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NUMERICAL COMPUTATIONS OF ELASTIC TORSION USING THE FINITE-VOLUME METHOD

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ABSTRACT

Analytical and numerical studies of Saint-Venant's torsion for bars of arbitrary cross-section is still a topic of interest for reasearchers and engineers. Numerical studies are mainly based on Finite Element Methods (FEM), or more recently on Boundary Element Methods. In this paper, elastic torsion problems are studied numerically using the Finite Volume Method (FVM), and three general approaches in torsion formulation [1] are analyzed: **(a)** Laplace partial-differential equation (PDE) with non-homogeneous Neumann boundary conditions (BC) for warping function w (displacement formulation); **(b)** Poisson PDE with homogeneous Dirichlet BC for Prandtl stress function ϕ and **(c)** Laplace PDE with non-homogeneous Dirichlet BC for conjugate function ψ. When computational domain (cross-section of the bar) is simply connected region, all three approaches are feasible in numerical sense, where the (b) is the simplest in terms of implementation. On the other hand, with multiply connected regions there is a big issue for (b) and (c). For arbitrary, multiply connected region, the values of ϕ and ψ on inner contours are not known in advance. These values are related to compatibility condition of single-valued displacement, which leads to known value of stress circulation around each inner contour. From that condition it is not possible to determine the value or ϕ or ψ on inner contours, since the stress distribution is obtained after the solution of the equations in (b) and (c). For such cases, approach (a) is the only option.

Finite volume method is predominantly used in computational fluid dynamics (CFD), and it is very rarely used in computational solid mechanics. But essentially, it is just one approach for numerical solution of PDEs (FVM discretises strong conservative form of momentum equation, while FEM uses weak form). OpenFOAM [2], free and open-source CFD software is used as the tool for implementation of numerical solutions of equations (a), (b) and (c). General FVM solver, based on the (a) is implemented using OpenFOAM libraries, together with appropriate Neumann BC. The solver is then tested on many examples, starting from the problems with known analytical solution (excellent agreement is found). Additional post-processing is implemented within the solver, where after determination of shear stress (calculated from the gradient of warping function field), Prandtl stress function is calculated as the stream function of stress vector field. It is shown that boundary-fitted grid which is characteristic for FVM is very suited for considered computations. Finally, analogy between elastic torsion and some classical fluid mechanics problems governed with the same type of the equations is analyzed.

REFERENCES

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