Dimensioning A PTC Systems Using Parabola Properties

Miša Stojićević[1], Branislav Popkonstantinović[1] ,Zorana Jeli[1], Ivana Cvetković[1], Boris Kosić[1]

[1]University of Belgrade, Faculty of Mechanical Engineering, Department of Theory of Mechanisms and Machines, 11000 Belgrade, Serbia

mstojicevic@mas.bg.ac.rs

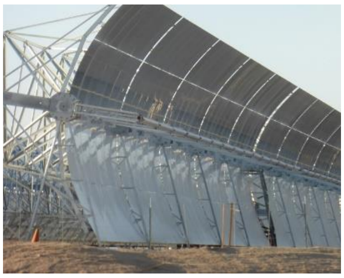
ABSTRACT

Out of many geometric curves, parabola stands out in usage in concentrated solar power (CSP). Its reflective properties allow reflecting light into one single point. This paper gives insight into the geometry of parabolic trough concentrators from the point of designing a size of a collector regarding two inputs. The first input is a parameter of the parabola which, provides the shape and size of the reflecting surface. A second input is a deviation of the angle of incoming light. Usually, this angle is perpendicular to the parabola's directrix, but the sizing of the collector tube must consider that because manufacturing or any other inaccuracy reflected ray might disperse from the focal point of the parabola. A detailed view of this dissipation of reflected rays depending on the entering ray's angle will be explained in the paper. A geometrical and algebraic display will be given, and a formula for sizing collector will be derived. In this equation two input values will be combined: parameter of parabola and value of the deviation of entering ray of light. This gained formula can later be used to show dependence between these two inputs and provide the answer to a question: is there a way to construct a mechanism that will move the focal point in accordance to changing entering angle and parameter of a parabola?

Keywords: parabolic trough collector; parabola; concentrated solar power; focal point; applied geometry, accuracy; design

1 INTRODUCTION

In geometry of concentrated solar power (CSP) parabola is one of the most used curve. Its property to concentrate ray into one focal point is used in several types of CSP installations (Stojicevic et al.., 2019). One of those installation is parabolic trough solar thermal power (PTSTP) system. In core of this system is parabolic trough collector (PTC), shown in Fig 1a and Fig. 1b, which is made out of parabolic-shaped mirrors with a tube as receiver placed in focal point. Principle of work is very simple: Sun rays are reflected from parabolic shaped long mirrors and reflected into one point were collector is placed. PTC is the most proven and lowest cost large-scale solar power technology available today (Price¨ et al.., 2002) and this concept has been proven in many countries (Fernández-García, A. et al.., 2010). In 1870, the first practical experience with PTCs belonged to John Ericsson (a Swedish engineer immigrant to the United States), who designed and constructed a collector with an aperture area of (3.25 m2) to produce steam for drive a small (373 W) engine. He also built (from 1872 to 1875) seven similar systems with air as working fluid (Abdulhamed, A.J. et al.., 2018) (Pytilinski, J. T. et al.., 1978).There are several directions of research in this area described in many research papers. One direction of research in this area is usage of different materials for better effectiveness of system (Liang, H. et al..,2015) and other is geometry design. Authors in (Jebasingh, V.K. et al.., 2016) have given an insight in performance of PTC and it applications in industrial and domestic utilizations. Two similar technologies, Fresnel and PTC, are compared in (El Gharbi, N. et al.., 2011) where authors give a better performance characteristics to PTC in aspect of optical quality and thermal efficiency. Three different models were analyzed for PTC by authors (Liang, H. et al.., 2016) from aspect that solar flux is not evenly distributed on the absorber. Research of geometry of parabola is done in (Ada, Tuba et al.., 2015) and (Glaeser G. et al.., 2016) in aspect of definition of Euclidean and Taxicab geometry. Papers (Hoseinzadeh, H. et al.., 2018) and (Cheng, Ze-Dong et al.., 2015) have done research in aspect of optimization of design of PTC which gave result in decreasing size of collector's tube. Step by step design of designed parabolic trough is given in (Masood, R. et al.., 2016) where it can be seen its basic geometry. One more concept will be shown in this paper. Design of PTC, will be done in regard of entering rays of sunlight in angle that is not perpendicular in directrix of parabola. By changing this angle it will be show how collector's tube can be sized so that it can compensate errors that are produced from changing angle of entering ray. Similar subject is mentioned in (Macedo-Valencia et al.., 2014) where the dimensions of the collector can be seen with calculus.Aim of this paper will be on dimensioning parabolic trough collector starting with assuming that entering ray of light will deviate from its track that usually is used in designing PTC systems. Depending of angle of deviation a receiver tube will be dimensioned so it can absorb errors that are made during this deviation process.

