ANALYTICAL AND NUMERICAL CALCULATION OF THE EQUIVALENT STRESS OF OPEN SECTION THIN-WALLED "U" PROFILE AT CONSTRAINED TORSION

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Abstract

The purpose of this work is to present analytical and numerical determination of the equivalent stress of the open section thin-walled "U" profile subjected to the constrained torsion. This work can be divided into two parts. In the first part of this paper equivalent stress was obtained with analytical calculation. In the second part the finite element method was applied for calculation of equivalent stress. At the end, the results of analytical method were compared with numerical method.

Keywords:

Thin-walled beam, "U" profile, equivalent stress, torsion, constraints, strain

1. Introduction

Thin-walled beams find a wide application in construction and machinery industry, as they enable obtaining any shape of the beam cross-section. Due to their low weight, thin-walled open section beams are widely applied in many structures. Many modern metal structures are manufactured using thin-walled elements (shells, plates, thin-walled beams) which are subjected to complex loads [1]. In most constructions such as, for example, automotive, railway vehicles, boats and similar constructions, they are installed in thin-walled elements. Thin-walled elements can be disparate shapes, can have greater or lesser bending and torsional rigidity, but their common property is that they have a low weight compared to other possible constructive shapes [2,3].

2. Analytical calculation

Analytical calculation of the equivalent stress of the open section thin-walled "U" profile was performed according expressions (1)-(14). Properties of material used in this paper are given in Table 1.

Table 1. Properties of Č 0360

Materials	Young's modulus	Poisson's ratio
Č 0360	20000 kN/cm^2	0,3

Cross section of the thin-walled „U" profile is given in *Figure 1.* , where the flanges $b_1=b_3=8$[cm] and the web $b_2=10$[cm] are widths and $t=0,3$[cm] is thickness of this profile.

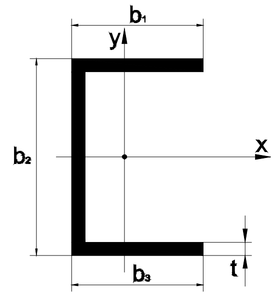

Figure 1. Cross-section U-beam

Area size of the "U" cross-section was calculated using the expression [3]:

$$A=\sum_{i=1}^{3} b_i t_i = 2b_1t_1 + b_2t_2 \tag{1}$$

Moments of inertia of the cross-sectional area about the centroidal axes x and y are given by expression [3]:

$$I_x=\sum_{i=1}^{3} t_i \int y(s)y(s)ds = \frac{b_1b_2^2t_1}{2}+\frac{t_2b_2^3}{12} \tag{2}$$

$$I_y=\sum_{i=1}^{3} t_i \int x(s)x(s)ds = \frac{t_1b_1^3}{3}\cdot\frac{1+2\lambda}{2+\lambda} \tag{3}$$

Sectorial moment of inertia is given by expression [3]:

$$I_\omega=\int_A \omega^2 dA=\sum_{i=1}^{3} t_i \int_S \omega(s)\omega(s)dS \tag{4}$$

Torsional moment of inertia is given by expression [3]:

$$I_t=\frac{\eta}{3}\sum_{i=1}^{3} b_i t_i^3, \tag{5}$$

where η is cofficient of safety.

Torsional section moduli is given by expression:

$$W_t=\frac{I_t}{t_{\max}} \tag{6}$$

Schematic representation of constrained torsion of the console is given in *Figure 2.*

