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Modelling the Container Yard as an Operational System in a Port: A Methodological Approach

The paper presents $M^X/M/c$ batch queues in which the group size is given by the shifted-Poisson and Poisson-like distributions. This queue model deduced the expressions for the specific cost ratio involving the state probabilities, the utilization factor, the mean and the variance of the group size. Proposed model discusses the total queuing system costs of container at container yard (CY) and specific cost ratio to improve the best values for container performances at CY. The analytical approach make the model appropriate to analyze. The special cases can be solved exactly, which is shown in another paper dealing with the application of this methodological approach.

Keywords: *Container yard, Operational system, Modelling process, Specific cost ratio, Queuing theory.*

1. INTRODUCTION

The paper provides a modelling process at the Container Yard (CY) as an Operational System. The problems concerning CY are considered by applying a $M^X/M/c$ queue. Using this model for defining the strategies at CY and to calculate the total cost of the system. It is obvious that the arrival and service processes of containers at CY must give the input data in the shape of some statistical distribution.

On the basis of a comprehensive consideration of various complex factors that affect CY operation, this mathemathical approach was proposed to modeling and analyzing the operation processes of the CY. A CY in the port can be considered as a queuing system defined by basic parameters: the container arrival rate and the container service rate in an observed period of time. It is evident that the optimal number and capacity of servers must be of greater importance in real system. The total cost of the system can be also determined by the specific number of servers. As servers at CY are used specific types of yard cranes (YCs). There are a few types of YCs at the container terminals (CTs): rubber tyred gantry crane, rail mounted gantry crane, straddle carriers, some of forklift types in small CYs and so on. Therefore, the costs of YCs make important point for obtaining total costs of containers at the terminal.

The analytical modelling conducted here offers advantages through more structural insights in relations between parameters. Also, it helps in the modelling conceptualization phase (see more in [35]).

The literature review is presented in Section 2, whilst Section 3 shown the mathematical models with the specific cost ratio by a queuing system. Section 4 contains conslusions.

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2. LITERATURE REVIEW

It is well known that containers arrive at CY in batches and its behavior must fit some statistical distribution. Considering that, it is noted that for valid analysis some studies used batch arrival queues and batch arrival multi-server queues. Markov theory and reliability models for the estimation of the associated risks and costs that can result in delays due to machinery breakdowns in CTs were used in [22]. Markov chain method for defining the arrival in batches to a single channel with arbitrary service time distribution was applied in [12]. Also, using renewal theorem of batch arrival, in [1] was solved a single-server queuing system. All these authors have carried out their own theory about queues, specifying the best approach for modeling CTs in port. Speaking of batch arrival of customers, in [17] has made a comparison of analytical and simulation planning models of CTs.

The analysis of a queue with batch arrivals and batch-dedicated servers is explained in [14]. In [31] was developed an analytical methodology of bulk queuing system. The authors deal with the port storage locations as queuing systems with bulk arrivals and a single service in [32]. The optimal number of servers in queuing system with bulk arrivals by minimizing the total costs of system are determined in [33]. In [10] and [37] authors discussed about the anchorage-ship-berth link utilizing queuing theory with bulk arrivals. In [20] was analyzed a multi-server queue with bulk arrivals and finite-buffer space. In [19] was used birth-and-death modelling in order to examine the improvement of CT operations. Dynamic system performance evaluation in the port utilizing queuing models with batch arrivals was studied in [8]. More recently, a few very interesting papers based on stochastic approach and queuing network model (QNM) are published. Solving semi-open queuing networks with time-varying arrivals are applied at CT in [3]. A stochastic modeling approach for parallel process flows and solution methods for closed QNM with general two-phase servers with applications in automated CTs have been studied in [18]. Model-driven decision support system for integrated container handling and QNM for resource blocking, locking and vehicle interactions are developed in [21]. In [34] was developed an integrated QNM for overlapping operations at a CT. Model the interactions among train and external truck containers at shared cranes and model synchronization of train bulk arrival containers with transport vehicles were presented in [35]. Otherwise, the remarshaling problem for inbound containers is addressed and two new integer linear programming models are proposed to solve the problem at an automated CT in [15].