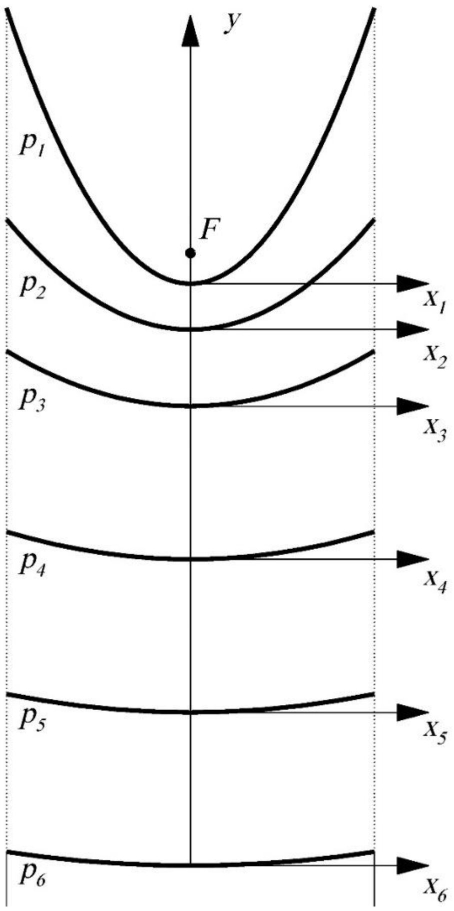


(a)

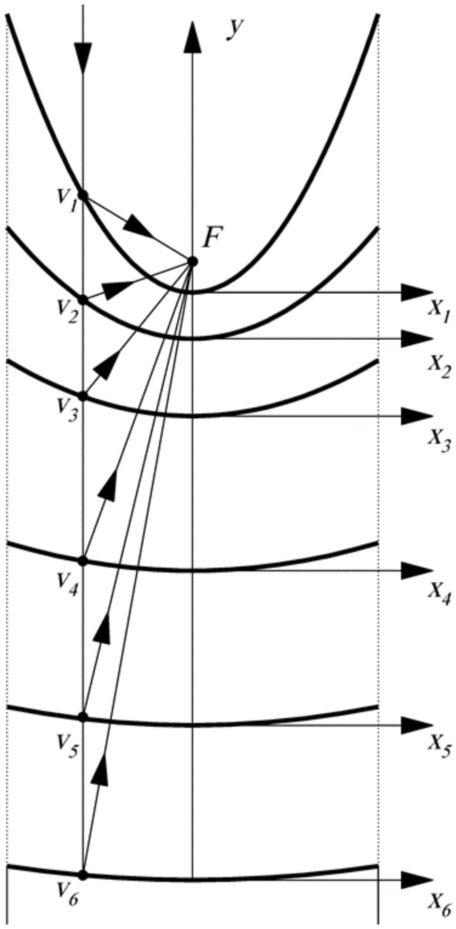
(b)

Figure 1: (a) Parabolic trough collector and receiver tube and (b) Field of PTC in work
(Source: Z22 - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=27881587 and ArséniureDeGallium - United States Department of Energy: http://www.nrel.gov/solar/parabolic_trough.html, Public Domain, https://commons.wikimedia.org/w/index.php?curid=671862

2. REFLECTING PROPERTIES OF PARABOLA



(a)



(b)

