

Modeling of fragmentation of rapidly expanding cylinders

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Abstract

The paper considers fragmentation process of high-explosive projectile casings, i.e. rapidly expanding cylinders loaded by extreme internal pressure generated by detonation of explosives. The classical, physically based, Mott model of the ring fragmentation is examined and the adequate computer program is realized. The fragment size (or mass) distribution is analyzed and the average fragment size is related to the characteristics of expansion and the casing material properties. The influence of the fragmentation process parameters on the nature of fragment length distribution is analyzed. The theoretical distributions are compared with experimental data and good correspondence is obtained.

Keywords: fragmentation, high-explosive projectile, Mott fragmentation model, fragment distribution law

1 Introduction

Fragmentation is a process of the structural disintegration of a body, i.e. formation of a certain number of particles, caused by the multiple fracturing of the fragmenting body material. The strains and fractures

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in the material can be induced by static forces (grinding, crushing), or by dynamic loads (collision of elementary particles, cosmic bodies, impact of a projectile into a target etc.). We will consider here one special case of a dynamic fragmentation of a body due to the action of an impulse load – fragmentation of the ring (cylinder) subjected to extremely high internal pressure generated by gaseous products of an explosive detonation. The study of this process is of great importance for the design and efficiency analysis of fragmentation warheads.

There are two substantially different approaches to the fragmentation problem – the statistical and the physically based approach. The statistical modeling is based on the analysis of experimental data, which enables the establishment of a mathematical description of the distribution of size, mass and shape of generated fragments. The papers of Grady and Kipp [1], Held [2], Silvestrov [3], and Dehn [4], gave a survey of different statistical models of the fragment distribution. The present authors [5] analyzed these distribution laws, using the comprehensive experimental database.

In this paper we will consider the physical approach to the fragmentation process based on the classical study of Mott [6]. In the paper [6], Mott formulated the original and consistent fragmentation model, which was the basis for further investigations and more complex approaches to the modeling of fragmentation (e.g. [7]). It is interesting that the model, being completely physically based, is not related to the previous important investigation of the author, originating from consideration of geometrical statistics, which resulted in a well-known Mott formula [8].

2 Fragmentation model

The model [6] is related to the fragmentation of shell cases, i.e. to the thin cylindrical casings of fragmenting warheads. In order to assure one-dimensionality of the model, it is assumed that the shell is consisted of a number of coaxial circular rings stacked to each other. So, we can consider that the cracks are perpendicular to the axis of the shell, and therefore all fragments have the same thickness (case thickness) and the same width (width of the ring). The aim is to calculate the distribution of the length of generated fragments.

2.1 Model of case fracturing

The treatment of criteria of the case material fracture has a very important role in the fragmentation modeling. The mechanical properties of material, such as fracture stress, strain, and contraction at the moment of fracture obviously have a statistical nature. They depend on irregularities in the material structure (voids, inclusions, micro-cracks) that vary from one sample to another. The basic assumption in the Mott model is that elementary probability that an unfractured specimen of unit length will fracture when the strain increases from ε to $\varepsilon + d\varepsilon$ has the exponential form

$$C e^{\gamma \varepsilon} d\varepsilon, \quad (1)$$

where parameters C and γ represent the characteristics of material. The chosen exponential function provides a rapid increase of fracture probability with the increasing strain¹. Using assumption (1), it is possible to determine the fracture probability p for the strain ε , having in mind that $(1 - p)$ is the probability that there is no fracture for strain values less than ε . The corresponding differential equation has the form

$$dp = (1 - p) C e^{\gamma \varepsilon} d\varepsilon, \quad (2)$$

and considering initial condition ($\varepsilon=0, p=0$), it gives

$$p = 1 - \exp \left[-\frac{C}{\gamma} (e^{\gamma \varepsilon} - 1) \right]. \quad (3)$$

The expected (average) value of fracture strain is determined by

$$\bar{\varepsilon} = \int_0^{\infty} \varepsilon dp \approx \frac{1}{\gamma} \left[\ln \frac{\gamma}{C} + \int_0^{\infty} e^{-x} \ln x dx \right], \quad (4)$$

where the assumption that $\gamma?C$ is used. The definite integral in the last equation is equal to the negative value of Euler-Mascheroni constant $\gamma_e=0.5772$, so expression (4) gets the form

