Romeo Meštrović
Professor
Maritime Faculty Kotor
University of Montenegro

Branislav Dragović
Professor
Maritime Faculty Kotor
University of Montenegro

Nenad Zrnić
Professor
University of Belgrade
Faculty of Mechanical Engineering

Dražen Dragojević
PhD Student
University of Montenegro
Maritime Faculty Kotor

Marko Popović
University of Montenegro
Maritime Faculty Kotor

On Specific Cost Ratio in a Port Modelled as $M/G/1$ Queue

In this paper we deduce related expression for the specific cost ratio concerning the $M/G/1$ queue using the known general expression for the specific cost ratio and the Pollaczek–Khintchine formula for the $M/G/1$ queue. Moreover, we also present related numerical and graphical results including the cases concerning the $M/M/1$ and $M/D/1$ queues. The deduced formula gives a possibility for comparison of values of specific cost ratio of different $M/G/1$ type queues. The corresponding optimization is considered in the sense of minimization of values of the appropriate specific cost ratio under some constraints to the values ρ (the utilization factor). The proposed method is exposed by some numerical experiments.

Keywords: *Specific cost ratio in port, $M/G/1$ queue, Pollaczek–Khintchine formula, Utilization factor*

1. INTRODUCTION

The specific cost ratio, usually denoted as R, is defined as a ratio of total daily cost of a port queueing system and average daily cost of a ship in port. The optimal/mimimal numbers of berths and related optimal intervals (the so-called ranges of optimal server capacities; see, e.g., [1] and [18]) in the sense of minimization of R for certain single arrival and bulk arrival queueing systems were studied in [16]-[18] and [20]. Explicit expressions for R concerning certain single arrival and bulk arrival queuing systems were given in [1], [3], [4], [6]-[14], [18] and [19].

Using the known general expression for the specific cost ratio and the Pollaczek–Khintchine formula concerning the $M/G/1$ queue, we deduce related expression for the specific cost ratio related to the $M/G/1$ queue. Moreover, we also present related numerical and graphical results including the cases concerning the $M/M/1$ and $M/D/1$ queues. The deduced formula gives a possibility for comparison of values of specific cost ratio of different $M/G/1$ type queues.

The remainder of the paper is organized as follows. In Section 2, the expression for the specific cost ratio related to $M/G/1$ queue is presented. This formula is suitable for analyzing the corresponding specific cost ratio, considered as a function of coefficient of variation of service time and the appropriate utilization factor ρ. Some numerical experiments for specific cost ratio by $M/G/1$ queue with different values of ρ and coefficient of variation of service time are demonstrated in Section 3. These numerical results are followed by graphical results in the same section. In Section 4, concluding remarks are given.

Correspondence to: Dr. Branislav Dragović, Professor
Maritime Faculty Kotor, University of Montenegro
Dobrota 36, 85330 Kotor, Montenegro
E-mail: branod@ucg.ac.me

2. THE EXPRESSION FOR THE SPECIFIC COST RATIO RELATED TO THE $M/G/1$ QUEUE

Consider port queuing models with n_b berths, where ships arrive according to a Poisson process with the rate λ and the mean service time equals to $1/\mu$ (μ is the average service rate). The total cost of a port queuing system for considered period consists of a cost related to berths and the cost related to the ships. Then the specific cost ratio, R, is defined as a ratio of total cost (defined as a sum of a cost per ships and a marginal cost of a port system) for considered period (in which λ ships arrive in a port), AC, and the total daily cost of a port queuing system and average daily cost of a ship in port, c_s, that is,

$$R = \frac{AC}{c_s} = \frac{\lambda}{c_s} AC_s$$

where $AC_s = AC/\lambda$ is a total daily cost per ship for considered port system. For this definition, see e.g., Dragović et al. [1], Noritake [11], Noritake and Kimura [12] and Radmilović et al. [13]. Following [8], using a routine calculation, with the notation $c_b/c_s = r$ (the quotient of average daily cost of a berth, c_b, and the average daily cost of a ship in port, c_s), we find that

$$R = rn_b + \left(L_q + \frac{2\lambda}{\mu} \right) + \lambda \frac{dL_q}{d\lambda}, \qquad (1)$$

where L_q is the average number of ships waiting in a queue. After the substitution $\lambda/\mu = \rho$, ρ is called the utilization factor which is equal to the traffic intensity θ, into (1), we find that [11]

