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RESOLUTION OF REDUNDANCY USING LOCAL OPTIMIZATION: A SYNERGY APPROACH

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Abstract. *The major aim of this study is to promote a synergy approach which allows controlling redundant robotic mechanism. It will be shown that new robot control can be established using an appropriate biological analog where introducing the hypothetical control and applying the synergy approach it will be possible to resolve the kinematic redundancy. Also, in this paper the possibility of switching synergies within a single movement according to the task requirements is treated and the synergy approach with the logical controlis proposed to solve this problem.*

Key words: *control, kinematic redundancy, optimization, synergy*

1. INTRODUCTION

Industrial robots perform various tasks improving the quality and efficiency of manufacturing. They also replace human workers in tasks that may jeopardize human safety and health. Some complex industrial - and especially nonindustrial tasks -induced recently a new approach to robot design and control in order to achieve very stable, fast, and accurate systems. For a example, some of these tasks are: industrial assembly, high speed manipulation, service and home robotics, robotized surgery, etc. The approach is based on anthropomorphic redundant configurations. A redundant robots are called kinematically redundant if they have more degrees of freedom (DOF) then required for realization of a prescribed task in a task space (1). Recent research offer a new view to redundant robots. In many tasks, kinematics can be solved with a nonredundant robot configuration, but difficulties arise with the dynamic effects and accuracy. Redundancy may help to solve these problems (2). But, redundant mechanisms also have a disadvantage of the difficulty in controlling them. The main question is how to choose a suitable mechanism configuration from the infinite number of possible configurations called "self-motions" which match each position of the manipulation object, for a

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prescribed point of the end-effector in a task space. For solving this problem, several technical cost functions were proposed. The standard method to deal with this problem are divided into two groups, i.e. global and local methods, according to the criterion of what is needed to know in advance about the operational space trajectory in order to find an appropriate solution (2). In both types of methods some additional performance criterion (e.g., energy consumption) is adopted and a search is made for joint motions that execute the desired end-effector´s trajectory while optimizing the criterion (3). The global methods require the prior knowledge about the full end-effector´s trajectory, since they look for joint motions that provide the global optimum of the adopted criterion. On the contrary, local methods perform only local optimization of the criterion, and need only information about the instantaneous pose of the end-effector (4).

The other idea is to imitate the human behavior. Thus, it is necessary to examine the way humans perform complex motions, find the biological analog, and apply it the robot. There is a striking contrast between the apparent ease with which humans perform multi-joint movements in the environment rich in obstacles, changing targets, and unpredictable forces, and the very modest progress in understanding how the central nervous system (CNS) controls such movements. The existence of invariant features in the execution of functional motions points out that the CNS uses *synergy* Bernstein, (5) (i.e rule(s) that can be developed by the CNS based on some principles). In that way, control of a redundant system is obtained using biomechanical principle - synergy with introduced hypothetical control which imposes a specific constraint(s) on the control variables. Also, the possibility of switching synergies within a single movement according to the task requirements may be an essential component of acquiring motor skill (6). Applying the synergy approach with local optimization of a suitable dynamic criterion with respect to the logical control one can solve the problem of redundancy with the possibility of obtaining two synergies control within a single movement. This paper deals with the problem of how to control a robot to a make it move like a human, find biological analog and thus use the advantages of this behavior. This problem is interesting from two points of view, i.e first the robot specialists may finds ways to improve robot control and second the workers in the field of biomechanics may find mathematical models to simulate the systems they are investigating. In this paper the main concern is supporting the first attitude.

2. HUMAN-LIKE MOTION AND CONTROL: MAIN IDEAS

Human arm movements are considered to be stable, fast and accurate due to the properties of muscles, musculo-skeletal structures and hierarchical control. However, this the flexibility and the large number of degrees of freedom in the motor system are the problem when same motor task is realized by movements. However, the benefits of human-like motion cannot be gained without the knowledge of the principles that the *central nervous system* (CNS) employs during the generation of the human arm movements. There is a striking contrast between the apparent ease with which humans perform multijoint movements in the environment rich in obstacles, changing targets, and unpredictable forces, and the very modest progress in understanding how the CNS controls and which principles employs during the generation of the human arm movements. Particularly, it is important to understand how the CNS resolves the redundancy problem of the arm, and selects one particular solution for joint motions from an infinite set of possible motions. Unfortunately, a thorough explanation of these principles is not yet available (7). But, in (8) it is suggested the hypothesis that it is possible to associate some cost function to each human arm joint. The arm performs movements that optimize these cost functions. The joint motion time is an example of a kinematic cost function. Examples of dynamic cost functions are the following: quadratic norm of joint control torques, kinetic energy, jerks in joints (third time derivative of the joint position) (9). Some neuro-physiological and psychophysical cost functions are the following (10): (i) "input energy" (defined as a quadratic norm of input neural signals of motor unit (muscles)), "input fatigue" (denoting the magnitude of such neural signals.) The fact that a variety of cost functions has been used to explain the principles of the human arm motor control indicates that the CNS does not obey any particular cost function, but also does not violate the general physical and technical principles of the optimality from which the particular cost functions come out. Hence, additional efforts in searching for new appropriate and effective cost functions are justified. They contribute to a better understanding of the biological principles of motor control.