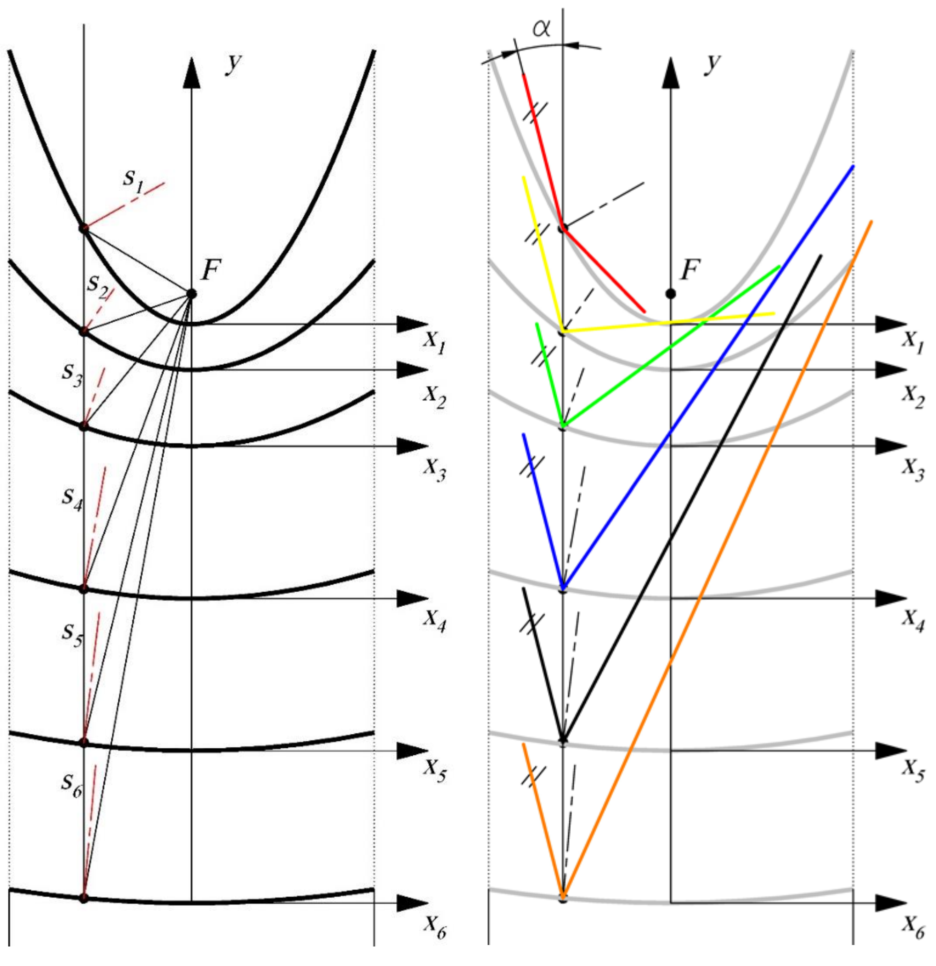
Figure 2: (a) Virtual model of the location, and (b) An image example of different nature

In core of all this research is parabola. A parabola is a non-central second-order curve where any point on that curve is at an equal distance from the focus of parabola (F) and a fixed straight line called the directrix. Equation of parabola can be presented as: $x^2 = 2py$.

Considering that p is parameter of parabola and when p is large number parabola become narrow and steep, and when it gets smaller it flattens a parabola and distances it of its focus point F. This is shown in Fig. 2a where $p_1>p_2>p_3>p_4>p_5>p_6$. For fixed point, focal point F, parabola is flattening for same width of parabola.

To see this problem graphically dependence between shape of parabola and parameter of parabola a Fig 1b is given. Here, an entering ray passes trough points on parabola labeled with V. When hits reflecting surface in certain point a ray is redirected into focal point F.

When designing PTC systems a parameter of parabola is very important. All six parabolas are receiving a same amount of light it can be noted that 5th and 6th parabola are giving a potential trouble for design for its focal point is much more distanced from mirror surface.



(a) (b)

Figure 3: (a) Virtual model of the location, and (b) An image example of different nature

Assumption is that every point V, where ray is reflected of surface, there must be a line that passes trough point V and which is a symmetry for entering and exiting ray. For all 6 parameters of parabola, a symmetry line can be seen of Fig 3a. With rise of parameter a symmetry line will be steeper and steeper. This symmetry will remain even if entering ray is deviates by angle α as can be seen in Fig3b. In parabolic trough CSP systems a collector is placed in center of focal point F. All rays that are reflected from surface must at least be tangent to collector's diameter. This way, collector tube will absorb all reflected ray within allowed angle α. All this point to a conclusion that there must be a formula that binds parameter of parabola, position of point V and entering angle α in order to gain a diameter of collector tube for parabolic trough.