$$R = rn_b + L_q + 2\rho + \rho \frac{dL_q}{d\rho}. \qquad (2)$$

As noticed above, in several port systems it is of interest to study the specific cost ratio (see [1]-[10], [13] and [15]). Here we consider a related problem concerning the queue port model $M/G/1$. Let us recall that in this model customers (ships) arrive according to a Poisson process with rate λ (that is, the interrarival time is exponential with rate λ) and the service times of the customers are independent random variables with a common general probability distribution function $B(x)$ with $B(0) = 0$. There is a single server and an infinite waiting room, while the queue discipline is FCFS. Denoting by S the random variable which is the service time of a customer, it is assumed that the utilization factor (the server utilization) ρ is smaller than 1 and equal to λ/μ, where $1/\mu = E(S)$ is the mean service time ($E(S)$ denotes the expected value of random variable S). By the well known Pollaczek–Khintchine formula (see, e.g., Tijms [21] - Subsection 9.2, p. 348),

$$L_q = \frac{\lambda^2 E(S^2)}{2(1-\rho)}, \qquad (3)$$

where $E(S^2)$ is the second moment of random variable S, i.e., $E(S^2) = \sigma^2(S) + (E(S))^2$ ($\sigma^2(S)$ usually denotes the variance of S). Let $C_v = \sigma(S)/E(S) = \sigma(S)\eta$ be the coefficient of variation of S ($0 \le C_v \le 1$). Then

$$E(S^2) = \sigma^2(S) + (E(S)^2) = (1 + C_v^2)(E(S))^2 = \frac{1 + C_v^2}{\eta^2},$$

which substituting into (3) together with $\lambda/\mu = \rho$ immediately yields

$$L_q = \frac{(1 + C_v^2)\rho^2}{2(1-\rho)}. \qquad (4)$$

Notice that $C_v^2 = 1$ for exponentially distributed service times, and so, the expression (4) becomes

$$L_q(\exp) = \frac{\rho^2}{1-\rho}.$$

This is the well known formula for the average number of customers waiting in $M/M/1$ queue (see, e.g., [22]). Furthermore, for deterministic service times we have $C_v^2 = 0$, and then the expression (4) becomes

$$L_q(\det) = \frac{\rho^2}{2(1-\rho)} = \frac{1}{2} L_q(\exp), \qquad (5)$$

which is the well known formula for the average number of customers waiting in $M/D/1$ queue (see, e.g., [22]). Now assuming that C_v does not depend on ρ, differentiating the expression (4) with respect to ρ, and substituting this into (2) together with $n_b = 1$, we find that

$$R = r + 2\rho + \frac{(1 + C_v^2)\rho^2(3 - 2\rho)}{2(1-\rho)^2}. \qquad (6)$$

The expression (6) is useful for our computational purposes, as it is presented in the next section.

$$L_q(\det) = \frac{\rho^2}{2(1-\rho)} = \frac{1}{2} L_q(\exp), \qquad (7)$$

$$L_q(\det) = \frac{\rho^2}{2(1-\rho)} = \frac{1}{2} L_q(\exp), \qquad (8)$$

3. NUMERICAL EXPERIMENTS

Using the expression (6), here we present some numerical and graphical results for the specific cost ration concerning different queue of type $M/G/1$. More precisely, for any value $r > 0$ (the first term on the right hand side of (6)), here we present some numerical and graphical results for the difference $R - r$, where R is given by (6), as a function of C_v ($0 \le C_v \le 1$) and ρ ($0.1 \le \rho < 0.9$). Accordingly, the expression (6) can be written as

$$R = r + f(C_v, \rho),$$

where

$$f(C_v, \rho) = 2\rho + \frac{(1 + C_v^2)\rho^2(3 - 2\rho)}{2(1-\rho)^2}. \qquad (9)$$

The graphic of function $f(C_v, \rho)$ for $\rho \in [0.1, 0.9]$ and $C_v \in [0,1]$ is presented in Figure 1.

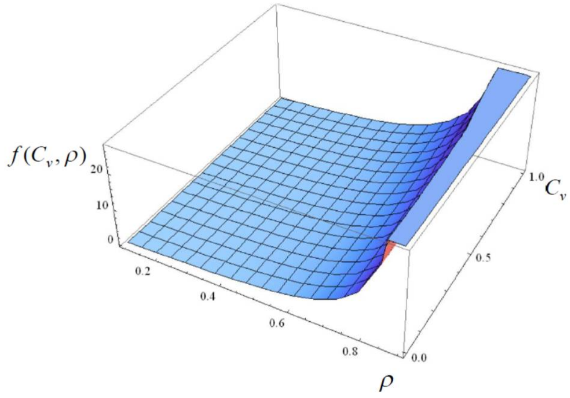

Figure 1. The graphic of function $f(C_v,\rho)$ for $\rho \in [0.1,0.9]$ and $C_v \in [0,1]$

The values of $f(C_v,\rho)$ for different values of C_v and ρ are presented in Tables 1 and 2.