3. MATHEMATICAL MODELS

We consider a batch arrival multi-server queue $M^X/M/c$ described as follows. Containers arrive at a port container yard (CY) in batches according to a time-homogeneous (stationary) Poisson process with mean arrival rate λ. A CY is a single or multi-channel system with c yard cranes (YCs) for the service. The c YCs have independent, exponentially distributed service times with common average service time $1/\mu$ (μ is the service rate). The queue discipline is first come first served by tows batch and random within the tow batch. The number of containers that arrive for service at the same time is a discrete random variable X with distribution given by $a_k = P(X=k)$, $k \geq 1$ (whereas k = number of containers in group), whose mean is $E(X)=\bar{a}$ and the variance $\sigma^2(X)=\sigma^2$. Furthermore, the interarrival times, the batch sizes and service times are mutually independent. The service times of service batches (containers) are independent of the arrival process. The utilization factor or server occupancy of considered queue model is $\rho = (\lambda\bar{a})/(c\mu)$, and its traffic intensity is usually defined as $\theta = \lambda/\mu$.

Let P_n $(n=0,1,2,\ldots)$ be the (steady-state) probability that there are n containers in a port CY. Let $L=L(c)$ be the average number of containers at the CY, and let $L_q = L_q(c)$ be the average number of containers in queue. Then by [2] we have

$$L - L_q = c + \sum_{n=0}^{c}(n-c)P_n \cdot \tag{1}$$

Further, by [2],

$$L - L_q = \theta\bar{a} = \frac{\lambda}{\mu}\bar{a} = c\rho \cdot \tag{2}$$

Substituting (2) into (1) we obtain

$$\sum_{n=0}^{c-1}(c-n)P_n = c(1-\rho) \tag{3}$$

From (3) we immediately get

$$P_0 = 1 - \rho - \frac{\sum_{n=1}^{c}(c-n)P_n}{c} \cdot \tag{4}$$

Furthermore, the probabilities P_0 and P_n $(n=1,2,\ldots)$ satisfy the following Kabak's recurrence relations [9] and [16]:

$$P_n = y(n)\sum_{k=0}^{n-1}P_k A_{n-k}, \quad n = 1, 2, \ldots, \tag{5}$$

where

$$y(n) = \frac{\lambda}{\mu(n)} \text{ with } \mu(n) = \mu\min\{n, c\}, \; n = 1, 2, \ldots \tag{6}$$

and the coefficients A_j are defined as

$$A_j = 1 - \sum_{i=0}^{j-1}a_i, \quad a_i = P(X=i) \tag{7}$$

for each $j = 2,3,\ldots$, and $A_1 = 1$.

Recall that the average number of containers present in considered queuing system with c yard cranes is usually defined as a sum

$$L = L(c) = \sum_{n=0}^{\infty} nP_n \cdot \tag{8}$$

For our purposes, it is suitable the following formula for $L = L(c)$ related to arbitrary queue model $M^X/M/c$ described above [2]:

$$L(c) = \frac{\theta(\bar{a} + A''(1)/2) + \sum_{n=0}^{c-1}n(c-n)P_n}{c - \theta\cdot\bar{a}}, \tag{9}$$

where $A(z) = \sum_{k=0}^{\infty} a_k z^k$ is the probability generating function of the random variable X (with the mean $E(X)=\bar{a}$ and the variance $\sigma^2(X)=\sigma^2$). Since by the well-known identity, $A''(1) = \sigma^2 + (\bar{a})^2 - \bar{a}$, substituting this into (9) gives

$$L(c) = \frac{\theta}{2}\cdot\frac{\sigma^2 + \bar{a}^2 + \bar{a}}{c - \theta\cdot\bar{a}} + \frac{\sum_{n=0}^{c-1}n(c-n)P_n}{c - \theta\cdot\bar{a}}. \tag{10}$$