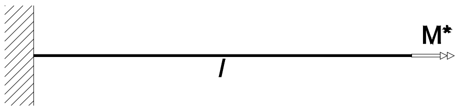

Figure 2. Constrained torsion of the cantilever beam

Console is loaded with a torsion moment according to the expression:

$$M^* = 31{,}4\left[\text{kNcm}\right] \tag{7}$$

The reduced Young's modulus is given by expression:

$$\overline{E} = \frac{E}{1-\nu} \tag{8}$$

The bending – torsional characteristic is given by expression [3-5]:

$$k = \sqrt{\frac{GI_t}{\overline{E}I_\omega}} \tag{9}$$

Bi-moment and the maximum normal stress are given by expressions (10) and (11), respectively [3]:

$$B_{\max} = -\frac{M^*}{k}th\left(kl\right) \tag{10}$$

$$\sigma_{\max} = \frac{B_{\max}}{I_\omega}\omega_{\max} \tag{11}$$

In the case of loads by concentrated torsion moment on the free end of the console, moment of pure torsion on the free end is given by expression [3]:

$$M_{t\max} = M^*\left(1-\frac{1}{ch\left(kl\right)}\right) \tag{12}$$

Shear stress is given by expression:

$$\tau_{\max} = \frac{M_{t\max}}{W_t} \tag{13}$$

In the case of a complex load (normal stress and the shear stress are taken together in the calculating), can be define the equivalent stress that is calculated by the Hencky-Mises hypothesis [6]:

$$\sigma_e = \sqrt{\sigma_{\max}^2 + 3\tau_{\max}^2} = 31{,}111\frac{\text{kN}}{\text{cm}^2} \tag{14}$$

3. Numerical analysis using finite element method

Calculations were performed using the Finite element method [7]. The material properties used for the simulations are shown in *Table 1*. Numerical simulations [7,8] were performed using ABAQUS and KOMIPS software .

First, the analysis was done in the software package ABAQUS, and then in the software package KOMIPS and finally this numerical results were compared with analytically obtained results.

Model was composed of a plates. All dimensions of plates are given in centimeters. The best results are obtained with a given mesh of finite elements. Generated finite element mesh model of "U" profile, in ABAQUS software, is shown in *Figure 3.*

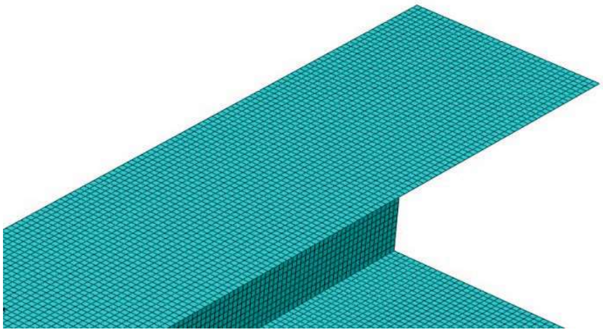

Figure 3. Mesh of finite elements on the model of "U" profile

Constraint, i.e. boundary condition was encastre at the one end of the beam. The torque is represented by the couple produced by two parallel horizontal forces introduced at the free end of the cantilever beam.

Boundary condition and load of model of "U" profile in software ABAQUS are given in *Figure 4.*

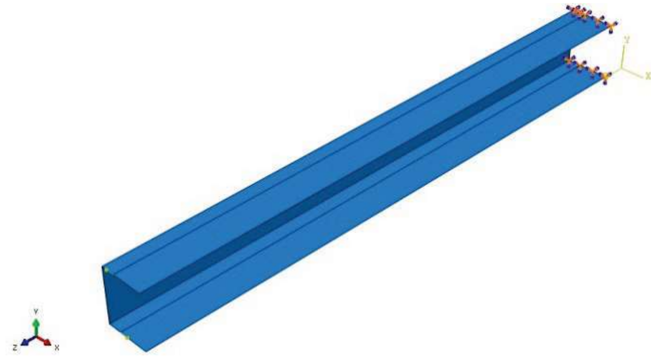

Figure 4. Load and boundary conditions

Figure 5 shows the values of the equivalent stress. The maximum value of the stress occurs at the last finite element. The displayed stress values are given in kN/cm².