Table 1. The values of $f(C_v,\rho)$ with $C_v = 0,0.2,0.4$

	$f(C_v,\rho)$ for $C_v = 0, 0.1, 0.3$		
ρ	0	0.1	0.3
0.20	0.482	0.482	0.489
0.30	0.821	0.823	0.840
0.40	1.289	1.293	1.333
0.50	2.010	2.090	2.160
0.60	3.245	3.407	3.549
0.70	5.800	6.148	6.452
0.80	12.912	13.808	14.592

Table 2. The values of $f(C_v,\rho)$ with $C_v = 0.5,0.7,0.9$

	$f(C_v,\rho)$ for $C_v = 0.5,0.7,0.9$		
ρ	0.5	0.7	0.9
0.20	0.501	0.521	0.547
0.30	0.876	0.928	0.999
0.40	1.411	1.528	1.685
0.50	2.250	2.490	2.810
0.60	3.731	4.217	4.865
0.70	6.844	7.890	9.284
0.80	15.600	18.288	21.872

Notice that the values in the second column of Table 1 (for $C_v = 0$) correspond to the function $f(C_v,\rho)$ concerning the $M/D/1$ queue, while the values in the last column of Table 1 (for $C_v = 1$) correspond to the function $f(C_v,\rho)$ concerning the $M/M/1$ queue. The graphics of functions $\rho \mapsto f(0,\rho)$ (concerning the $M/D/1$), $\rho \mapsto f(1,\rho)$ (concerning the $M/M/1$ queue) and $\rho \mapsto f(0.5,\rho)$ are presented in Figure 2.

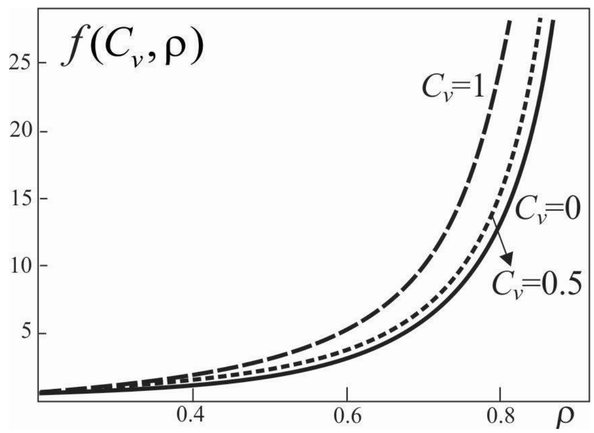

Figure 2. The graphic of functions $\rho \mapsto f(C_v,\rho)$ with $C_v = 0,0.5,1$ with $\rho \in [0.1,0.9]$

4. CONCLUSION

Notice that the analysis of total cost for different port systems was extensively studied in the last ten years. In particular, queuing modeling approach to the study of specific cost ratio can be very significant for understanding various kinds of port operations. Using the famous Pollaczek–Khintchine formula for the average number of ships waiting in $M/G/1$ queue, the expression for specific cost ratio of this queue is derived in this paper. The presented numerical and graphical results give the possibilities of discussing and comparing the values of specific cost ratio of the $M/G/1$ queues with different distributions of service time, as well as with specific cost ratio of the queues investigated in earlier authors' papers.

The obtained numerical results in this paper are compared and discussed. Notice that the obtained expression for specific cost ratio also gives the possibilities of discussing and comparing the values of specific cost ratio of the $M/G/1$ queues with specific cost ratio of the queues investigated in earlier authors' papers. Moreover, we believe that our results would be useful for further research in this research field. In particular, this concerns the problem of minimization of specific cost ratio.

Also, our results should be useful for further study in this direction, especially to study optimal number of berths in port and to the optimization of specific cost ratio in port logistics.

REFERENCES

[1] Dragović, B., Zrnić, Dj. and Radmilović, Z.: *Ports & container terminals modeling*, Research monograph, University of Belgrade, Faculty of Transport and Traffic Engineering, 2006.

[2] Dragović, B., Zrnić, Dj. N. and Park, N.K.: *Container Terminal Performance Evaluation*, Research Monograph, University of Belgrade, Faculty of Mechanical Engineering, 2011.

[3] Dragović, B. and Meštrović, R.: An Analysis of Specific Cost Ratio in a Port: Part II, in: Proceedings of 11th International Conference ″Research and Development in Mechanical Industry″ (RaDMI-2011), Vol. I, Sokobanja, Serbia, September 2011, pp. 382-385.

[4] Dragović, B., Meštrović, R., Mikijeljević, M. and Markolović, S.: A $M/M/n_b$ queue model-based study of specific cost ratio in a port, in: Proceedings of XX International Conference on Material Handling, Constructions and Logistics, MHCL 2012, University of Belgrade, Faculty of Mechanical Engineering, 2012, pp. 369-372.

