p_0(0) - p_1(0)}{1+\rho} \cdot e^{-(\lambda+\mu)\cdot t};$$

$$p_1(t) = \frac{\rho}{1+\rho} - \frac{\rho \cdot p_0(0) - p_1(0)}{1+\rho} \cdot e^{-(\lambda+\mu)\cdot t};$$

while the probabilities of the stationary working order have the following form:

$$p_0 = \frac{1}{1+\rho}; \; p_1 = \frac{\rho}{1+\rho};$$

where $\rho=\lambda/\mu$ is offered load.

If we change the expression that represents the solution of the system (1) and (2) to the expression (3), the times of continuous work of the system are obtained. After those times the relative error of the given probability at that moment and in the stationary working regime will be less then demanded criterion limits (δ_i).

$$t_0 = \ln\left[\frac{|\rho \cdot p_0(0) - p_1(0)|}{\delta_0}\right]^{\frac{1}{\lambda+\mu}}$$

$$t_1 = \ln\left[\frac{|p_0(0) - \frac{1}{\rho}\cdot p_1(0)|}{\delta_1}\right]^{\frac{1}{\lambda+\mu}}$$

while the time of reaching the stationary regime is obtained as:

$$t_{stat} = \max\{t_1, t_2\}$$

The approximate moment of reaching the stationary regime of the system order depends on limits of the suggested criterion. As the limits of the criterion are narrower, period needed for reaching the stationary regime is longer.

Duration times of the nonstationary regime caused by initial conditions, by the application of the suggested criterion, will be determinate for the multiserver system with batch arrival of the units, complete help between the servers, limited waiting queue and cancellation of the whole group, which are the subject that will be discussed in the next chapter.

1.2. Multiserver model with batch arrival of units to the system, complete help between the servers, limited waiting queue and cancellation of the whole group

When there exists only one unit within the system, it would be served by all servers (if it is possible physically), when there are two units, one half of the servers would serve the first, and the other half the second unit, etc. That way, the servers would be equally disposed for the unit serving,

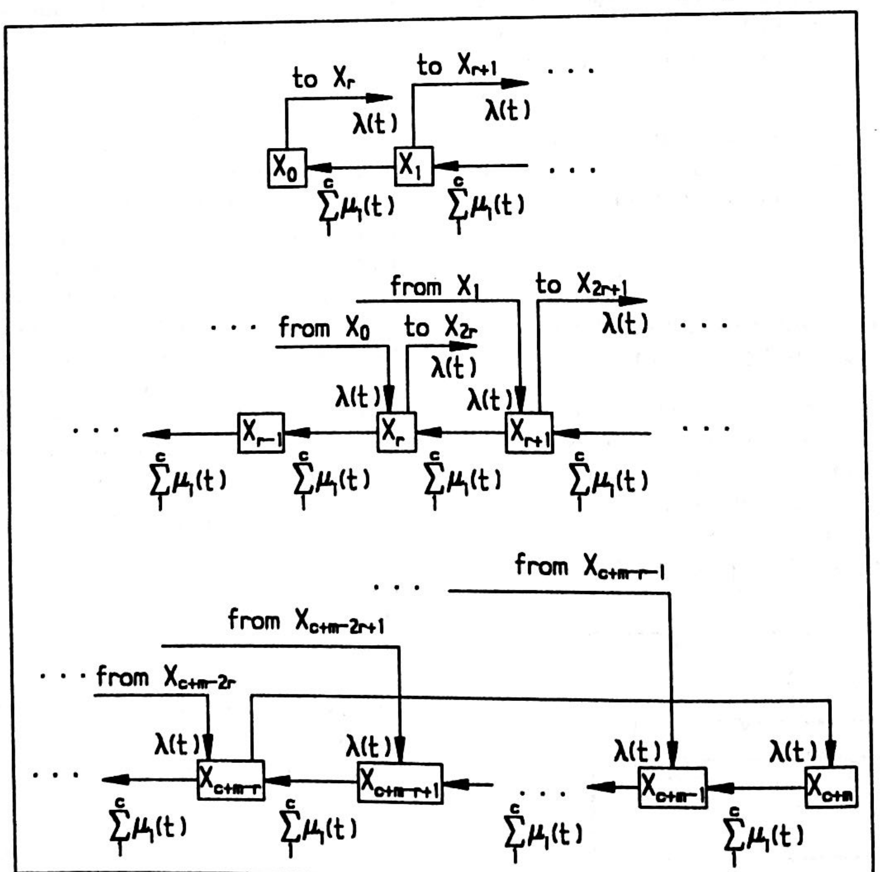

Figure 2. General graph of system condition

until the moment when the number of units does not reach number of the servers, when each server would serve one unit.