¹It should be noted that it is possible to choose different increasing functions for modeling the increase of fracture probability with the increasing strain. Grady [7] used the power function in modification of the basic Mott model

$$\bar{\varepsilon} \approx \frac{1}{\gamma} \left[\ln \frac{\gamma}{C} - 0.5772 \right]. \quad (5)$$

The standard deviation of the critical strain value is

$$\sigma_{\varepsilon} = \left(\int_0^{\infty} (\varepsilon - \bar{\varepsilon})^2 dp \right)^{1/2} = \frac{1}{\gamma} \frac{\pi}{\sqrt{6}} \approx \frac{1.282}{\gamma}. \quad (6)$$

Equation (6) shows that, according to the model, the dispersion of actual fracture strain values is inversely proportional to the parameter γ . The perfectly definite value of fracture strain (i.e. its zero dispersion) would correspond to infinite value of the parameter γ .

In the detailed theoretical consideration Mott [6] shows that the parameter γ , which is directly related to the fracture strain deviation, can be approximated by the function of the mechanical characteristics of material

$$\gamma \approx 160 \frac{\sigma_P}{\sigma_F(1 + \varepsilon_F)}, \quad (7)$$

where σ_F and ε_F are true fracture stress and strain, while the parameter σ_P is the proportionality coefficient in the strain-hardening law for the material at high strains $\sigma = \sigma_0 + \sigma_P \ln(1 + \varepsilon)$.

According to the model, the parameter γ can be determined by formula (7) if all necessary material characteristics are known, or by (6) if the results of multiple experimental tests of the material are available. Finally, the parameter C is determined from equation (5), and the law of fracture probability (1) is completely defined.

2.2 Releasing wave and fragment forming

According to the considered fracture model, the first crack (and then fracture) is generated at the strain value close to $\bar{\varepsilon}$, and with further increase of the strain, the probability of the new cracks generation increases exponentially. The whole process of crack, fracture and fragment generation occurs in the very narrow interval of strains. If we have in mind that high strain rates at the impulse load on the ring brought

about extreme pressure of detonation products, it is clear that the fragmentation process occurs in a very short time interval.

For a qualitative description of the essence of physical model, suppose that the first crack is generated in position A at the initial moment, and at corresponding value of the strain (Fig.1). Simultaneously, the stress is released in the neighbourhood of A; the propagation of released region is modeled by Mott wave whose velocity can be determined. Further fractures can no longer take place in these unstressed zones of the ring. The development of a new crack must occur at the part of ring that is still stressed. The formation of each new crack triggers the new Mott waves which increase unstressed region of the ring. The process of crack formation, i.e. fragment generation, proceeds until the whole ring material is stress free.

Due to the short duration of the process, it can be assumed that the radial ring expansion velocity v , as well as the ring radius r , do not significantly change during the fragmentation. The tension strain rate of the ring can be assumed approximately constant with the value

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{v}{r}. \quad (8)$$

In order to determine the velocity of releasing wave, the motion of the front of Mott wave B (and the whole unstressed region AB) relative to section C which is still stressed, can be considered (Fig.1). The velocity of region AB, relative to section C that is close to the wave front (in the sense that central angle $(a - x)/r$ should be small), can be expressed as

$$v_{rel} = \frac{a - x}{r} v, \quad (9)$$

where x and a are the distances of the wave front B and section C from the point of fracture A, respectively.