[5] Dragović, B., Zrnić, N.Đ. and Škurić, M.: New and old results of queuing models for port modeling, in: Proceedings of European Conference on Shipping & Ports, ECONSHIP 2011, 2011, pp. 1-14.

[6] Dragović, B., Meštrović, R., Zrnić, N.Đ. and Grandis, B.: An analysis of specific cost ratio for $M/M/1/k$ queue in a port, in: Proceedings of 12th International Conference ″Research and Development in Mechanical Industry″ (RaDMI-2012), Vol. II, Vrnjačka Banja, Serbia, 2012, pp. 960-964.

[7] Dragović, B., Meštrović, R., Zrnić, N.Đ. and Škurić, M.: Specific cost ratio in a port: analytical and computational approaches, in: Proceedings of 7th International Conference on Ports and Waterways - POWA 2012, Zagreb, Croatia, 2012, pp. 1-8.

[8] Dragović, B., Meštrović, R., Zrnić, N. and Dragojević, D.: Specific cost ratio in a port modelling by $M/E_k/1$ queue, FME Transactions, Vol. 46, No. 3, pp. 355-359, 2018.

[9] Dragović, B. and Meštrović R.: The estimation of the specific cost ratio in a port, in: Proceedings of Annual Conference International Association of Maritime Economics – IAME 2018, Mombasa, Kenya, 2018, pp. 1-12.

[10] Dragović B., Meštrović R., Zrnić N. and Dragojević, D.: An analysis of specific cost ratio in a port by $M/E_k/1$ queue: Part I, Second part of Proceedings of XXII International Conference on Material Handling, Constructions and Logistics, MHCL 2017: Special Session – Maritime and Port Logistics, Bar, Montenegro, 2017, pp. 341-344.

[11] Meštrović, R. and Dragović, B.: An Analysis of Specific Cost Ratio in a Port: Part I, in: Proceedings of 11th International Conference ″Research and Development in Mechanical Industry″ (RaDMI-2011), Vol. I, Sokobanja, Serbia, September 2011, pp. 93-97.

[12] Meštrović, R., Dragović, B., Grandis, B. and Petranović, M.: Estimates of specific cost ratio in a port for $M/M/1/k$ queue, in: Proceedings of XX International Conference on Material Handling, Constructions and Logistics, MHCL 2012, University of Belgrade, Faculty of Mechanical Engineering, 2012, pp. 369-372.

[13] Meštrović, R., Dragović, B., Zrnić, N.Đ. and Škurić, M.: A multi-server queuing model study of specific cost ratio in a port, Technical Gazette, Vol. 20, No. 5, pp. 781-786, 2013.

[14] Meštrović, R., Dragović, B., Zrnić, N. and Dragojević, D.: A relationship between different costs of container yard modelling in port using queuing approach, FME Transactions, Vol 46, No. 3, pp. 367-373, 2018.

[15] Meštrović, R., Dragović, B., Zrnić N. Đ. and Dragojević, D.: An analysis of specific cost ratio in a port by $M/E_k/1$ queue: Part II, in: Second part of Proceedings of XXII International Conference on Material Handling, Constructions and Logistics, MHCL 2017: Special Session – Maritime and Port Logistics, Bar, Montenegro, 2017, pp. 345-348

[16] Noritake, M.: Congestion cost and pricing of seaports, Journal of Waterway, Port, Coastal, and Ocean Engineering, Vol. 111, No. 2, pp. 354-370, 1985.

[17] Noritake, M. and Kimura, S.: Optimum number and capacity of seaport berths, Journal of Waterway, Port, Coastal, and Ocean Engineering, Vol. 109, No. 3 (1983), pp. 323-339.

[18] Radmilović, Z., Dragović, B. and Meštrović, R.: Optimal number and capacity of servers in $M^{X=\bar{a}}/M/c(\infty)$ queuing systems, International Journal of Information and Management Sciences, Vol. 16, No. 3, pp. 1-17, 2005.

[19] Škurić, M.**,** Dragović, B. and Meštrović, R.: Some results of queuing approaches at container yard, International Journal of Decision Sciences, Risk and Management, Vol. 3, Nos. 3/4, pp. 260-273, 2011.

[20] Taniguchi, E., Noritake, M., Yamada, T. and Izumitani, T.: Optimal Size and Location Planning of Public Logistics Terminals, Transportation research part E, Vol. 35, No. 3, pp. 207-222, 1999.

[21] Tijms, H.C.: *A First Course in Stochastic Models*, John Wiley and Sons, 2003.

[22] Vukadinović, S.: *Elements of Queuing Theory* (in Serbian), University of Belgrade, Faculty of Transport and Traffic Engineering, Belgrade, 1983.