In the case that group of units finds all the servers free, it would be accepted into the system on the following way: if the number of servers is higher or equal to the number of units, the whole group would be accepted immediately for serving, while the number of units in the group is higher then the number of servers, as many unit as there are servers would be accepted, while all other units would stand into the waiting queue. In the case when all servers are busy, and the number of vacant places in the waiting queue higher or equal to the number of units in the group, the whole group should be placed into the waiting queue. In the case when all servers are busy and the number of vacant places in the waiting queue less then the number of units in the group, the whole group would be cancelled. The condition of the system should be determinate on the basis of the number of the units within the system. The general condition graph is shown on figure 2.

System of the differential equations which describes change of system condition probabilities in the time has the following form: (4)

$$p_0'(t) = -\lambda(t)\cdot p_0(t) + \left[\sum_{i=1}^{c}\mu_i(t)\right]\cdot p_1(t);$$

$$p_k'(t) = -\left[\lambda(t) + \sum_{i=1}^{c}\mu_i(t)\right]\cdot p_k(t) + \left[\sum_{i=1}^{c}\mu_i(t)\right]\cdot p_{k+1}(t);$$

for k=1,2,...,(r-1),

$$p_k'(t) = -\left[\lambda(t) + \sum_{i=1}^{c}\mu_i(t)\right]\cdot p_k(t) + \left[\sum_{i=1}^{c}\mu_i(t)\right]\cdot p_{k+1}(t) + \lambda(t)\cdot p_{k-r}(t);$$

for k=r, r+1,..., c+m-r,

$$p_k'(t) = -\left[\sum_{i=1}^{c}\mu_i(t)\right]\cdot p_k(t) + \left[\sum_{i=1}^{c}\mu_i(t)\right]\cdot p_{k+1}(t) + \lambda(t)\cdot p_{k-r}(t);$$

for k=c+m-r+1,..., c+m-1,

$$p_{c+m}'(t) = -\left[\sum_{i=1}^{c}\mu_i(t)\right]\cdot p_{c+m}(t) + \lambda(t)\cdot p_{c+m-r}(t);$$

where c is number of servers, r number of units in a group, m number of places in waiting queue, $\lambda(t)$ -arrival rate of group at time t, $\mu_i(t)$ -service rate at time t for server i, $p_i(t)$ probability that the i units are in the system at time t.

As was said before, when $t\to\infty$ and when $\mu_i(t)$=const, (i=1,2,...,c) and $\lambda(t)$=const, system of the differential equations (4) will transit to system of the algebraic equations (5), i.e. the system will transit to the stationary working regime. (5)

$$0 = -\lambda\cdot p_0 + c\cdot\mu\cdot p_1;$$

$$0 = -(\lambda + c\cdot\mu)\cdot p_k + c\cdot\mu\cdot p_{k+1};$$

for k=1, 2, ..., (r-1),

$$0 = -(\lambda + c\cdot\mu)\cdot p_k + c\cdot\mu\cdot p_{k+1} + \lambda\cdot p_{k-r};$$

for k=r, r+1, ..., c+m-r,

$$0 = -c\cdot\mu\cdot p_k + c\cdot\mu\cdot p_{k+1} + \lambda\cdot p_{k-r};$$

for k=c+m-r+1, ..., c+m-1,

$$0 = -c\cdot\mu\cdot p_{c+m} + \lambda\cdot p_{c+m-r};$$

By such defined model, offered load by server is defined as:

$$\rho_c(t) = \frac{r\cdot\lambda(t)}{c\cdot\mu(t)}.$$

Characteristics of the system are shown in the table 1.