The equation of motion of the zone AB can now be written in the form

$$\sigma_F = -\rho x \frac{d}{dt} \left(\frac{a - x}{r} v \right), \quad (10)$$

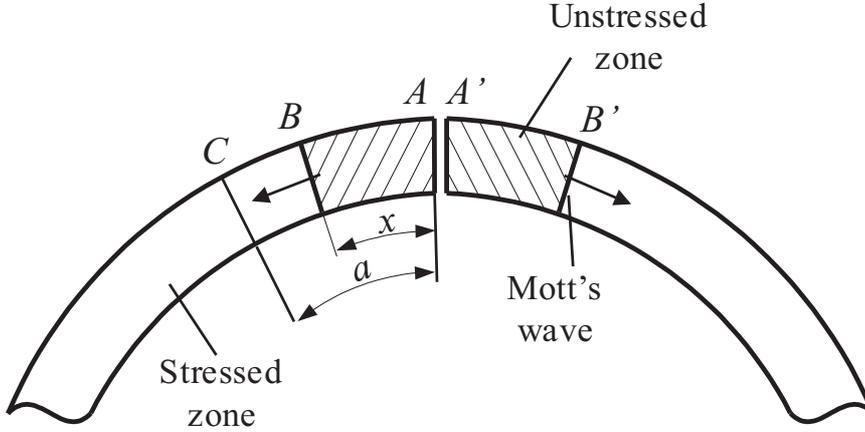


Figure 1: Release wave propagation in the crack neighbourhood

where σ_F is the value of stress at the moment of fracture at a corresponding strain rate, and ρ is material density. The integration of differential equation (10), taking into account the initial condition ($t=0, x=0$) and the assumption that v/r is constant, leads to

$$\frac{x^2}{t} = \frac{2\sigma_F r}{\rho v}, \quad (11)$$

which gives the time dependance of the width of the releasing zone²

$$x = \sqrt{\frac{2\sigma_F r}{\rho v}} \sqrt{t}. \quad (12)$$

Expression (12) enables determination of the position of releasing wave front, i.e. boundaries of unstressed zone as a function of time elapsed from the crack formation.

²Obtained relation $x = x(t)$, due to the neglected elasticity of the ring material, yields an infinite value of wave velocity at the initial moment. This physically unrealistic result can be corrected by taking into account the material elasticity. The wave velocity in that case will be equal to the sound velocity in the ring material. With the increasing time, the velocity and the position of releasing wave have values close to those given by (12).

2.3 Fragment length determination

The exposed concepts of the ring material fracture and the propagation of releasing zones enable the numerical determination of fragments' length. Regarding the assumption (1), the increase rate of the number of cracks n at the ring of circumference $L=2\pi r$, is given by

$$\frac{dn}{d\varepsilon} = flCe^{\gamma\varepsilon}, \quad (13)$$

where f is the portion of the ring circumference that is still stressed. Introducing a new variable $\alpha=\gamma\varepsilon$, equation (13) becomes

$$\frac{dn}{d\alpha} = \frac{flC}{\gamma}e^{\alpha}. \quad (14)$$

If the crack formation occurs at the strain ε_1 (i.e. corresponding value α_1), the time interval to the moment with the strain $\varepsilon > \varepsilon_1$ is

$$t = \frac{\varepsilon - \varepsilon_1}{\dot{\varepsilon}} = \frac{r}{v}(\varepsilon - \varepsilon_1) = \frac{r}{\gamma v}(\alpha - \alpha_1). \quad (15)$$

Replacing equation (15) in (12), the position of releasing zone can be determined, i.e. the length of crack surrounding region where further fracture is prevented can be calculated by

$$\Delta x = x_0\sqrt{\alpha - \alpha_1}, \quad (16)$$

where the parameter x_0 is defined by expression

$$x_0 = \sqrt{\frac{2\sigma_F}{\rho\gamma}} \frac{r}{v}. \quad (17)$$

The procedure for determination of the length of the fragments implies the known necessary parameters of the process that include characteristics of the ring material (density ρ , fracture stress σ_F , and fracture probability parameters γ and C), the fragmentation strain rate $\dot{\varepsilon} = v/r$, and the ring radius r .

The position of the first crack at the ring circumference ($L=2\pi r$) can be arbitrarily chosen, assuming that on the basis of (14) the corresponding value of the parameter $\alpha = \alpha_1$ is determined by

$$\frac{lC}{\gamma}e^{\alpha_1} = 1. \quad (18)$$

First, the length of releasing zone neighbouring the first crack at the moment of the second crack formation is calculated. According to equations (14) and (18), the increase of parameter α is $\alpha_2 - \alpha_1 = 1$, so expression (16) gives the length of releasing zone $\Delta x_1 = x_0$. The position of unstressed zone of the length Δx_1 is marked on the ring circumference on both sides of the crack. The position of the second crack is randomly picked on the remained, still stressed part of the ring. The value of parameter f is now $(L - 2\Delta x_1)/L$, which enables the determination of parameter α at the moment of the third crack formation, using equation (14), which can be approximated with