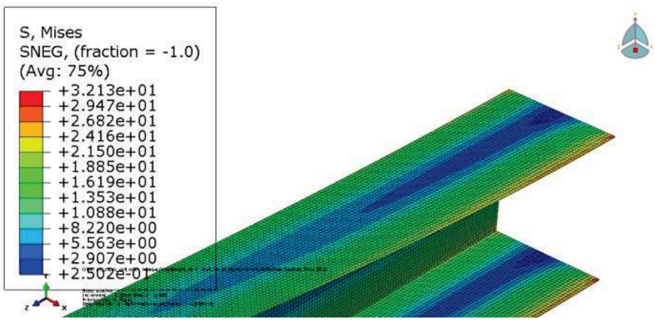

Figure 5. Values of the equivalent stress software package ABAQUS

In Figure 6 the finite element is marked at the end of the considered cantilever thin-walled beam of the chosen shape. In this finite element the value of stress $\sigma_e = 32{,}13 \left[\frac{\text{kN}}{\text{cm}^2} \right]$ corresponds to analytically obtained values.

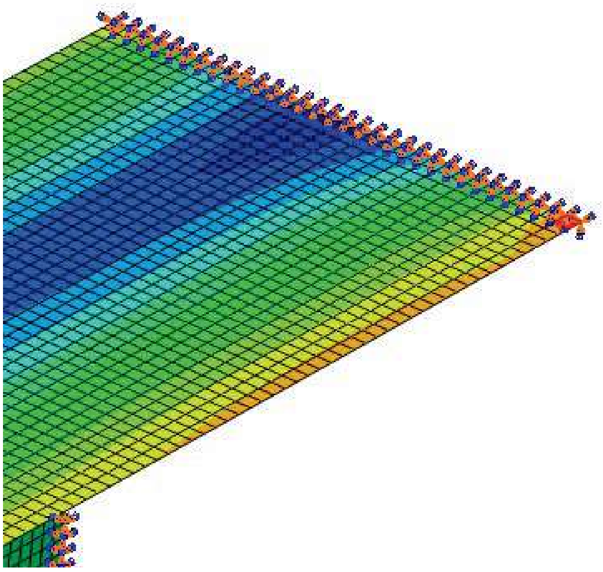

Figure 6. Equivalent stress of the finite element at the end of flange

The numerical calculation was made in software package KOMIPS [7] to verify the results obtained with the analytical calculation and numerical analysis in the software ABAQUS.

Figure 7 shows the constraint, i.e. boundary condition was encastre at the one end of the beam. The torque is represented by the couple produced by two parallel horizontal forces introduced at the free end of the cantilever beam in the software package KOMIPS [7]. The mesh of finite elements on the model of "U" profile is also given in *Figure 7.*

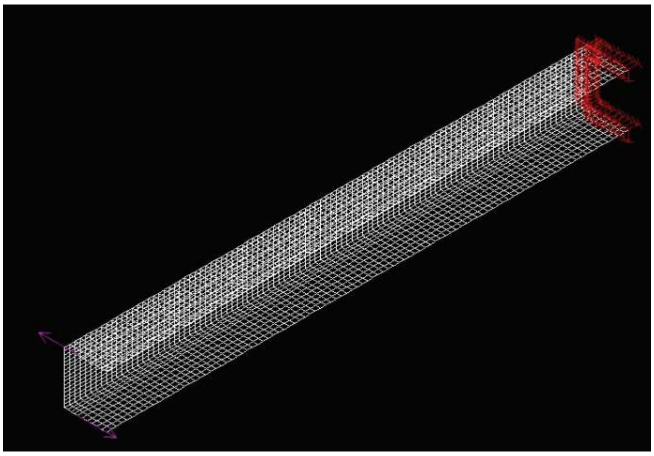

Figure 7. Load, boundary condition and mesh of finite elements in software package KOMIPS

Figure 8 shows the values of the equivalent stress. The maximum value of the stress occurs at the last finite element.

