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Some Further Consideration of Modeling Process at the Seaport Automobile Terminals

In this paper we consider the loading processes at seaside link of Seaport automobile Automobile Terminal (SAT) as a bulk queuing model of the form $M^{X\equiv g}/D/1/\infty/s$. We deduce the recursive equation for the steady-state probabilities concerning this queue model. Using this recursive equation, we obtain the explicit expressions for the steady-state probabilities with indices up to four. All these steady-state probabilities are expressed as a function on the utlization factor (ρ) and the size of every arriving group at the ramp (g). Related numerical and graphical results are also presented. As applications, it is considered and discussed the modeling process at the SAT in the Port of Bar.

Keywords: *SAT, Port of Bar, Traffic modeling, $M^{X\equiv g}/D/1/\infty/s$ bulk queuing model, Steady-state probabilities, Utilization factor*

1. INTRODUCTION

This paper deals with the traffic modeling of Seaport Automobile Terminal (SAT) in the port of Bar, which faced a fast growth in the past few years. It presents a continuation of our studies which are given in [3]-[6], [10] and [11]. As in these previous published studies, the main aim of the paper concerns to the operational policies at the SAT because the statistical analysis shows the difference in ships' size (see [9] and [16]). Furthermore, during last eight years it is presented a bigger number of ships going at SAT berths.

The defined analytical model corresponds to the bulk queuing model $M^{X\equiv g}/D/1/\infty/s$ with one ship ramp (server) in which the size X of every arrival group at the ramp is a constant random variable.

In [14] the authors paid attention on the calculation of the level of service in a Ro-Ro/Pax terminal to its capacity from the ship's point of view. Moreover, similar studies were given in [15]. In [17] some quality indicators of a Ro-Ro terminal are estimated, while in [18] authors discussed about the problem of the storage yard of seaport automobile terminal, where import vehicles will be stored temporarily, while third one developed a complete taxonomy of the disturbances that happened in a Ro-Pax terminal. A framework to evaluate their consequences on the normal performance of the terminal, in terms of severity and frequency, has been built. On the other hand, an integral optimization model and estimated manpower planning at the Bremerhaven port represented a very important study that was done in [13] where it was derived an integral decision model as a complex combinatorial problem. In [7] the authors discussed about the planning of transshipment of vehicles based on a multi-agent system (MAS). Some other studies are also used to applied related bulk queuing model at SAT such as [1], [2], [8] and [12]

The rest of the paper containts some the following Sections. A description of the $M^{X\equiv g}/D/1/\infty/s$ model is given in Section 2 with modeling methodology for considered loading process in the Port of Bar. Using a recurrence method for solving this system, in Section 3 we give some numerical and graphical results involving the expressions for the steady-state probabilities with indices up to four for the considered bulk queuing model. Concluding remarks are given in Section 4.

2. A DESCRIPTION OF RELATED BULK QUEUING MODEL

As noticed above, this paper discusses the loading process at seaside link of SAT as a queuing model with bulk arrivals. The stochastic characteristics and assumptions of loading operations are as follows (see [8] and [9]):

- time of arrival of a single automobile or in bulk (automobiles' group) at the ship ramp cannot be precisely given;
- the loading time through the ship ramp is a constant service time;
- the ship's ramp is not always occupied; in some periods there are no automobiles (the capacity is under utilized) and there are the time intervals of high utilization when the queue is formed;
- the loading operation at SAT via ship ramp includes the following facts: automobiles are moved from storage yard to the ship ramp; waiting in front of ship ramp if the ramp is occupied; loading at the ship ramp, parking on the assigned slots inside ship; returning of drivers by accompanying cars to storage yards to take another group of automobiles. This cycle is called the

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turnaround time for the loading process of automobiles onto ships, $\bar{t}_c$.

The seaside operations at SAT, i.e. the loading process of ships may be considered as a bulk queuing model. In this case, customers are groups of automobiles, and the service channels are ships' ramps operating for the loading/unloading of automobiles. In the seaside link of SAT is assumed as follows:

- the applied queuing model is a stationary with infinite waiting area at ship ramp;
- the sources of arriving pattern are not integral parts of loading process of ships via ramps;
- the service channel is the ship ramp with similar or identical and independent handling capacities;
- the units arrivals can be single automobile or automobile groups;
- automobiles are loaded via ship ramp or waiting to be loaded and in this case none can be rejected (accordingly, the queue length is assumed to be infinite);
- the size of an arriving group of automobiles is a random variable;
- the queue discipline is first come first served by group's bulk and random within the group's bulk.