$$\frac{3 - 2}{\Delta\alpha} = \frac{f l C}{\gamma} e^{\alpha_3}. \quad (19)$$

The last equation can be reduced to the form

$$f e^{\alpha_3 - \alpha_1} (\alpha_3 - \alpha_2) = 1, \quad (20)$$

and α_3 can be numerically determined. Now, the length of releasing zone around the second crack is

$$\Delta x_2 = x_0 \sqrt{(\alpha_3 - \alpha_2)}, \quad (21)$$

and the expanded unstressed zone surrounding the first crack is

$$\Delta x_1 = x_0 \sqrt{(\alpha_3 - \alpha_1)}. \quad (22)$$

Equations (21) and (22) define the new releasing zones, and on the reduced length of still stressed portion of the ring the position of the third crack can be randomly selected. The parameter f decreases to the value $(L - 2\Delta x_1 - 2\Delta x_2)/L$, and the procedure continues analogously to the previously explained, using the general expressions for determination of the parameter α_n and the lengths of all released zones

$$\begin{aligned} f e^{\alpha_n - \alpha_1} (\alpha_n - \alpha_{n-1}) &= 1, \\ \Delta x_k &= x_0 \sqrt{\alpha_n - \alpha_k}, \quad k = 1, 2, \dots, n. \end{aligned} \quad (23)$$

The calculation terminates when the whole ring becomes unstressed. At that moment, the positions of formed cracks determine the length of generated fragments.

On the basis of a certain number of such numerical experiments, analyzing example³ where $L=20x_0$, Mott obtained the histogram (Fig.2) which shows empirically determined distribution of fragment length in length intervals of $0.4x_0$.

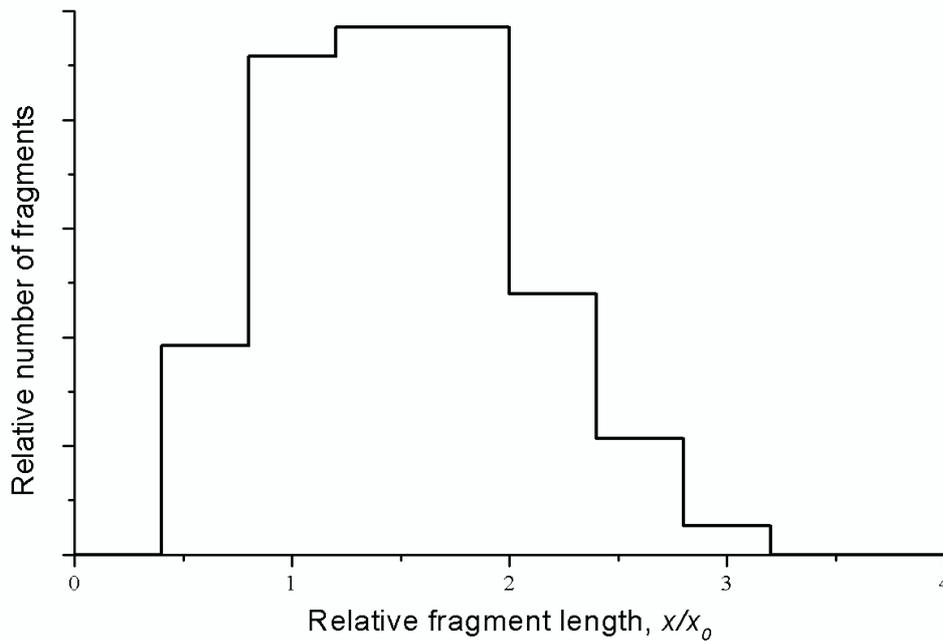


Figure 2: Histogram of the fragment length distribution according to Mott [6]

On the basis of the fragment length distribution, Mott concluded as follows:

- (i) the length of the most fragments is in the interval $[x_0, 2x_0]$; the average fragment length is close to the value $1.5x_0$;

³In the original paper [6], neither the ring material properties nor strain rate are specified. It is important to note that the considered approach, based on “experimental statistics”, without the application of a computer, requires enormous effort and a lot of time for realization.

- (ii) since the parameter x_0 is proportional to the ring radius r , the average fragment size will be proportional to the value of this parameter;
- (iii) according to equation (17), the average fragment length is inversely proportional to the fragmentation velocity v .