3. EQUATIONS OF PARABOLA

Parabola can be represented as set of vertices that are on an equal distance from directrix and focus point (d_P: $y = \frac{p}{2}$), p>0 where p is parameter of parabola with coordinates:

$$F = (0, \tfrac{p}{2}) \quad \text{(Eq. 1)}$$

Equation of parabola shown in Fig. 2 can be described using equation:

$$x^2 = 2py \quad \text{(Eq. 2)}$$

For any line, represented with general equation as $= kx + n$, that passes through parabola $x^2 = 2py$ as a tangent of that parabola, in point V(X_v, Y_v), equation 2 must be satisfied:

$$p = 2kn \quad \text{(Eq. 3)}$$

Then a tangent in point V(X_v, Y_v) on parabola can be described as:

$$y_t \cdot y_v = p(X_t + X_v) \quad \text{(Eq. 4)}$$

For a point V on parabola it can be said that a tangent line can be represented as

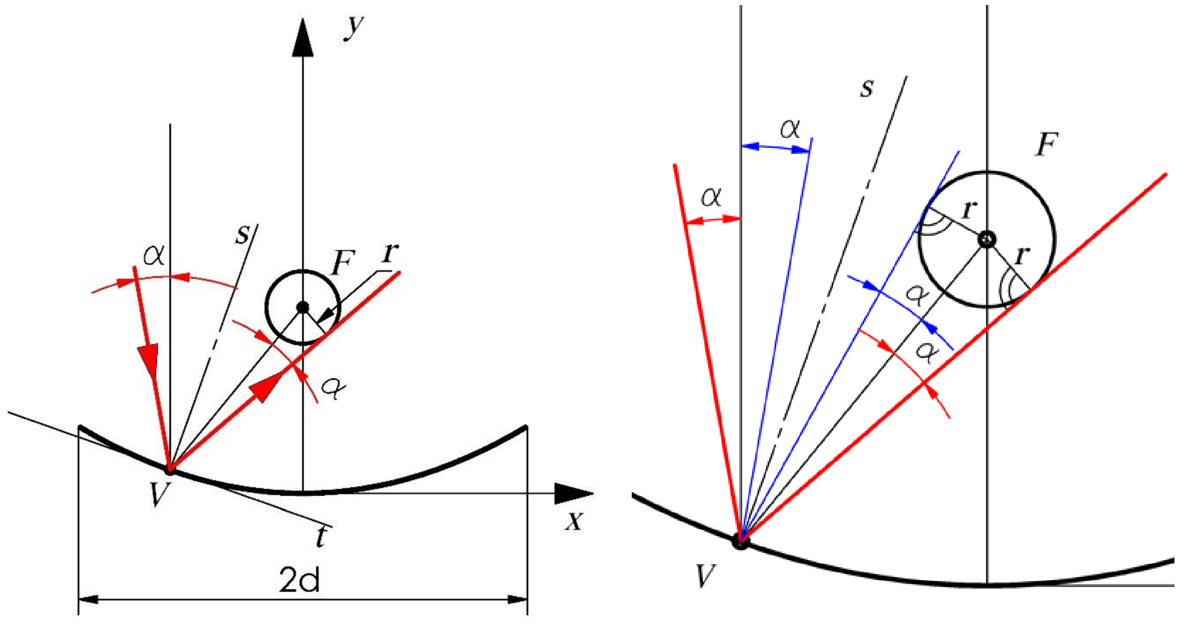
$$Y_t = \frac{p}{Y_v} X_t + \frac{pX_v}{Y_v} \quad \text{(Eq. 5)}$$

For line $y = kx + n$ to be rotated for 90 degrees it can be presented as:

$$y = -\frac{1}{k} x + n \quad \text{(Eq. 6)}$$

Then Equation 5 becomes:

$$y_s = -\frac{Y_v}{p} x_s + \frac{pX_v}{y_v} \quad \text{(Eq. 7)}$$



(a) (b)

Figure 4: (a) Virtual model of the location, and (b) An image example of different nature

Equation 6 is a line that passes through point V on parabola and represents a symmetry between input and output ray on reflective surface. Changing angle of input ray, for some alpha, will change output ray by same angle alpha.