In this paper a new robot control which is obtained using the synergy approach is suggested. It was observed in the execution of the functional motions that certain trajectories are preferred among the infinite number of options (11). Such behavior of organisms can be only explained by the existence of inherent optimization laws in the self-organized systems governing the acquisition of motor skills. Existence of invariant features in the execution of the functional motions points out that the CNS uses *synergy* (5) (i.e rule(s) that can be developed by the CNS based on some principles). In fact, such behavior implies that it obeys the optimization at the coordination level where the goal is to minimize efforts in terms of synergy patterns. Speaking mathematically, the synergy imposes specific constraints on the control variables of joints which are related to the task dependent functions pertaining to classes of motor acts. Term "synergy" according to (5) and (11) means a set of rules that unite hypothetical control signals to individual joints into an equation helping to solve the problem of kinematic redundancy. Also, the control of the arm movement in humans relies very much on distributed usage of different joints, and inherent optimization of muscles which are active. Arm muscles are grouped in pairs about simple hinge joints where muscles should be regarded as functional units with more than one control or activation parameter. The existence of motor-unit subpopulations in the muscle reflects a neural organization rather than differences in the mechanical effect of motor units. Understanding coordination of the muscle task-dependent activation patterns requires analysis of the activation of their patterns of motor unit-subpopulations. The actuator redundancy comes from the possibility to use several motor units for the same movement of any arm joint. In that way, the ability of the human arm to rearrange its motions is enabled by the existence of both the kinematic and the actuator redundancy. However, actuator redundancy is not studied here, although its role in performing the movements must be pointed out. The main objective is to resolve the kinematic redundancy and to achieve the control of the robot in a human-like fashion.

3. ROBOT ARM KINEMATICS AND DYNAMICS: DISTRIBUTED POSITIONING CONCEPT

Here, a robot arm is considered as an open linkage consisting of n rigid bodies interconnected by n one-degree-of-freedom joints Fig.1, and so the arm has n degrees of freedom. The approach based on the biological analog is also applied, i.e, the modeling is based on the separation of the prescribed movement into two motions: smooth global, and fast local motion, called the distributed positioning (DP). Distributed positioning is an inherent property of biological systems. In humans highly inertial arm joints (shoulder and elbow) provide smooth global motion, and low inertial hand joints (fingers) perform fast and precise local motions (12) (13).

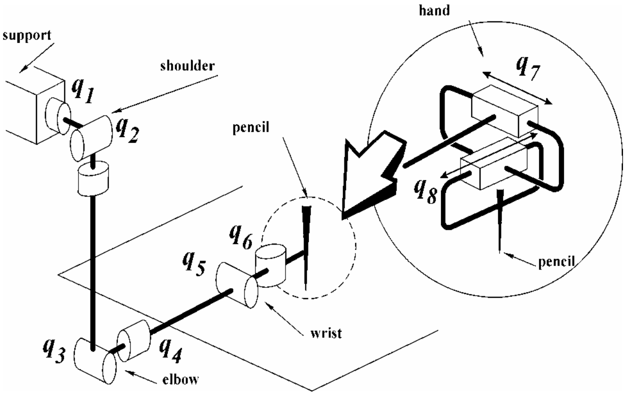

Fig. 1. Eight-DOF arm-hand complex

Let the position of the arm Fig. 1 be defined by the vector of joint (internal) coordinates of dimension $n = 8$: $q = [q_1 q_2 ... q_8]^T$. The position of the terminal device is defined by the vector of external coordinates of dimension $n_e = 5$: $X = [xyz\theta\varphi]^T$, where x,y,z define the tip position and angles θ, φ define the pencil axis. The kinematic model of the arm-hand complex i.e. the transformation of coordinates (internal to external and vice versa) is highly nonlinear :

$$r = f(q), \tag{1}$$

where f is the function : $R^8 \to R^5$. The inverse kinematics (calculation of $q(t)$ for given $r(t)$) has an infinite number of solutions since (1) represents a set of 5 equations with 8 unknowns. This is due to the presence of redundancy. The dimension of the redundancy is $n_r = n - n_e = 8 - 5 = 3$.The kinematic model can be written in the Jacobian form of the first or of the second order:

$$\dot{r} = J(q)\dot{q}, \ \ddot{r} = J(q)\ddot{q} + A(q, \dot{q}), \tag{2}$$

where J is $n_e \times n$ (i.e. 5x8) Jacobian matrix and A is $n_e \times 1$ (i. e. 5x1) adjoint vector containing the derivative of the Jacobian. Let X_1 be the subvector containing the accelerated motions (dimension n_a), and X_2 the subvector containing the smooth motions ($n_e - n_a$). Now :

$$r = [X_1 \ X_2]^T \tag{3}$$

The redundant robot ($n = 8$ DOFs) is now separated into two subsystems. The subsystem with $n_e = 5$ DOFs with greatest inertia is called ***the basic configuration.*** The other subsystem is ***the redundancy*** having $n_r = 3$ DOFs. It holds that $n = n_e + n_r$. Analysing the plane writing task one finds that there are $n_a = 2$ accelerated external motions : $x(t)$ and $y(t)$. The others (z, θ, φ) are constant or smooth. According to the DP concept we introduce $X_1 = [x\ y]^T$ and $X_2 = [z\ \theta\ \varphi]^T$. The basic configuration as a mechanism $q_b = [q_1\ q_2 \ldots q_5]^T$ can be defined. The resting joints, one wrist joint (q_6) and "fingers" (q_7,q_8), form the redundancy and $q_r = [q_6\ q_7\ q_8]^T$ defines the position of the redundancy. The DP concept solves the inverse kinematics of a redundant robot in two steps. At the first step the motion of the basic configuration is calculated (q_b) using the kinematic model and the properties of the DP concept, and at the second step the motion of the redundancy (q_r) is determined, (14). Also, one can see that dynamical model of the robotic system is described by n linear differential equations which can be represented by the matrix equation:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = [M_b(q)\ M_r(q)]\begin{bmatrix}\ddot{q}_b \\ \ddot{q}_r\end{bmatrix} + D(q,\dot{q}) = Q\,, \tag{4}$$

where $M(q) \in R^{nxn}$ is the matrix of inertia, vector $D(q,\dot{q}) \in R^{nx1}$ covers gravitational and other effects, and $Q \in R^{nx1}$ is the vector of driving torques/forces acting upon the robots joints.

4. Resolving Redundancy Using the Local Optimization of the Criterion– Synergy Control

a) With respect to parameters of the hypothetical control

Here, one new way to the resolution of redundancy has been shown, by introducing hypothetical parameter of the control following an appropriate biological analog. The term " synergy", according to (5) means a set of rules that unite hypothetical control signals to individual joints into an equation helping to solve the problem of the kinematic redundancy. In the biological control of the synergic systems one can conclude with a great certainly that the central control (hypothetical) exists (11) (15). Here, a next dynamic criterion appropriate for the on-line use in robotics is suggested.

$$I_l = \frac{1}{2}\ddot{q}_r^{\,T}(M_r - M_b^{-1}J_r)^T(M_r - M_b^{-1}J_r)\ddot{q}_r \to \underset{u_v}{ext} \tag{5}$$

where the equality constraints are given:

$$\begin{aligned}\ddot{r} &= J(q)\ddot{q} + A(q,\dot{q})\\ M_b(q)\ddot{q}_b &+ M_r(q)\ddot{q}_r + D(q,\dot{q}) = Q\end{aligned} \tag{6}$$

First, the generalized forces are introduced:

$$Q(t) = u(t) + Q_v(t) = u(t) + \alpha(t)u_v \tag{7}$$

where u_v is the hypothetical parameter of the control appropriate dimensions. Vector $\alpha(t)$ is obtained (through training) at a higher level of control: coordination level if the synergy has a general character or at actuator level if the synergy has a local character). Now, equation (6) can be written as:

$$\ddot{r} - A = J_b\ddot{q}_b + J_r\ddot{q}_r, \quad M_b\ddot{q}_b + M_r\ddot{q}_r + D = Q \tag{8}$$

or,

$$\underbrace{\{M_r - M_bJ_b^{-1}J_r\}}_{M_r^*}\ddot{q}_r = Q - \underbrace{D - M_bJ_b^{-1}(\ddot{r} - A)}_{D^*} \tag{9}$$

so, one can obtain augmented objective cost function as:

$$I_l = \frac{1}{2}(Q - D^*)^T(Q - D^*) \to \min_{u_v} \tag{10}$$

Necessary conditions for the optimality are:

$$\frac{\partial I_l}{\partial u_v} = 0 \Rightarrow \frac{\partial Q^T}{\partial u_v}\cdot(Q - D^*) = 0 \tag{11}$$

or

$$\alpha^T Q = \alpha^T D^*\big|_{u_v=0} \Rightarrow \alpha^T u = \alpha^T D^* \tag{12}$$