The preceding analysis clearly shows that the parameter x_0 determines the average fragment length. If we take into account equation (7), which relates the parameter γ to the material properties, it is clear that the average fragment length is proportional to the expression $\sigma_F \sqrt{(1 + \varepsilon_F) / \rho \sigma_P}$. Higher values of the fracture stress and corresponding strain increase the fragment length, whereas increasing density and parameter σ_P (which defines the strain-hardening of material) lead to a decrease of the average fragment size.

3 Analysis of the fragmentation model

The computer program that simulates the ring fragmentation was realized on the basis of the presented model. The program completely follows the described procedure with the determination of cracks' position on the stressed part of the ring, using the generator of uniformly distributed random numbers. Performing of sufficient number of simulations (i.e. numerical experiments, analogously to the Mott graphical procedure), a characteristic fragment length distribution can be established. Because of the comparison with the original Mott diagram, the simulation of steel ring fragmentation is accomplished, using the input values as follows: ring material density $\rho=7800\text{kg/m}^3$, fracture stress and strain $\sigma_F=800\text{MPa}$, $\varepsilon_F=0.63$, the parameter in the strain-hardening law $\sigma_P=450\text{MPa}$ (the last three values determine the parameter $\gamma=55$), strain rate $\dot{\varepsilon} = v/r = 10^4 \text{ s}^{-1}$, and the ring circumference $L=20x_0$.

The fragment length distribution, obtained by the program using 1000 simulations, is shown in Fig.3. There are certain differences between the results of numerical simulation and Mott graphic (which is the same as in Fig.2). The numerical histogram is shifted to the right (to higher fragment lengths) relative to the Mott diagram. This discrepancy can be explained by both the way of random selection of the position of

cracks and the number of performed “experiments” used for the Mott diagram.

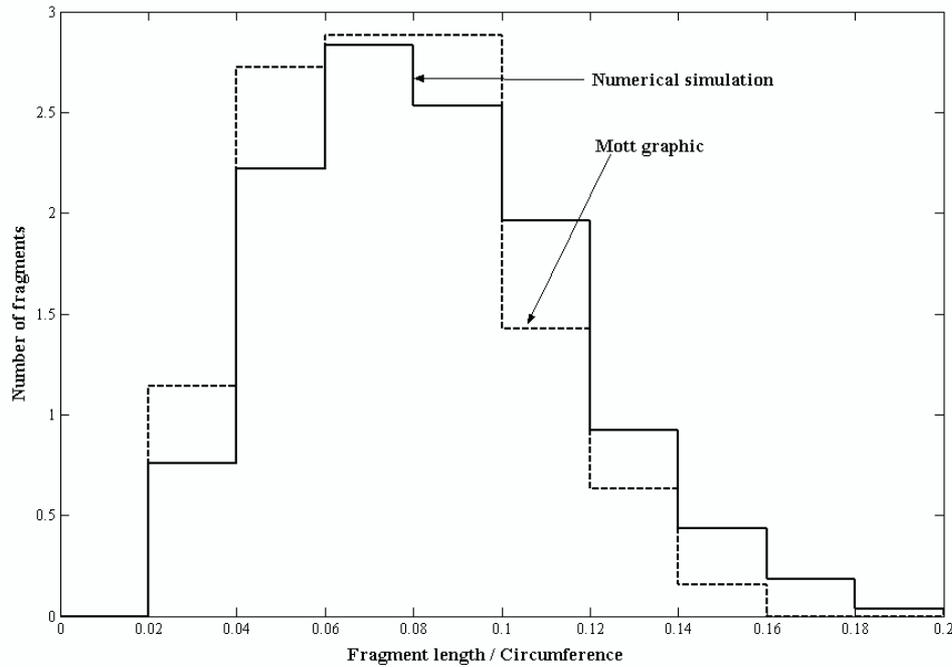


Figure 3: Fragment length distribution according to the Mott model – the results of numerical simulation and the original Mott graphic are shown

The analysis of program results shows that at the constant ratio of the ring circumference to the characteristic length L/x_0 , variation of the process parameters σ_F , ρ , $\dot{\epsilon}$ and γ influences the parameter x_0 , but the distribution of relative fragment length remains unchanged (Fig.3). This confirms Mott’s conclusion that the parameter x_0 substantially defines the fragment length distribution.