3.1 The specific cost ratio by a queuing system

The analysis of total cost for different port system is extensively studied by many authors. In particular, this concerns the problem of determination and minimization of the so called the specific cost ratio which is rarely investigated in the literature. Namely, the determination of specific cost ratio and the optimal numbers of berths in a port in the sense of minimization of related specific cost ratio for certain single arrival and bulk arrival queueing systems were studied in [4]-[10], [23]-[33], [36] and [37]. By considering the total annual cost for queuing systems with c yard cranes, it is derived in [13] and [36] the following expression:

$$C_{qs} = OT_{yc}\cdot T_e \cdot c + OT_c \cdot T_e \cdot L(c), \tag{11}$$

where

C_{qs}	- total annual cost for queuing system with c YCs;
OT_{yc}	- the daily operating cost of YC;
T_e	- defined period of year (365 days);
c -	- number of YCs;
OT_c	- the daily cost of containers;
T_e	- defined/considered period of year (365 days);
OT_c	- daily cost of containers and
$L(c)$	- the average number of containers that are present in a queuing system.

Dividing the equation (11) by $OT_c \cdot T_e$, we obtain (cf. [36])

$$R_c = \frac{C_{qs}}{OT_c \cdot T_e} = \frac{OT_{yc}}{OT_c} \cdot c + L(c) = r_{cy} \cdot c + L(c) \qquad (12)$$

where R_c is the *specific cost ratio* (total annual cost for queuing system with c yard cranes/total annual cost of containers) and $r_{cy} = OT_{yc} / OT_c$ is the daily yard crane–container cost ratio (the daily operating cost of yard crane/the daily cost of containers).

From (12) we see that the value of specific cost ratio is closely related to the average number of containers at the yard, $L = L(c)$. Here we consider the case when containers arrive and service at a port container yard in batches according to batch arrival multi-server queue $M^X / M / c$ described in Section 3. Then substituting (10) into (12) we obtain (cf. [36])

$$R_c = r_{cy} \cdot c + \frac{\theta}{2} \cdot \frac{\sigma^2 + \bar{a}^2 + \bar{a}}{c - \theta \cdot \bar{a}} + \frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c - \theta \cdot \bar{a}}. \qquad (13)$$

Notice that in [36] was studied the specific cost ratio concerning the queue model $M^X / M / c$, where X is a constant or geometric distribution. As applications, related numerical examples are presented in [35] for the port of Bar, Montenegro.

The shifted-Poisson distribution X

Here we consider the case when a group of containers that arrive at yard has the *shifted-Poisson distribution* X with the parameter a [11], that is,

$$P(X = i) = a_i = e^{-a} \cdot \frac{a^{i-1}}{(i-1)!}, \quad i = 1,2,\ldots; \; a > 0, \qquad (14)$$

Notice that $X = Y + 1$, where Y is the *Poisson distribution* with the same parameter a, and hence, the mean of X is

$$\bar{a} = m(X) = m(Y) + 1 = a + 1, \qquad (15)$$

and the variance of X is

$$\sigma^2 = \sigma^2(X) = \sigma^2(Y) = a. \qquad (16)$$

Substituting (15) and (16) into (13), we find that

$$R_c = r_{cy} \cdot c + \frac{\theta}{2} \cdot \frac{a^2 + 4a + 2}{c - \theta(a+1)} + \frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c - \theta(a+1)}. \qquad (17)$$

From $\rho = \theta \bar{a} / c = \theta(a+1)/c$ it follows that $\theta = c\rho/(a+1)$, which putting in (17) gives the following formula:

$$R_c = r_{cy} \cdot c + \frac{(a^2 + 4a + 2)\rho}{2(a+1)(1-\rho)} + \frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c(1-\rho)}. \qquad (18)$$