Table 1: Characteristics of the system

Name	Value
Probability of serving	$P_{ser}(t) = \sum_{k=0}^{c+m-r} p_k(t)$
Average number of units in waiting queue	$N_w(t) = \sum_{k=1}^{m} k\cdot p_{c+k}(t)$
Average time that unit spends in waiting queue	$t_w(t) = \frac{N_w(t)}{r\cdot\lambda(t)}$
Average number of units in system	$N_{ws}(t) = \sum_{k=1}^{c+m} k\cdot p_k(t)$
Average time that unit spends in system	$t_{ws}(t) = \frac{N_{ws}(t)}{r\cdot\lambda(t)}$

1.3. Experimental strategy

Since the given model, as was emphasised before, has been developed for analysis of the concrete, real system (bulk cargo terminal), for the reasons of simplicity, the analysis of duration time of the nonstationarity caused by initial conditions, as well as its influence to characteristics of the system, will be conducted for the real condition of the unloading terminal Zrnić (1986) [8].

Duration time of the nonstationary regime was defined for the following parameters of the system;

- number of servers (cranes) c=1,2 and 3 and for offered loads per server identical in all analysed cases ρ_c=0.75,
- number of places in the waiting queue (anchorage size) m=32,
- number of units in a group (barge composition) r=2,4 and 6,
- limits of the suggested criterion $\delta_i\le 1\%$, i.e. $\delta_i\le 5\%$,
- for the case when the system is empty at the beginning ($p_0(0)$=1, $p_i(0)$=0 for i=1,2, ..., 34), but all that does not diminish generality of the conclusions.

Change of the system condition probability in the time, change of system characteristics during the nonstationary regime and the influence of duration of work of the system to its characteristics are defined for the following case: c=2, m=32, r=6 and ρ_c=0.75, while for the same system and different intensities of load of the servers, certain areas of the stationary and nonstationary regime of the system work, as well as the change of system characteristics in the stationary or nonstationary working regime have been defined.

System of the differential equations (4) with convenient parameters has been solved numerically, by using Runge-Kutta method of the fifth order, with defined preciseness (Scheid (1968) [5]). The original software has been written for solution of system of the differential equations.

2. RESULTS OF THE ANALYSIS

2.1. Duration time of the nonstationary regime

Diagrams on the figures 3 and 4 show duration time of the nonstationary regime for case of the empty system at the beginning of the analysis, for limits of the given criterion $\delta_i \leq 1\%$ and $\delta_i \leq 5\%$.

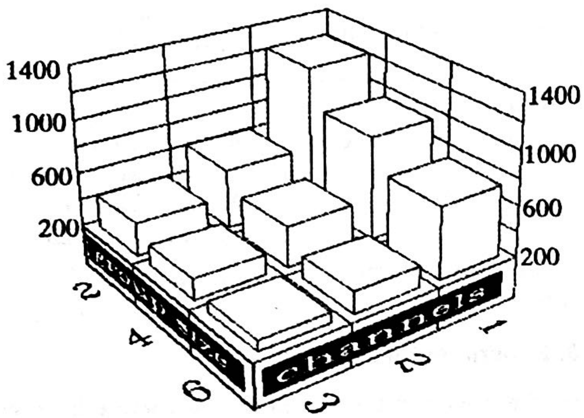

Figure 3: Duration time of the nonstacionary regime
$\delta_i \leq 1\%$

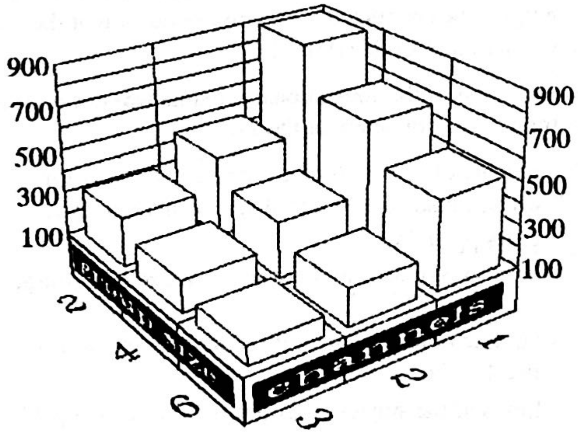

Figure 4: Duration time of the nonstacionary regime
$\delta_i \leq 5\%$

2.2 Change of system characteristics during the nonstationary regime

On the figure 5. change of a couple of first probabilities of system condition during the time, i.e. change of system condition probability in the stationary and nonstationary working regime has been shown.