Mott [6] analyzed the model results and determined the histogram (Fig.2) for $L/x_0=20$, regarding that the histogram does not essentially change for different values of L/x_0 . The fragment length distributions

for the different ring circumferences ($L/x_0=10, 30, 50$), under the same other conditions, are shown in Fig.4. The distribution obviously depends of the ring circumference – with increasing ratio L/x_0 , the distribution is still unimodal, but it is closer to a uniform distribution. However, the average fragment length is independent of the ring length (i.e. L/x_0 ratio), and the computations show that it is very close to $1.7x_0$. Using the modified probability law (1) in the Mott model, Grady [7] analytically showed that the average fragment length is $\sqrt{\pi}x_0$, which is approximately equal to the result based on numerical approach. Therefore, Mott's evaluation of the average fragment length ($1.5x_0$) is improved.

On the other hand, the average number of fragments is the linear function of L/x_0 ratio ($0.59L/x_0$); e.g. for the Mott's experiment, $L/x_0=20$, we got average fragment number 11.8 from the model.

The results of simulation (Fig.4) also show that the minimum fragment length decreases with the increasing ring circumference; for $L/x_0 = 20$, the number of fragments with the length lower than $0.5x_0$ is negligible. Also, the number of relatively large fragments (longer than $3x_0$) increases with increasing ratio L/x_0 .

Considering the influence of the fragmentation velocity v and the expanded ring radius r on the fragment length distribution, it should be noted that these two values are not mutually independent. The ring fragmentation velocity can be approximately determined by Gurney equation [9] for a cylindrical shell

$$v = \frac{v_G}{\sqrt{\frac{1}{2} + \frac{m}{C}}}, \quad (24)$$

where m and C are masses of ring and explosive charge, respectively, while v_G denotes the Gurney velocity that represents the property of an explosive. Now, the characteristic length x_0 can be expressed by

$$x_0 = \sqrt{\frac{2\sigma_F}{\rho\gamma} \frac{(1 + \varepsilon_F)R}{v_G}} \sqrt{\frac{1}{2} + \frac{\rho}{\rho_e} \frac{2\delta}{R}}. \quad (25)$$

where R is the initial radius, δ is the ring thickness, and ρ_e is the explosive density. If we take into account approximate formula (7) for the parameter γ , the expression (25) gets its final form

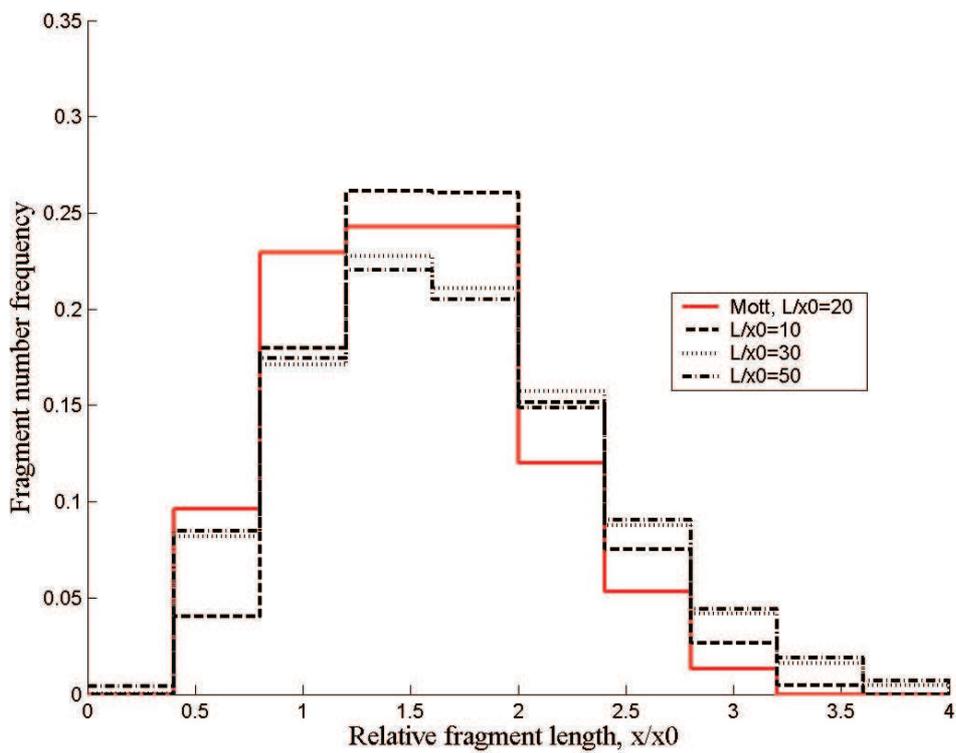


Figure 4: Fragment length distributions obtained by simulations based on Mott model for the different ring circumferences ($L/x_0 = 10, 30, 50$)