More formally, the described model is known as a $M^X/D/1$ queue. The automobiles allocated on storage areas arrive in groups of size g at the ramp according to a time-homogeneous Poisson process with (the mean arrival) rate λ in a considered unit time (in view of the facts that the arrivals of groups of automobiles in all cycles are mutually independent). The size of every arriving group at the ramp is a constant random variable X (with the distribution $P\{X=g\}=1$). The ramp is in fact a single service that is loading of automobiles via the ship ramp. We suppose that a related service time is determined, that is, the mean loading rate per ship ramp is μ. Since in each considered case the mean arrival rate λ with respect to the considered unit time (of a group) is relatively small with the respect to the mean loading rate per ship ramp, we can assume that each automobile will find a waiting place available upon arrival; so that, we can suppose that the related queue model possesses an infinite capacity.

Accordingly, the arrivals of automobiles from storage areas in groups at the ramp and loading stages via ship ramp can be viewed as the $M^{X\equiv g}/D/1/\infty/s$ queue. Unfortunately, in our knowledge does not exist formulae in closed form for performances of such a type of bulk queue with a finite population. However, since the size of group g is usually small at SAT with respect to the size of the population (total number of automobiles) s (in fact, $g/s<1/100$), our $M^{X\equiv g}/D/1/\infty/s$ queue may be well approximated by the $M^{X\equiv g}/D/1$ queue with infinite population (cf. [10] and [11]).

Then, in accordance to the above notations, we have:

- The arrival rate of every group in a particular gang is $\bar{t}_c$ and it is actually equal to the previously defined turnaround time; hence, the mean arrival rate λ of a considered queue model is $\lambda = h/\bar{t}_c$. In order to ensure sustainable operations, g drivers are grouped into h gangs. In every cycle are involved h gangs, where in each of these gangs are engaged g drivers. (Here it is used the fact that the arrivals of groups of cars are independent, and the well known fact that if X_1 and X_2 are two independent Poisson variables with mean λ_1 and λ_2, respectively, then $X = X_1 + X_2$ is also a Poisson variable with mean $\lambda_1+\lambda_2$).
- The related service time is a constant $1/\mu = 5s$, that is, the service rate (of passages of cars over the ramp) is $\mu=12$ per one minute.
- The utilization factor (the server utilization) ρ is $\rho=\lambda g/\mu = hg/(\bar{t}_c\mu)=\alpha hg$ with $\alpha = 1/(\bar{t}_c\mu)$ for a particular embarkation process).
- Total number of operational gangs that are particularly engaged in each cycle, h, is 4. Number of cars that are transported in each operational gang, g, is 6.

Notice that in recent paper [4] Dragović et al. analyzed the traffic modeling of operations at Ro-Ro automobile terminal (also known as the Seaport Automobile Terminal [7]) in the Port of Bar. In particular, using some analytical results for the batch queue system $M^X/D/c$ established in [8] (also see [1] and [2]), Dragović et al. [4] deduced some suitable formulae for certain basic stochastic characteristics (performances) related to the loading operations at terminal over ship ramp at seaside link of Ro-Ro automobile terminal in the Port of Bar. These performances are derived without the use of notion of the state probabilities of related queue model. Here we focus our attention to the determination of steady-state probabilities P_n, $n=0,1,2,....$ (P_n denotes the probability that n customers are in the system).

Now consider the above described stationary queue $M^{X\equiv g}/D/1$ with related parameters. Furthermore, let $Y(t)$ denote the total number of arrivals during the period $(0,t)$, and let

$$\pi_n(t) = P\{Y(t)=n\}. \qquad (1)$$

Then under a general assumption that the size of every arriving group at the system is a random variable X (with $P\{X=n\}=a_n$, $n=0,1,2,....$, the following recurrence formulae are satisfied (see [1] and [6]):

$$\pi_0(t) = e^{-\lambda t}, \qquad (2)$$

$$\pi_{n+1}(t) = \frac{\lambda t}{n+1}\sum_{j=0}^{n}(n-j+1)a_{n-j+1}\pi_j(t), \qquad (3)$$

$$n=0,1,2,....$$

Since in our case we have $a_g = 1$ and $a_i = 0$ for each $i \neq g$, by using mathematical induction, the formulae (2) and (3) easily yield (see [10] and [11])

$$\pi_{kg}(t) = e^{-\lambda t}\frac{(\lambda t)^k}{k!},\ k = 0,1,2,\dots, \quad (4)$$

and

$$\pi_n(t) = 0, \text{ otherwise.} \quad (5)$$

For our bulk queuing model the steady-state Chapman-Kolmogorov equations for the probability distribution $\{P_j, j = 0,1,2,\dots\}$ are given by (see Equations (1) and (2) in [1])

$$P_0 = \pi_0\left(\frac{1}{\mu}\right)(P_0 + P_1) \quad (6)$$

$$P_n = \pi_n\left(\frac{1}{\mu}\right)(P_0 + P_1) + \sum_{m=2}^{n+1} P_m \pi_{n+1-m}\left(\frac{1}{\mu}\right), \quad n = 1,2,\dots \quad (7)$$