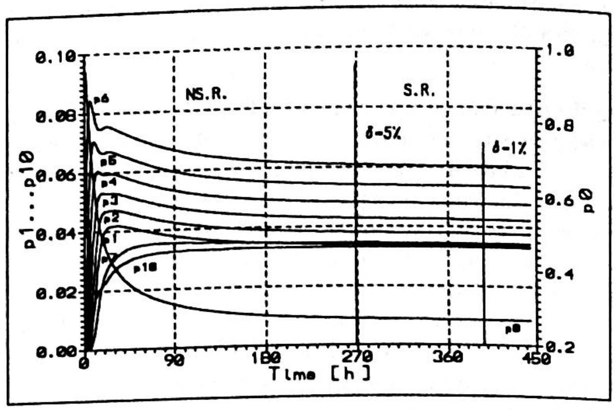

Figure 5: Change of condition probability in the time

On the figures 6-10 changes of the system characteristics during the nonstationary regime have been shown. On the figure 11 areas of the stationary and nonstationary regime, depending on offered load of the server have been shown.

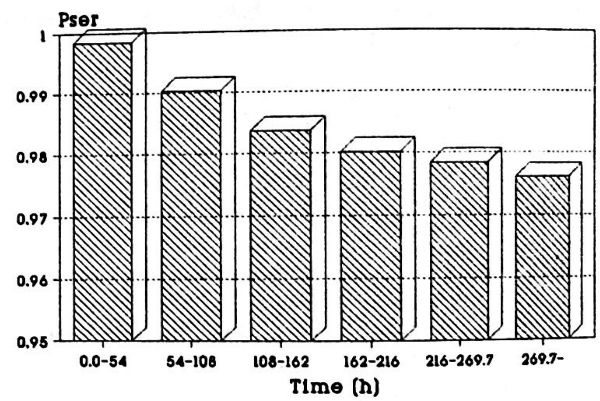

Figure 6 Change P_{ser} in the nonstationary regime

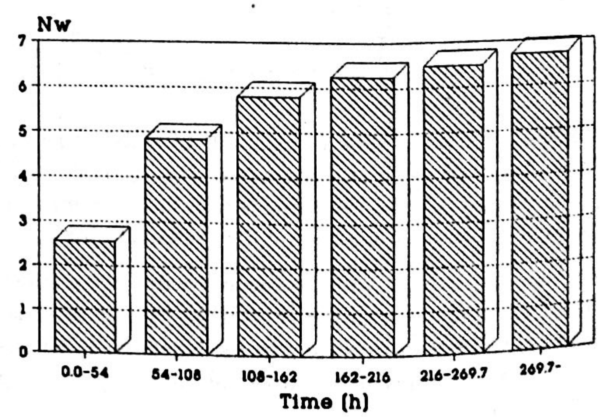

Figure 7 Change N_w in the nonstationary regime

It is possible to approximate characteristics of the systems N_w, N_{ws}, t_w and t_{ws} (Y) by the least square polynomial approximation (Scheid (1968) [5]), as a rational function of time (X) with following form:

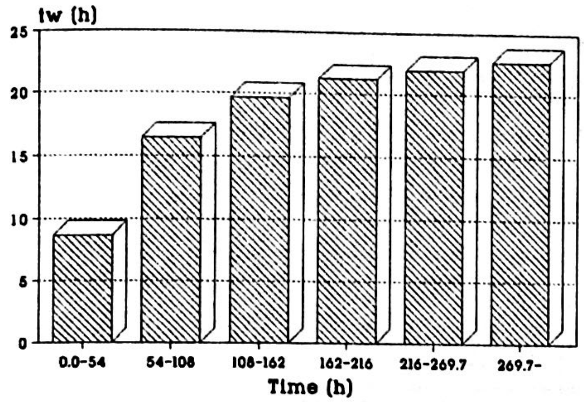

Figure 8 Change t_w in the nonstationary regime

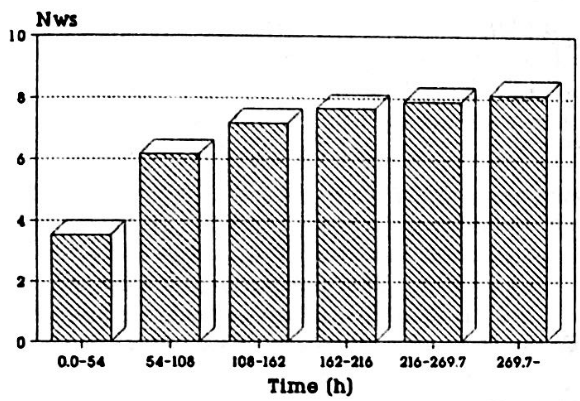

Figure 9 Change N_{ws} in the nonstationary regime

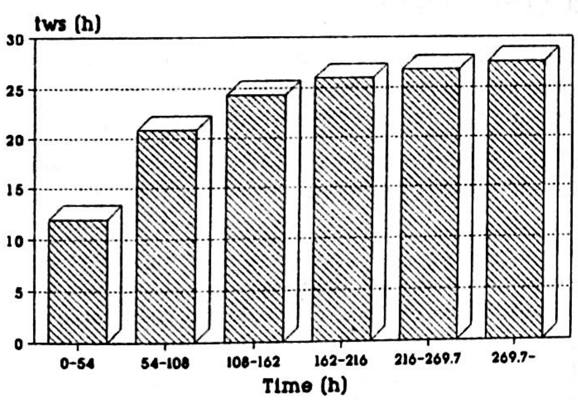

Figure 10 Change t_{ws} in the nonstationary regime

$$Y = \frac{X}{a + b \cdot X};$$

while it is possible to approximate the value P_{ser} by the least square polynomial approximation, as a square polynom:

$$Y = a \cdot X^2 + b \cdot X + c.$$

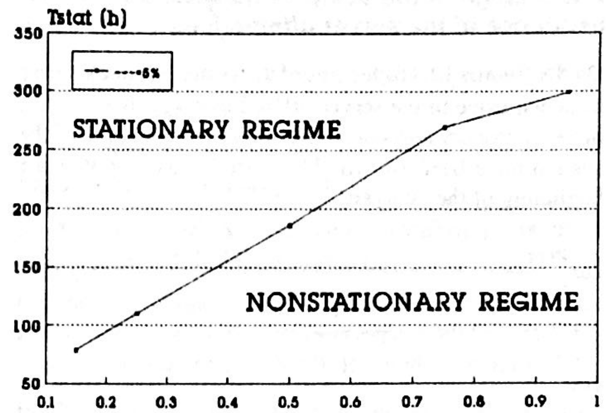

Figure 11 Work regimes

2.3. Influence of the duration time of the system to its characteristics

The general idea of this analysis is to determine duration time of the continuous work of the system, after which it is possible to neglect influence of the nonstationarity caused by defined initial conditions, i.e. to determine coefficients ("real state") to multiply with defined characteristics of the system, if the duration time of system work is less then needed to neglect the nonstationarity.

In the table 2 the change of the relative error of difference of average number of units in the waiting queue (N_w) at the given moment and at the stationary working regime ($N_{w(s)}$, as well as appropriate "real state" coefficient (K) for the case when the system was empty at the starting moment, in dependence to the continuous working time of the system, has been shown. By using the same method, it is possible to determine the time after which it is possible to neglect influence of the nonstationarity caused by initial conditions to the other characteristics of the system.

It is clearly seen, from the table 2, that if the system works longer then $4 \cdot t_{stat}$, it is not needed to consider the nonstationarity, that originates because of initial conditions in the case of average number of units in the waiting queue.

Table 2: "Real state" coefficients (K) for N_w

Working time of system	$\frac{N_w(t) - N_{w(s)}}{N_{w(s)}} \cdot 100[\%]$	$K = \frac{N_w(t)}{N_{w(s)}}$
$0.25xt_{stat}$	56.22	0.4378
$0.50xt_{stat}$	38.54	0.6146
$0.75xt_{stat}$	28.49	0.7151
$1.00xt_{stat}$	22.21	0.7779
$1.50xt_{stat}$	15.13	0.8487
$2.00xt_{stat}$	11.39	0.8862
$2.50xt_{stat}$	9.11	0.9089
$3.00xt_{stat}$	7.60	0.9240
$3.50xt_{stat}$	6.51	0.9350
$4.00xt_{stat}$	5.70	0.9430
$4.50xt_{stat}$	5.06	0.9494
$5.00xt_{stat}$	4.56	0.9544

2.4. Change of the system characteristics in dependence of the server offered load

On the figures 12-16 changes of the system characteristics in dependence to the server offered load (ρ_c=0.15÷0.95), in the stationary and nonstationary working regime of the system have been shown. The system was empty at the beginning of the analysis.