$$x_0 = 0.11 \frac{\sigma_F}{\sqrt{\rho\sigma_P}} \frac{(1 + \varepsilon_F)^{3/2} R}{v_G} \sqrt{\frac{1}{2} + \frac{\rho}{\rho_e} \frac{2\delta}{R}}. \quad (26)$$

The last equation enables the analysis of influence of all involved process parameters on the average fragment length ($\approx 1.7x_0$). The main conclusions are: (i) increasing of an initial ring radius causes the increase of an average fragment length, and (ii) an average fragment length is inversely proportional to the Gurney velocity (as well as the detonation velocity) of an explosive.

4 Comparison with experimental data

The experiments related to the one-dimensional fragmentation of a ring (or cylinder) are very rare. Fig.5 shows the diagrams that represent the comparison between experimental data of Wesenberg and Sagartz [10] (Fig.5a) and Grady and Benson [11] (Fig.5b) with the theoretical fragment length distribution based on the presented model.

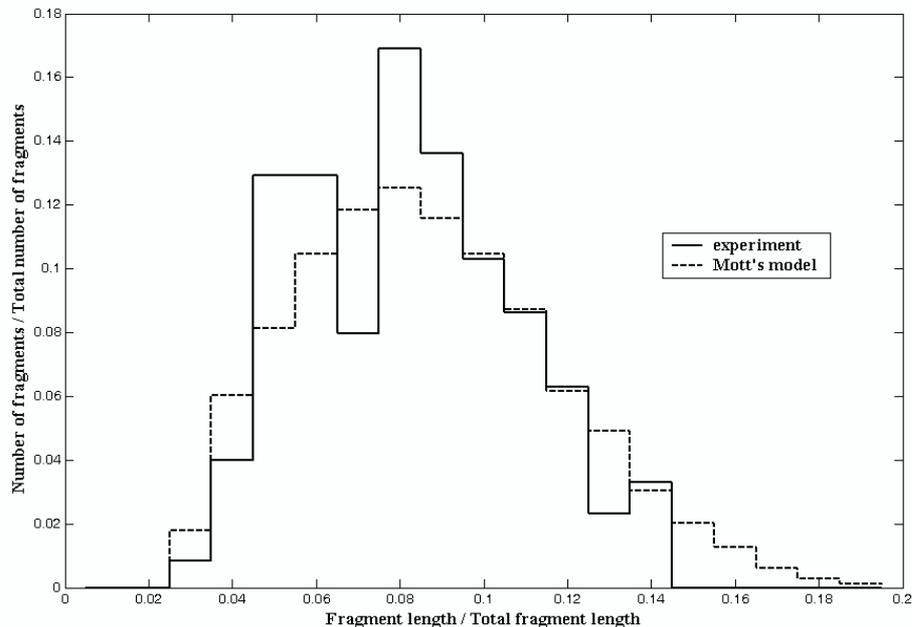
Both experiments relate to the aluminum ring fragmentation (material properties used are $\rho=2700 \text{ kg/m}^3$, $\sigma_F=120 \text{ MPa}$, $\varepsilon_F=0.45$) by an electromagnetic loading. Measured fragmentation velocities (up to 300 m/s) are lower than in the case of fragmenting warheads; however, because of the smaller ring dimensions, the strain rates are of the same order of magnitude (10^4 s^{-1}).

The theoretical prediction, as can be seen in Fig.5a, fits in very well the data of Wesenberg and Sagartz experiment.

Having in mind that the experimental histogram in Fig.5b is based on a relatively small number of tests (four tests), agreement between the model results and experimental data can be characterized as satisfactory.