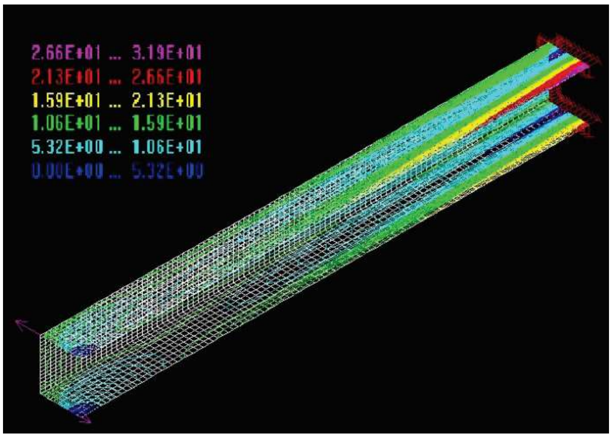

Figure 8. Values of the equivalent stress in software package KOMIPS

In Figure 9 the finite element is marked at the end of the considered cantilever thin-walled beam of the chosen shape. In this finite element the value of stress $\sigma_e = 31{,}19 \left[\frac{\text{kN}}{\text{cm}^2} \right]$ corresponds to analytically obtained values.

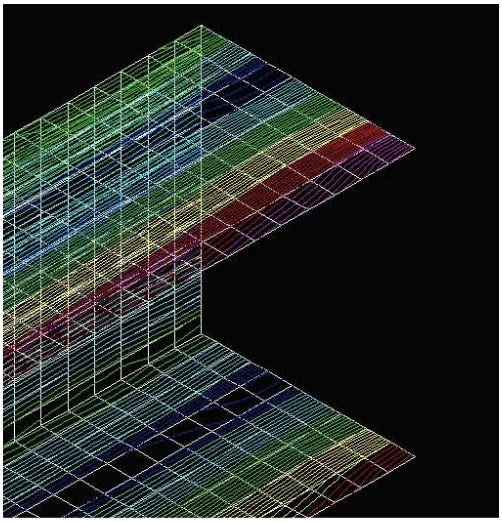

Figure 9. Equivalent stress finite element at the end of the flange

4. Conclusion

The paper shows that the equivalent stress obtained by numerical analysis using finite element method has approximately the same value like equivalent stress obtained with analytical calculation.

Also, it has been shown that using the two different softwares obtains approximately the same values of equivalent stress. Accuracy of results depends on the choice of the finite elements mesh.

It should be noted that the finite element method clearly shows where exactly the maximum value of stress is, and that the value of stress is not the same at the whole cross section.

It is prepared physical model of thin-walled "U" profile and still plans to experimentally test this profile. It is also plan to comparing the experimental results with the previous three calculations.

5. References

[1] N. Anđelić, Vesna Milošević-Mitić, T. Maneski, M. Milovančević, Đ. Đurđević, „Optimum design of open section thin-walled structural elements according to stress constraint",the paper is accepted for the symposium SIE 2015, University of Belgrade, *Faculty of Mechanical Engineering*, Belgrade 2015.

[2] D. Ružić, „Strength of Structures", (in Serbian),University of Belgrade, *Faculty of Mechanical Engineering*, Belgrade 1995.

[3] Andjelić N., Milošević-Mitić V.: *Optimum design of thin-walled I-beam subjected to stress constraint***.** Journal of Theoretical and Applied Mechanics 50, no. 4, 553-571 (2012).

[4] C. F. Kollbruner, N. Hajdin, „ Dunnwandige Stabe", Band 1, *Springer Verlag*, 1972.

[5] C. F. Kollbruner, N. Hajdin, „ Dunnwandige Stabe", Band 2, *Springer Verlag*, 1975.

[6] D. Ružić, R. Čukić, M. Dunjić, M. Milovančević, N. Anđelić, V. Milošević-Mitić, „Tables of Strength of Materials", University of Belgrade, *Faculty of Mechanical Engineering*, Belgrade 2003.

[7] T. Maneski, „Computer modeling and calculation of structures", *University of Belgrade,* University of Belgrade, *Faculty of Mechanical Engineering*, Belgrade 1998.

[8] T. Maneski, Vesna Milošević-Mitić, Davor Ostrić, „Settings Structural Strength", University of Belgrade *Faculty of Mechanical Engineering*, Belgrade 2002.