Then substituting (2), (4) and (5) into (6) and (7), and using the fact that after a routine calculation, in view of the fact that $\lambda/\mu = \rho/g$, the equations (6) and (7) becomes respectively (see [10] and [11])

$$P_0 = e^{-\rho/g}(P_0 + P_1), \quad (8)$$

$$P_n = \pi_n\left(\frac{1}{\mu}\right)(P_0 + P_1) + \sum_{i=0}^{[(n-1)/g]} P_{n+1-ig} e^{-\rho/g}\frac{(\rho/g)^i}{i!}, \quad (9)$$

for all $n = 1,2,\dots,$ where, as usual, $[a]$ denotes the greatest integer not exceeding a, and in (9) $\pi_n(\rho/g)$ and $\pi_n(1/\mu)$ may be replaced by a suitable expression given by (4) or (5).

Furthermore, it is known that in the case of any single-server queueing system (see, e.g., [2])

$$P_0 = 1 - \rho. \quad (10)$$

For computational purposes, it would be useful the following result.

Proposition. *Under previous conditions, assumptions and notations, we have*

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \cdots = \frac{P_g}{P_{g-1}} = e^{\rho/g}. \quad (11)$$

In other words, the sequence $P_1, P_2, \dots, P_{g-1}, P_g$ *is a geometric progression with the quotient* $e^{\rho/g}$.

Proof of Proposition. First observe that

$$[(n-1)/g] = 0 \text{ for all } n = 1,2,\dots,g.$$

Hence, using the expression (9), we find that

$$P_n = \pi_n\left(\frac{1}{\mu}\right)(P_0 + P_1) + P_{n+1}e^{-\rho/g} \quad (12)$$

$$\text{for all } n = 1,2,\dots,g-1.$$

Furthermore, by (4) and (5), we have

$$\pi_n\left(\frac{1}{\mu}\right) = e^{-\lambda/\mu} = e^{-\rho/g}$$

$$\text{for all } n = 1,2,\dots,g-1,$$

which substituting into (12) gives

$$P_n = P_{n+1}e^{-\rho/g},$$

whence it follows that

$$\frac{P_{n+1}}{P_n} = e^{\rho/g} \quad \text{for all } n = 1,2,\dots,g-1.$$

This complete the proof of Proposition.

3. EXPERIMENTAL STUDIES

Here we give some numerical and graphical results for the steady-state probabilities P_n with $n = 0,1,2,\dots,7$ for the bulk queuing model $M^{X \equiv g}/D/1$ described in the previous section whose size of the arriving group is $g = 7$. Substituting $g = 7$ and $P_0 = 1 - \rho$ given by (10) into (8) we obtain

$$P_1 = (1-\rho)(e^{\rho/7} - 1) \quad (13)$$

Notice that by Proposition of the previous section,

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \frac{P_5}{P_4} = \frac{P_6}{P_5} = \frac{P_7}{P_6} = e^{\rho/7}. \quad (14)$$

The expressions (13) and (14) immediately yield

$$P_2 = P_1 e^{\rho/7} = (1-\rho)(e^{\rho/7} - 1)e^{\rho/7}, \quad (15)$$

$$P_3 = P_2 e^{\rho/7} = (1-\rho)(e^{\rho/7} - 1)e^{2\rho/7}, \quad (16)$$

$$P_4 = P_3 e^{\rho/7} = (1-\rho)(e^{\rho/7} - 1)e^{3\rho/7}, \quad (17)$$

$$P_5 = P_4 e^{\rho/7} = (1-\rho)(e^{\rho/7} - 1)e^{4\rho/7}, \quad (18)$$

$$P_6 = P_5 e^{\rho/7} = (1-\rho)(e^{\rho/7}-1)e^{5\rho/7}, \qquad (19)$$

and

$$P_7 = P_6 e^{\rho/7} = (1-\rho)(e^{\rho/7}-1)e^{6\rho/7}, \qquad (20)$$

By using the expressions (10) and (13) - (20), we obtain the numerical and graphical results given in Tables 1 and 2, and Figures 1 and 2, respectively.