When a symmetry can be established for certain parabola

$$r = L\,sin\ \alpha \qquad \text{(Eq. 8)}$$

Where L is distance between point $V(X_V, Y_V)$ on parabola and focal point $F(X_F, Y_F)$

$$L = \sqrt{(X_F - X_V)^2 + (Y_F - Y_V)^2} \qquad \text{(Eq. 9)}$$

For a point on parabola V, which is furtherd from focal point F, it can be considered that $X_V = d$ (or $-d$) and $Y_V = d^2/2p$

Equation 8 become:

$$L = \sqrt{(0-d)^2 + \left(\frac{p}{2} - \frac{d^2}{2p}\right)^2} \qquad \text{(Eq. 10)}$$

Further solving this equation:

$$L = \sqrt{d^2 + \left(\frac{p^2-d^2}{2p}\right)^2} \qquad \text{(Eq. 11)}$$

$$L = \sqrt{\frac{4p^2d^2+p^4-2p^2d^2+d^4}{4p^2}} \qquad \text{(Eq. 12)}$$

$$L = \sqrt{\frac{p^4+2p^2d^2+d^4}{4p^2}} \qquad \text{(Eq. 13)}$$

$$L = \sqrt{\left(\frac{p^2+d^2}{2p}\right)^2} \quad = \frac{p^2+d^2}{2p} \qquad \text{(Eq. 14)}$$

Formula for calculating a radius of collector depending from parameter of parabola, maximum distance point from y axis and value of deviation angle entering ray becomes:

$$\mathrm{r} = \frac{p^2+d^2}{2p}\sin\alpha \qquad \text{(Eq. 15)}$$

4. EXAMPLE

Equation 15 shows that angle α is sine function and its value can vary from 0 to 1 where for value of 1 angle α is 90 degrees. Considering this as a maximum that entering ray can deviate in most distant point on parabola equation 15 become dependent only on remaining two factors: parameter p and size of parabola d., radius of collector for few values of p and d is presented in Table 1.

p \ d	10	20	30	40	50	60	70	80	90	100
10	10	13	17	21	26	31	36	41	46	51
20	25	20	22	25	29	33	38	43	47	52
30	50	33	30	31	34	38	41	46	50	55
40	85	50	42	40	41	43	46	50	54	58
50	130	73	57	51	50	51	53	56	59	63
60	185	100	75	65	61	60	61	63	65	68
70	250	133	97	81	74	71	70	71	72	75
80	325	170	122	100	89	83	81	80	81	82
90	410	213	150	121	106	98	93	91	90	91
100	505	260	182	145	125	113	106	103	101	100

Table 1: Maximum size of r relative to p and d

Graphical representation of this is shown in Fig. 5 where a size of collector is on y axis and d is on x axis.

On this graph it can be seen that for a parameter p=10, maximum size of collector will rise much faster than case where parameter is larger. For three parameters equal to 150, 200 and 250 growth of collector's size is nearly same relative to much lower parameter of 10. Collector size will be better scaled if parameter of parabola is larger and it will absorb more errors that are made from changing angle of entering ray.

Table 1 and Fig 5 shows a values for a size of collector when part of equation 15 is at its maximum. This maximum is when entering angle is 90 degrees which is very large angle to be taken in consideration during design of PTC. In order to grasp a relative angle that can be considered an angle of entering ray is show it graphically. Fig. 6 shows a parabola with 11 equidistant points on it. For all this points, a symmetry line is drawn accordant to Equation 7. This can be visually confirmed as entering rays and reflected rays are symmetrical to each other relative to symmetric line.