Equations (8) can be represented in the following manner:

$$JM^{-1}Q = JM^{-1}(C(q,\dot{q}) + G(q)) + \ddot{r} - A(q,\dot{q}) \tag{13}$$

and using (12) now it is possible to obtain the vector of the synergy control for given redundant robotic system such as:

$$u = \begin{bmatrix} JM^{-1} \\ \alpha^T \end{bmatrix}^{-1} \begin{Bmatrix} JM^{-1}(C(q,\dot{q}) + G(q)) + \ddot{r} - A(q,\dot{q}) \\ \alpha^T D^* \end{Bmatrix} \tag{14}$$

b) With respect to parameters of the logical control

Also, the possibility of switching synergies within a single movement according to task requirements may be an essential component of acquiring motor skill (6). Applying the synergy approach with local optimization of a suitable dynamic criterion with respect to logical control one can solve the problem of the redundancy with the possibility of obtaining two-synergies control within a single movement. Here, the generalized forces are introduced in the following way:

$$Q(t) = u(t) + Q_L(t) = u(t) + th(\alpha(t)u_L)\cdot th(\beta(t)(u_L - 1)) \tag{15}$$

where $u_L \in \{0,1\}$ is the logical control and $th(u) = \dfrac{e^u - e^{-u}}{e^u + e^{-u}}$. Also, it is assumed that the vectors $\alpha(t)$, $\beta(t)$ are obtained (through training) at higher level of control. Applying, the same dynamic criterion (5)

$$I_l = \frac{1}{2}\ddot{q}_r^{\,T} W (M_r - M_b^{-1}J_r)^T (M_r - M_b^{-1}J_r)\ddot{q}_r \to \underset{u_L}{ext} \tag{16}$$

and using the constraints (6), one can obtain the criterion:

$$I_l = \frac{1}{2}(Q - D^*)^T (Q - D^*) \to \min_{u_L} \tag{17}$$

The necessary conditions for the optimality are:

$$\frac{\partial I_l}{\partial u_L} = 0 \Rightarrow \frac{\partial Q^T}{\partial u_L}(Q - D^*) = 0 \tag{18}$$

or

$$\left[\frac{\alpha}{ch^2(\alpha u_L)} th(\beta(u_L - 1)) + \frac{\beta}{ch^2(\beta(u_L - 1))} th(\alpha u_L)\right] \cdot (Q - D^*) = 0 \tag{19}$$

where is $ch(u) = \frac{e^u + e^{-u}}{2}$. In the same manner for the case $u_L = 0$ one obtains:

$$u_{|u_L=0} = \begin{bmatrix} JM^{-1} \\ \alpha th(-\beta) \end{bmatrix}^{-1} \begin{bmatrix} JM^{-1}(C(q,\dot{q}) + G(q) - A(q,\dot{q})) \\ \alpha th(-\beta) D \end{bmatrix} \quad \det\begin{bmatrix} JM^{-1} \\ \alpha th(-\beta) \end{bmatrix}^{-1} \neq 0 \tag{20}$$

and for the case $u_L = 1$

$$u_{|u_L=1} = \begin{bmatrix} JM^{-1} \\ \beta th(\alpha) \end{bmatrix}^{-1} \begin{bmatrix} JM^{-1}(C(q,\dot{q}) + G(q) - A(q,\dot{q})) \\ \beta th(\alpha) D \end{bmatrix}, \quad \det\begin{bmatrix} JM^{-1} \\ \beta th(\alpha) \end{bmatrix}^{-1} \neq 0 \tag{21}$$

5. CONCLUSION

In this paper a new approach to the control of redundant systems is presented. It is obtained using the biomechanical principle - synergy with introduced hypothetical control which imposes a specific constraint(s) on the control variables. Also, the possibility of switching synergies within a single movement according to the task requirements may be an essential component for acquiring the motor skill. Applying the synergy approach with the local optimization of a suitable dynamic criterion with respect to the logical control one can solve the problem of the redundancy with the possibility of obtaining two - synergies control within a single movement.

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RAZREŠENJE REDUNDANSE PRIMENOM LOKALNE OPTIMIZACIJE: SINERGIJSKI PRISTUP

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Glavni cilj ovog rada je da promoviše sinergijski pristup koji omogućuje upravljanje redundantnim robotskim mehanizmom. Pokazano je da novo robotsko upravljanje se može ostvariti primenom odgovarajućeg biološkog analogona uvodjenjem hipotetičkog upravljanja uz primenu prethodno navedenog sinergijskog pristupa, tako da je moguće razrešiti kinematičku redundansu. Takodje, u ovom radu je razmatrana i mogućnost uključivanja - prebacivanja sinergija u okviru jednog pokreta a prema zahtevima zadatka, što je realizovano korišćenjem logičkog upravljanja sa aspekta sinergijskog upravljanja.

Ključne reči: *upravljanje, kinematička redundansa, optimizacija, sinergija*