Table 1. The values of P_n ($n = 0,1,2,3$) as a function of utilization factor ρ

(ρ)	P_0	P_1	P_2	P_3	$\sum_{i=0}^{3} P_i$
0.4	0.6	0.035	0.037	0.040	0.754
0.5	0.5	0.037	0.040	0.046	0.665
0.6	0.4	0.036	0.039	0.043	0.564
0.7	0.3	0.032	0.035	0.039	0.448
0.8	0.2	0.024	0.027	0.034	0.316
0.9	0.1	0.013	0.016	0.018	0.167

Table 2. The values of P_n ($n = 4,5,6,7$) as a function of utilization factor ρ

(ρ)	P_4	P_5	P_6	P_7	$\sum_{i=4}^{7} P_i$
0.4	0.042	0.044	0.047	0.050	0.183
0.5	0.046	0.050	0.053	0.057	0.205
0.6	0.046	0.050	0.055	0.060	0.212
0.7	0.043	0.047	0.052	0.057	0.199
0.8	0.034	0.038	0.043	0.048	0.163
0.9	0.020	0.023	0.027	0.030	0.099

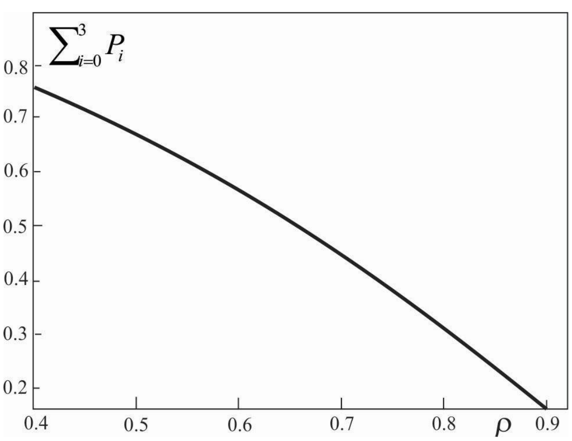

Figure 1. The sum $\sum_{i=0}^{3} P_i$ of steady-state probabilities as a function on ρ $(0.4 \le \rho \le 0.9)$

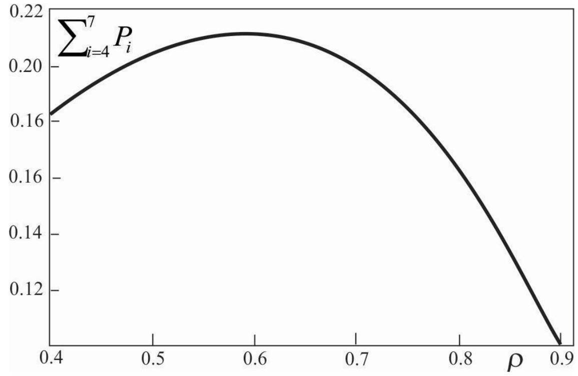

Figure 2. The sum $\sum_{i=4}^{7} P_i$ of steady-state probabilities as a function on ρ $(0.4 \le \rho \le 0.9)$

From Tables 1 and 2 we see that the all values P_n ($n = 1,2,\ldots,7$) are less than 0.06 for all considered values of ρ. Furthermore, from Table 1 we see that $\sum_{i=0}^{3} P_i$ attains a maximal value equals to 0.754 for $\rho = 0.4$, while from Table 2 we see that $\sum_{i=4}^{7} P_i$ attains a maximal value equals to 0.212 for $\rho = 0.6$.

As noticed in Section 2, the utilization factor (the service utilization) ρ is $\rho = \lambda g/\mu = hg/(\bar{t}_c\mu) = \alpha hg$ with $\alpha = 1/(\bar{t}_c\mu)$ (for a particular embarkation process). From this we see that for a fixed service time $1/\mu$ (which is in our considered case equal to 5s) and the arrival rate of a group of automobiles, the value of ρ is proportional with the related product hg (the total number of drivers/accompanying cars for operational gangs which are engaged in considered embarkation process in the Port of Bar). We believe that this fact should be useful for port authority to develop strategies and directions in order to improve some basic/important Ro-Ro automobile terminal's performances.

4. CONCLUSION

This study is based on previous investigations [3]-[6], [10] and [11] and it presents the modeling processes at the SAT in the Port of Bar. Analyzis of some stochastic and deterministic performances of related loading operations suggests a suitable queue model for describing arrivals and services of automobiles at the ship ramp (service). Accordingly, the loading processes at seaside link of SAT is desribed as a queuing model $M^{X\equiv g}/D/1$ with bulk arrivals and a constant size of an arriving group at the ramp. The proposed queuing model allows us to derive an infinite system of algebraic equations for related steady-state probabilites. Using a suitable recursive form of this system, it can be explicitly expressed these probabilities as a function of utilization factor (service utilization) and a size of an arriving group at the ramp. The obtained results give a possibility for further study on related topic in order to optimize some significant performances of modeling processes at the SAT.

ACKNOWLEDGEMENTS

The study was carried out within the Bilateral project (Montengro and Serbia) entitled ”Applying the concept of sustainable development to the intermodal transport connection of the Montenegro and Serbia (Green corridors on the intinirers since the Sea to Danube)", financed by the Ministry of Science of Montenegro and the Ministry of Education, Science and Technological Development of Serbia.

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