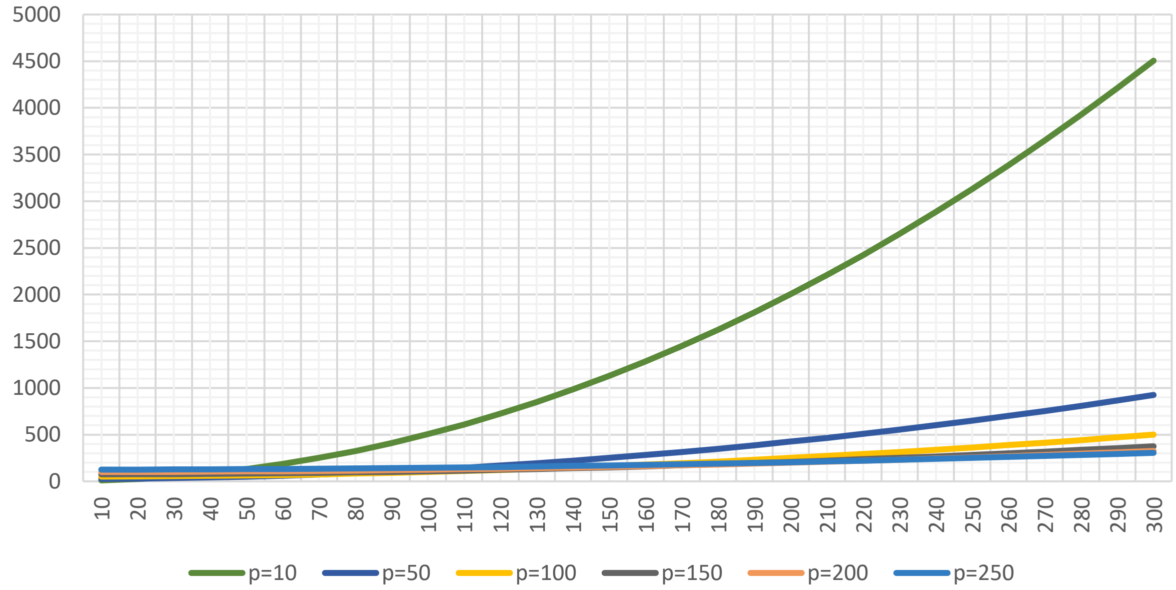


Figure 5: Size of collector depending of parameter p and distance d when $\sin\alpha = 1$

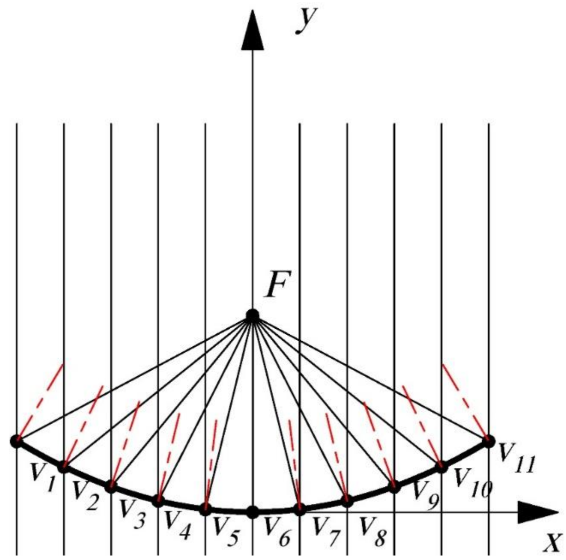


Figure 6: Parabola with symmetries in 11 points.

Fig. 7a to Fig. 7d is showing what is happening when angle α have four different values of this angle and it can be seen how a potential diameter of collector is growing and for angle large as much of 30 degrees. Most distant points will reflect entering ray back to parabola (Fig. 7c). For value of 45 degrees (Fig. 7d) dimension of collector that will absorb all errors is impossible.

Example can be made if values of parameter of parabola and most distant point are taken from Table 1. For values of p=100 and d= 50 a maximum radius size is r=125. In order to absorb all angles lesser than 10 degrees a size of collector is r=22. If a parameter of parabola is changed to be 3 times larger a size of collector will slightly change to r= 27.