The lack of comprehensive experiments related to the one-dimensional ring fragmentation, especially under the action of detonation products, disables definitive validation of the Mott model.

a)



b)

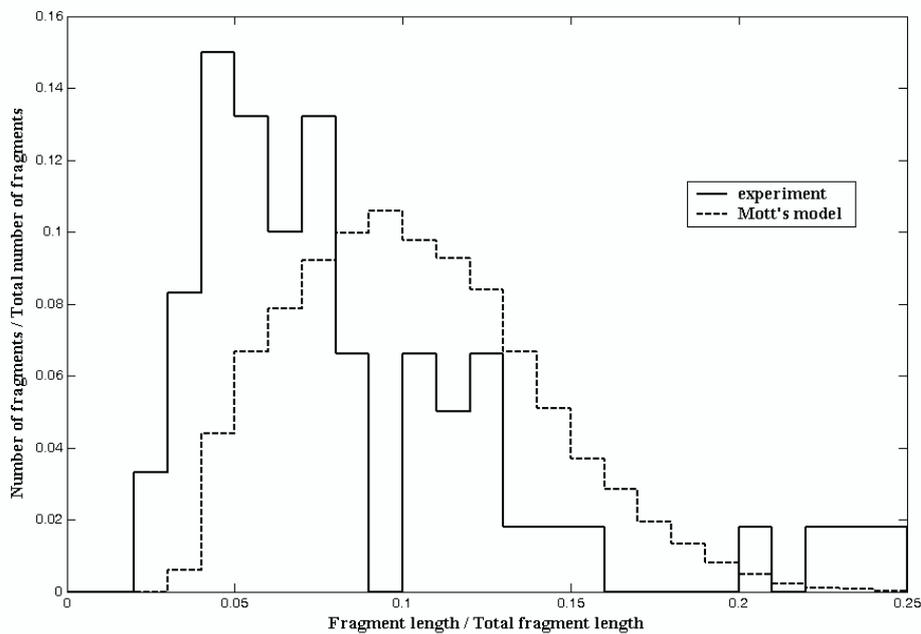


Figure 5: Comparison of theoretical fragment length distribution, based on the Mott model, with the experimental data for aluminum ring fragmentation from (a) Wesenberg and Sagartz [10], and (b) Grady and Benson [11]

5 Conclusion

The paper considers one-dimensional fragmentation of a ring (or a cylinder) under the action of a pressure generated by detonation of an explosive charge. The characteristics of classical, physically-based Mott model are studied in detail, including the fracture probability analysis, evaluation of the releasing wave velocity, and determination of the fragments' length. On the basis of this model, the computer program for the simulation of fragmentation process is realized. Using the program, the fragment length distribution is related to the parameters of fragmentation process. The distribution is substantially defined by the parameter x_0 that depends on the ring material properties and loading conditions. The average fragment length is found to be $1.7x_0$, improving the original Mott's result. It is also shown that the ring circumference affects the fragment length distribution. Introducing the Gurney relation for the final ring velocity, it is concluded that the average fragment size increases with increasing the initial ring radius, and with decreasing the Gurney velocity of an explosive.

The comparisons of experimental data with theoretical results give good correspondence.

Further investigation could be aimed at the modification of presented model in order to comprehend analysis of two-dimensional and three-dimensional fragmentation, which would enable the more realistic simulation of a case fragmentation.

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Modeliranje fragmentacije cilindra pod dejstvom visokog unutrašnjeg pritiska

UDK 532.526

U radu se razmatra proces fragmentacije košuljica projektila parčadnog dejstva, tj. generisanja parčadi rasprskavanjem metalnog cilindra koji je izložen visokim vrednostima unutrašnjeg pritiska usled detonacije eksploziva. Detaljno je razmotren klasični Mott-ov model fragmentacije metalnog prstena na osnovu koga je realizovan odgovarajući program za računar. Analiziran je zakon raspodele dužine (mase) fragmenata, kao i srednja dužina parčadi u funkciji osobina materijala prstena i karakteristika ekspanzije prstena. Razmotren je uticaj relevantnih parametara fragmentacionog procesa na karakter raspodele parčadi. Teorijska raspodela dužine fragmenata upoređena je sa dostupnim eksperimentalnim podacima pri čemu je dobijeno dobro podudaranje rezultata.