Notice that in view of (12) and (18) we have

$$L(c) = \frac{(a^2 + 4a + 2)\rho}{2(a+1)(1-\rho)} + \frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c(1-\rho)}. \qquad (19)$$

The Poisson-like distribution X

Motivated by the Poisson and the shifted-Poisson distribution, here we consider the case when a group of containers that arrive at yard has the *Poisson-like distribution* X with the parameter $a > 0$, whose probability law is defined as

$$P(X = i) = a_i = \frac{1}{e^a - 1} \cdot \frac{a^i}{i!}, \; i = 1,2,\ldots \qquad (20)$$

Notice that X is well defined random variable in view of the fact that

$$\sum_{i=1}^{\infty} \frac{1}{e^a - 1} \cdot \frac{a^i}{i!} = \frac{1}{e^a - 1}\left(-1 + \sum_{i=0}^{\infty} \frac{a^i}{i!}\right) = 1.$$

For our computational purposess we will need the following result.

Proposition. *The mean and the variance of* X *are respectively given by*

$$\bar{a} = E(X) = \frac{ae^a}{(e^a - 1)} \qquad (21)$$

and

$$\sigma^2 = \sigma^2(X) = \frac{ae^a(e^a - a - 1)}{(e^a - 1)^2}. \qquad (22)$$

Proof. We will use the well known facts that the mean and the variance of the following Poisson distribution Y with parameter $a > 0$

$$P(Y = i) = a_i = e^{-a} \cdot \frac{a^i}{i!}, \quad i = 0,1,2,\ldots; \; a > 0, \qquad (23)$$

are recpectively given by

$$E(Y) = a, \quad \text{and} \quad \sigma^2(Y) = a. \qquad (24)$$

By using the first equality of (23), from (20) we obtain

$$E(X) = \frac{1}{e^a - 1}\sum_{i=1}^{\infty} i \cdot \frac{a^i}{i!} = \frac{e^a}{e^a - 1}\sum_{i=1}^{\infty} i \cdot e^{-a}\frac{a^i}{i!}\frac{e^a}{e^a - 1}E(Y) = \frac{ae^a}{e^a - 1},$$

which yields (21).

From the equalities given by (24) we have

$$E(Y^2) = \sigma^2(Y) + (E(Y))^2 = a^2 + a.$$

Using the above equality, (21), (22) and comparing (20) and (23), we obtain

$$\sigma^2(X) = E(X^2) - (E(X))^2 = \frac{e^a}{e^a - 1}\sum_{i-1}^{\infty} i^2 e^{-a}\frac{a^i}{i!} - \frac{a^2 e^{2a}}{(e^a - 1)^2}$$

$$= \frac{e^a(a^2 + a)}{e^a - 1} - \frac{a^2 e^{2a}}{(e^a - 1)^2} = \frac{ae^a(e^a - a - 1)}{(e^a - 1)^2},$$

which yields (13). This completes proof of proposition.

From $\rho = \theta \bar{a} / c = \theta a / c$ it follows that $\theta = c\rho / a$, which together with (21) and (22) putting in (10) and then in (12) immediately gives the following expression for R_c:

$$R_c = r_{cy} \cdot c + \frac{(a+2)\rho}{2(1-\rho)} + \frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c(1-\rho)}, \qquad (25)$$

where

$$L(c) = \frac{(a+2)\rho}{2(1-\rho)} + \frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c(1-\rho)} \qquad (26)$$

4. CONCLUSIONS

Our modelling approach is useful for examining the implications of containers which arrive at a port container yard (CY) in batches as discrete random variable. We use $M^X / M / c$ queue in which the size X of arriving group is distributed as a constant and a geometric distribution in [36], while we assume here the shifted-Poisson and Poisson-like distributions.

This approach to analysis of specific cost ratio can be very significant for understanding various kinds of port operations inside CY. For these purposes, it is necessary to establish the expressions for specific cost ratio in terms of some basic port performances

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