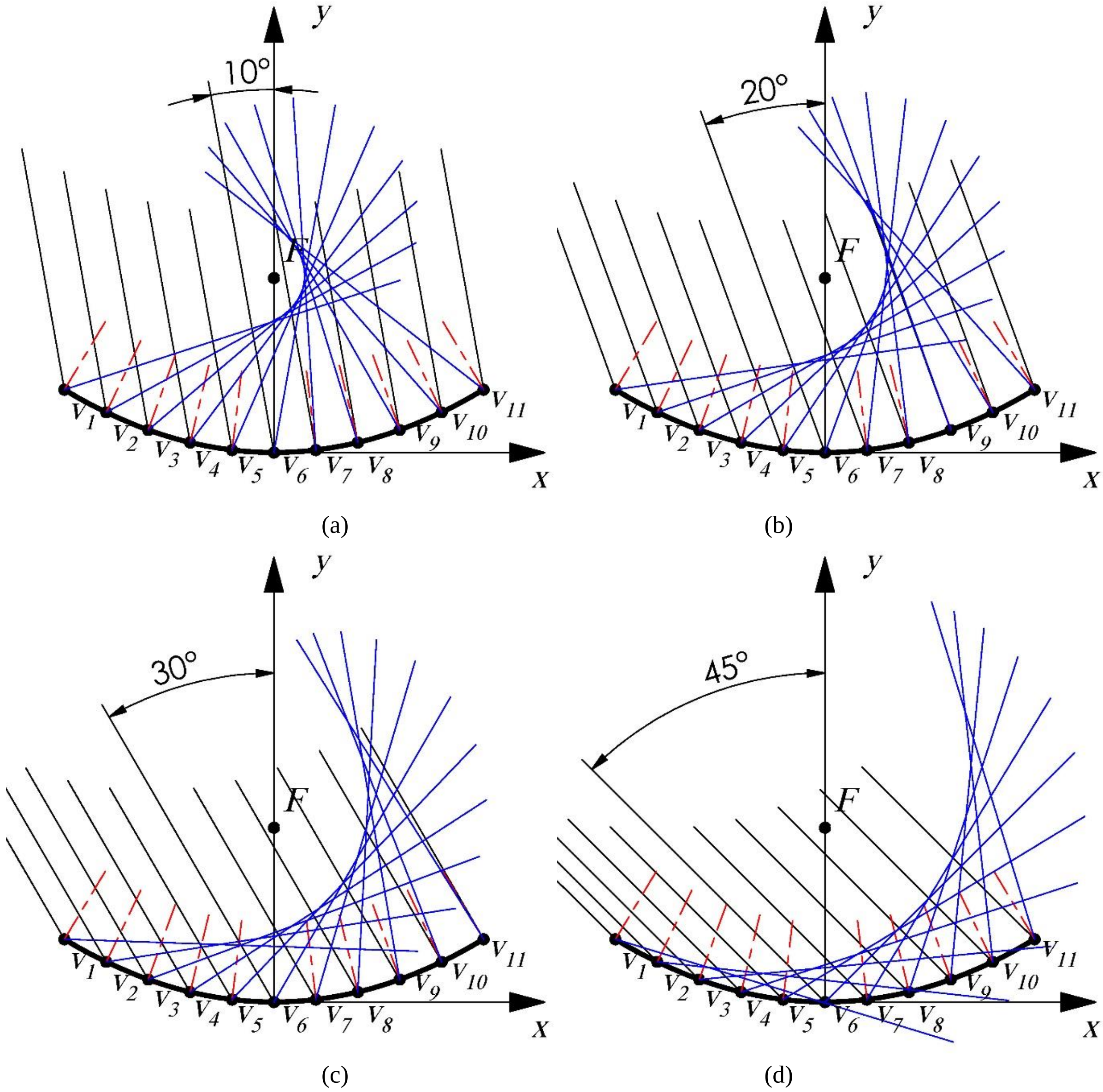


Figure 7: (a) Reflection of ray when $\alpha = 10$ deg (b) Reflection of ray when $\alpha = 20$ deg (c) Reflection of ray when $\alpha = 30$ deg (d) Reflection of ray when $\alpha = 45$ deg

5. CONCLUSION AND FURTHER RESEARCH

The parabolic trough solar thermal power (PTSTP) system has, among all other CSP systems, a lowest cost of exploration (Price¨ et al.., 2002), (Grena, Roberto et al.., 2010) but at this moment it is more expensive than fossil fuel plants of same capacity (Price¨ et al.., 2002, Cheng, Z.D. et al.., 2014, Cheng, Z.D. et al.., 2015). As a prices of using CSP systems drops, more interest will be shifted into green and renewable energy sources. Aim of this paper is to bring a small contribution in geometry of renewable technologies. It gives an insight in one seemingly insignificant problem that can be described as dimensioning a collector in PTC but it provides a start point for further research in this area. Algebra is provided so a potential user can check results in some software program for mathematical representations.

Further research, which will be based on this paper, can be best described as finding optimum size of collector and to do a thermodynamic calculations to see a heat transfer. Size of collector have a giant impact of efficiency of entire system in term of capacity of future plant. In this case collector will be static. Second path of research will be to look for a possible way to keep a size of collector as small as it can be in order to heat it up fast. In order to do so hypothesis is that a parabola will have a moving focal point that moves in way to absorb all reflected rays. Idea here is that a heavy large parabolic mirror will stay still or have a least possible movement. Meanwhile a much lighter and smaller collector will move by previously given trajectory. In overall this system should be in theory much more energy efficient than systems which required for movement of entire parabolic mirrors.