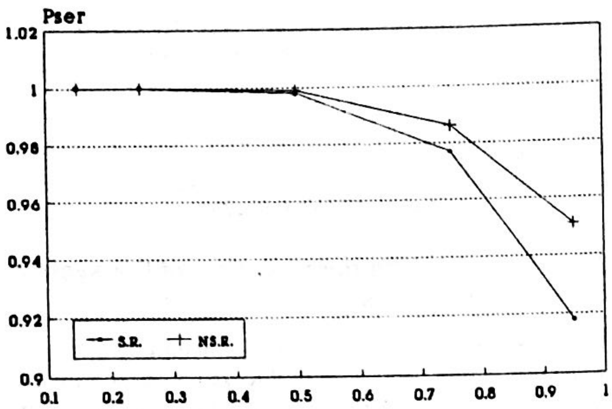

Figure 12 Change P_{ser} depending to ρ_c

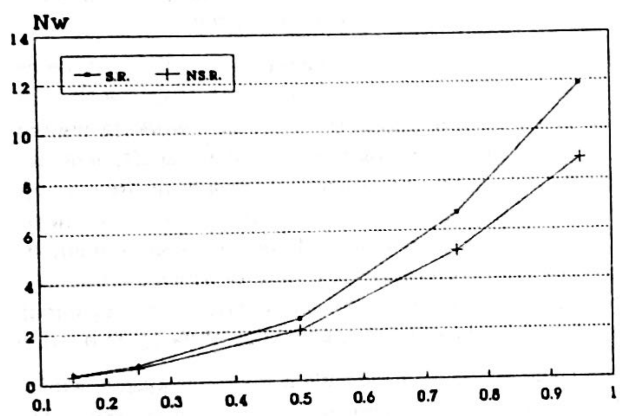

Figure 13 Change N_w depending to ρ_c

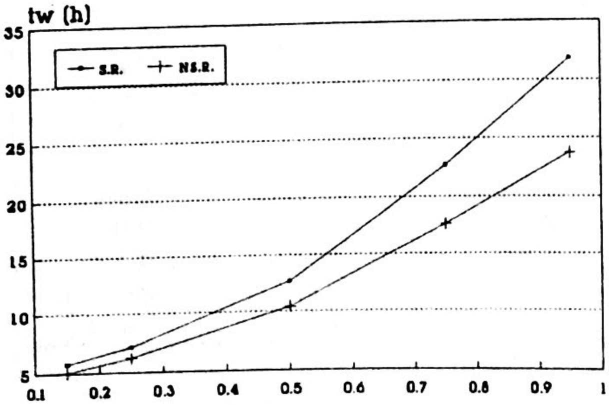

Figure 14 Change t_w depending to ρ_c

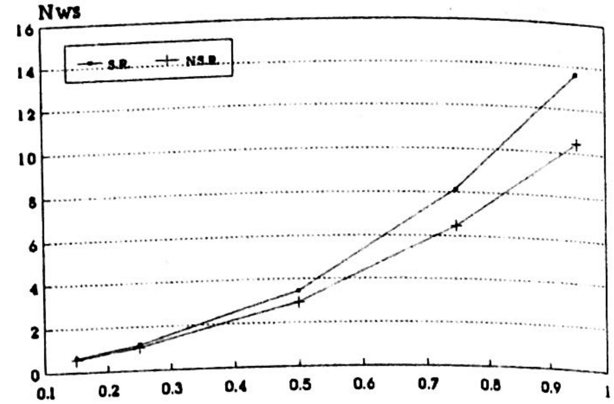

Figure 15 Change N_{ws} depending to ρ_c

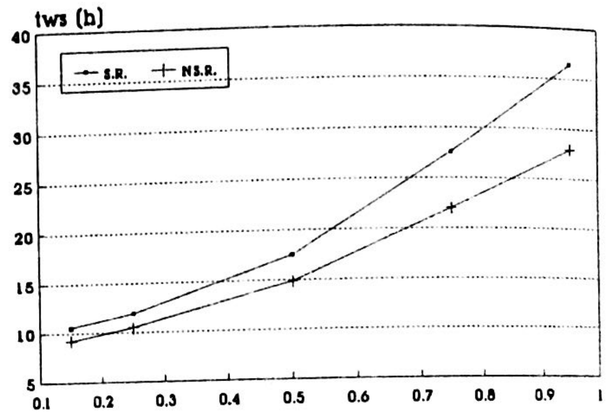

Figure 16 Change t_{ws} depending to ρ_c

3. CONCLUSION

The empirical results shown in this paper suggest the existence of the certain laws in the system behaviour in the nonstationary working regime caused by initial conditions.

