Gauss-type quadrature rules with respect to the external zeros of the integrand

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Abstract

In the present paper, we propose a Gauss-type quadrature rule into which the external zeros of the integrand (the zeros of the integrand outside the integration interval) are incorporated. This new formula with n nodes, denoted by $\mathcal{G}_n$, proves to be exact for certain polynomials of degrees greater than $2n-1$ (while the Gauss quadrature formula with the same number of nodes is exact for all polynomials of degrees less than or equal to $2n-1$). It turns out that $\mathcal{G}_n$ has several good properties: all its nodes belong to the interior of the integration interval, all its weights are positive, it converges, and it is applicable both when the external zeros of the integrand are known exactly and when they are known approximately. In order to economically estimate the error of $\mathcal{G}_n$, we construct its extensions that inherit the n nodes of $\mathcal{G}_n$, and that are analogous to the Gauss-Kronrod, averaged Gauss and generalized averaged Gauss quadrature rules. Further, we show that $\mathcal{G}_n$ with respect to the pairwise distinct external zeros of the integrand represents a special case of the (slightly modified) Gauss quadrature formula with preassigned nodes. The accuracy of $\mathcal{G}_n$ and its extensions is confirmed by numerical experiments.

Keywords: Gauss quadrature formula, External zeros of the integrand, Modified weight function, Quadrature error, Convergence of a quadrature formula