REFERENCES

1. Stojicevic, M., Jeli, Z., Obradovic, M., Obradovic, R. and Marinescu, G.C., 2019. Designs of solar concentrators. FME Transactions, 47(2), pp.273-278.

2. Price, H., Lu¨ pfert, E., Kearney, D., Zarza, E., Cohen, G., Gee, R. and Mahoney, R., 2002. Advances in parabolic trough solar power technology. J. Sol. Energy Eng., 124(2), pp.109-125.

3. Fernández-García, A., Zarza, E., Valenzuela, L. and Pérez, M., 2010. Parabolic-trough solar collectors and their applications. Renewable and Sustainable Energy Reviews, 14(7), pp.1695-1721.

4. Abdulhamed, A.J., Adam, N.M., Ab-Kadir, M.Z.A. and Hairuddin, A.A., 2018. Review of solar parabolic-trough collector geometrical and thermal analyses, performance, and applications. Renewable and Sustainable Energy Reviews, 91, pp.822-831.

5. Pytilinski, J. T. (1978). Solar energy installations for pumping irrigation water. Solar energy, 21(4), 255-262.

6. Liang, H., You, S. and Zhang, H., 2015. Comparison of different heat transfer models for parabolic trough solar collectors. Applied energy, 148, pp.105-114.

7. Jebasingh, V.K. and Herbert, G.J., 2016. A review of solar parabolic trough collector. Renewable and Sustainable Energy Reviews, 54, pp.1085-1091.

8. El Gharbi, N., Derbal, H., Bouaichaoui, S. and Said, N., 2011. A comparative study between parabolic trough collector and linear Fresnel reflector technologies. Energy Procedia, 6, pp.565-572.

9. Liang, H., You, S., & Zhang, H. (2016). Comparison of three optical models and analysis of geometric parameters for parabolic trough solar collectors. Energy, 96, 37-47.

10. Ada, Tuba, Aytaç Kurtuluş, and H. Bahadır Yanik. "Developing the concept of a parabola in Taxicab geometry." International Journal of Mathematical Education in Science and Technology 46.2 (2015): 264-283.

11. Glaeser, Georg, Hellmuth Stachel, and Boris Odehnal. The Universe of Conics: From the ancient Greeks to 21st century developments. Springer, 2016..

12. Hoseinzadeh, Hamed, Alibakhsh Kasaeian, and Mohammad Behshad Shafii. "Geometric optimization of parabolic trough solar collector based on the local concentration ratio using the Monte Carlo method." Energy Conversion and Management 175 (2018): 278-287.

13. Cheng, Ze-Dong, Ya-Ling He, Bao-Cun Du, Kun Wang, and Qi Liang. "Geometric optimization on optical performance of parabolic trough solar collector systems using particle swarm optimization algorithm." Applied energy 148 (2015): 282-293.

14. Masood, R., Gilani, S.I.U.H. and Al-Kayiem, H.H., 2016. A simplified design procedure of parabolic trough solar field for industrial heating applications. ARPN J. Eng. Appl. Sci, 11, pp.13065-13071.

15. Macedo-Valencia, J., Ramírez-Ávila, J., Acosta, R., Jaramillo, O.A. and Aguilar, J.O., 2014. Design, construction and evaluation of parabolic trough collector as demonstrative prototype. Energy procedia, 57, pp.989-998.

16. Grena, Roberto. "Optical simulation of a parabolic solar trough collector." International Journal of Sustainable Energy 29, no. 1 (2010): 19-36.

17. Cheng, Z.D., He, Y.L., Wang, K., Du, B.C. and Cui, F.Q., 2014. A detailed parameter study on the comprehensive characteristics and performance of a parabolic trough solar collector system. Applied thermal engineering, 63(1), pp.278-289.

18. Cheng, Z.D., He, Y.L. and Qiu, Y., 2015. A detailed nonuniform thermal model of a parabolic trough solar receiver with two halves and two inactive ends. Renewable Energy, 74, pp.139-147.