Characteristics of the system work, in the nonstationary regime, caused by different initial conditions differ among themselves. Characteristics of the system work, in the case when system was empty at the beginning, are more convenient regarding the stationary working regime, as well as nonstationary regime caused by different initial conditions. As the number of units in the system increases at the beginning, the characteristics of the system work in the nonstationary regime become more inconvenient, because the system was partially filled in at the beginning. The most inconvenient case, regarding the characteristics of the system, is when the system is completely full at the beginning, i.e. when the starting number of units in the system is equal to the sum of number of servers and number of places in the waiting queue (Zmić and Bugarić (1993) [7]).

Characteristics of the system work, by entrance of the system to the stationary working regime (moment of the system entrance to the stationary working regime is determinate by the application of the suggested criterion) differ from the characteristics of the system in the theoretical stationary working regime of the system (i.e. when working time of the system tends to eternity). That difference is included in the real state coefficient for each characteristic of the system. Working time of the system, after which it is possible to neglect influence of the nonstationarity to the given characteristic differs depending to the characteristic. Generally speaking, it is possible to determine final time period of the system work after which it is possible to neglect influence of the nonstationarity caused by initial conditions.

Limits of the suggested criterion for entrance of the system to the stationary working regime depend on the system. The criterion limits by the system such as the bulk cargo terminal, which has been analysed in this paper by shown model where the serving time and time elapsed between the arrival of the units are approximately couple of hours, i.e. the reply of the system is slow, can be wider (5%). By the system such as input/output zone of the high-bay warehouses where the times between arrival and serving of the units are not longer then couple of seconds and where the quick reply of the system is required, limits of the criteria should be narrower (for example 1%).

The results obtained by analysis of the multiserver model with batch arrival of the units, complete help between the servers, limited waiting queue and cancellation regarding the whole group, the fact that does not diminish the general character of the obtained conclusions, point out that it is needed to consider the nonstationarity caused by the initial conditions in the future engineering calculations of the system which can be exposed by Markov chains with constant Poisson input and exponential service times.

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LIST OF USED SIGNS:

c - number of channels,
K - "real state" coefficient,
m - number of places in waiting queue,
N_{ws} - average number of units in a system,
N_w - average number of units in queue,
p_i - probability i in stationary state of system,
$p_i(t)$ - change of i probability in time period,
P_{ser} - probability of servicing,
r - number of units in a group,
t_{stat} - time period after which system enters the stationary regime of work,
t_w - average time which units spends in a queue,
t_{ws} - average time which units spends in a system,
δ_i - relative error of difference of probabilities pi(t) and pi,
λ - arrival rate,
μ - service rate,
ρ_c - offered load by server.

INFLUENCE OF NONSTATIONARITY CAUSED BY THE INITIAL CONDITIONS OF THE SYSTEM TO ITS CHARACTERISTICS

Dj. Zrnić, U. Bugarić

In this paper the influence of the initial conditions of the system to the characteristics of the system itself and its changes during the caused nonstationary regime, as well as time of duration of the given nonstationary regime was discussed. The criterion for determination of the duration time of the nonstationary regime caused by the initial condition of the system has been suggested. Multiserver model of the queuing theory with the batch arrival of the units to the system, complete help between the servers, limited waiting queue and cancellation regarding the whole group has been described as well.

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UTICAJ NESTACIONARNOSTI IZAZVANE POČETNIM USLOVIMA RADA SISTEMA NA NJEGOVE KARAKTERISTIKE

Dj. Zrnić, U. Bugarić

U radu je razmatran uticaj početnih uslova rada sistema na njegove karakteristike kao i promena karakteristika sistema u toku, na ovaj način izazvanog, nestacionarnog režima. Takođe je razmatrano vreme trajanja nestacionarnog režima izazvanog početnim uslovima. U tu svrhu predložen je kriterijum za određivanje vremena trajanja ove vrste nestacionarnog režima rada sistema. U radu je takođe prikazan višekanalni model teorije redova čekanja sa grupnim dolaskom jedinica u sistem, potpunom pomoći među kanalima, ograničenim redom čekanja i otkazom koji se odnosi na celokupnu grupu.

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