# Development of Multifunctional Reconfigurable Desktop Machine Tool with Hybrid Kinematics 

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#### Abstract

This paper presents a desktop reconfigurable machine tool with hybrid kinematics for four types of machine tools, with a description of the applied mechanism and established modular system for their configuring. The postprocessor for five-axis machining presented in this paper is applied to the kinematic structures with table-tilting with two rotations(B,C). The paper also presents the calculation of the position of actuators $p_{1}$ and $p_{2}$ when the machine works with hybrid kinematics. Verification of postprocessor is realized on virtual prototype in CAD/CAM environment and experimentally on an available 3-axis machine tool. Experimental results confirmed the configured postprocessor which can be used for machine tools programming.


Key words: configuring, reconfigurable machine tool, postprocessor

## 1. INTRODUCTION

Research in the field of multifunctional and reconfigurable machine tools is intense and has a lot of completed results [1-4]. The subject of this work is the development of a reconfigurable desktop machine tool with serial and hybrid kinematics. The term concept of reconfigurable machine tool in this paper reffers to a system of constituent elements (modular system) by which multiple machine tools can be configured. Each of these machines is a new kinematic structure.

Basic functional requirements for the development of multifunctional and reconfigurable desktop machine tool with hybrid kinematics were: (1) make low-cost desktop machine tool, (2) establish a modular system for configuring four different machines, (3) control implemented on PC Linux platform with open architecture, (4) use step motors for feed drives, (5) machine is should be programmed using G-code, (6) ensure accessibility and safety at work, and (7) machine is seen as a resource for research and education. In this paper we have chosen the concept of machine shown in Figure 1.

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Figure 1 - Conceptual model of machine tool
The basic machine is 3 -axis desktop milling machine with portal construction, with VXYZ structure. Using additional 2-axis rotary/tilting table $(B, C)$ in front of the machine frame, such machine becomes five-axis milling machine, with WCBVXYZT structure [5]. By fixing of B -axes in $\mathrm{B}=0^{\circ}$ or $\mathrm{B}=90^{\circ}$ and keeping C axes active, such machine becomes a four-axis controlled, and can be used for
turning operations as well. The fourth possible conception assumes basic machine with hybrid kinematics, i.e.: two-axis parallel module M2, for realization of movement of point P in plane ( $\mathrm{X}, \mathrm{Y}$ ) via passive translation module ( $\mathrm{X}, \mathrm{Y}$ ), whereas Z axis remains serial. At this basic machine, two-axis rotary/tilting table remains, and in this way we obtain 5- axis machine with hybrid kinematics, which is discussed in this paper in more details.

For the machine that is the subject of analysis in chapter 2, we present the established system of the constituent elements for its configuring. Chapter 3 presents postprocessing for the 5 -axis machine tool with hybrid kinematics. Checking of the postprocessing calculations as well as workspace are also shown. Verification of the postprocessing calculations and configurated posprocessor for machine were realised using machining test by machining of a four-sided pyramid on the available CNC machine tool, and it is shown in Chapter 4.

## 2. CONFIGURATOR OF ONE CLASS RECONFIGURABLE MACHINE TOOLS

Analysis of the basic modules of the 5 -axis machine tools enabled the establishment of a modular system with reconfigurable hardware, which makes the basic concept of machine tool. Modules are shown in Figure 2 in the form of morphological matrix.


Figure 2 - Configurator for one class of reconfigurable machine tools
Integral part of this matrix are representations of realization of modular system: base, translatory axes, rotary tables, main spindle, tailstock and 2 -axis parallel mechanism as well as a part of the building program for machine tools of types S3D, S4D, S5D
and H5D. Passing through morphological matrix in Figure 2 and by completion of appropriate modules required for the basic functions of machines, a building program of possible machine tools was obtained. In Figure 2, the building program shows four examples of configured machine tools. Figure 3 shows more detailed view of the chosen sample of the building program from Figure 2, which includes and tags the controlled axes.

a) S 3 D
b) S 5 D

c) S4D

d) H5D

Figure 3-Samples of CAD models from Figure 2
These are the initial concepts of four machines: S3D (3-axis), as the main portal machine, S5D (5axis), as the classic 5 -axis portal machine, S4D (4axis) as a typical example of the 4 -axis portal machine and H5D (5-axis), as the main configuration with hybrid kinematics. Tag S indicates machine with serial kinematics, and tag H indicates machine with a hybrid (serial and parallel) kinematics. Desktop machines in building program shown in Figure 3 have the following characteristics:

- They are equivalent, since they can be configured from the same group of the modules and according to same program of building, i.e., according the same configurator.
- In the case of machining the shorter workpieces by using rotary tables, machine of H5D type is suitable: Long strokes of tool carrier are not needed, while the feedrates of tool axis will be large enough to follow rotations (simultaneous or one-by-one) of tables.
- For 4 -axis machining S4D is the best and most suitable machine. With sufficiently fast rotary table or equipped with additional spindle, it can work as a lathe, with appropriate tool holder interface which is blocked.
- 3-axis machine tool (S3D) is the initial for the other three machines from Figure 3.

Frame of the rotary/tilting table is mounted with its length aligned to cross direction of the machine in order to fit into the S3D conception of the machine, and such machines have basic structural formula WCVB XYZT.

## 3. POSTPROCESSING CALCULATION

Programming of machines with serial kinematics of WCBVXYZT type in some CAD/CAM software environment is routine after preparation of suitable postprocessor [6]. On the other hand, postprocessors for machines with parallel and hybrid kinematics are not so often. Therefore, there is no need for repeating the postprocessing calculations, characteristics of posprocessors for machines with serial kinematics. Instead, such calculations will be formalized and used for further formal checking and completion of program (written in G code), which will be executed on machines from Figure 3. Reason: If the machine of H5D type is used, then further postprocessing calculations for hybrid kinematics of this machine will be made in its control system, in order to make execution of interpreted G-code, created for machine WCVBXYZT (S5D), executable on the machine H5D as well. From this reason reconfiguring of the hardware from Figure 3 should be followed by reconfiguring of the software (control system), in order to establish one real sample of the machine from reconfiguring, with plan that includes the machine with hybrid kinematics (shown in Figure 4) as well. The formalism for postprocessing calculations for five-axis machine tools from Figure 3 is explained in further text, as well as the description of one test of validity of such calculations.

### 3.1 Formalism for postprocessing calculations for WCBVXYZT type of five-axis machines

For this kind of calculations preparing of the model is required. Basic preparation is shown in Figure 4, and it starts from establishing of coordinate systems of the basic machine $\left(\mathrm{O}_{\mathrm{R}}\right)$, of the tool $\left(\mathrm{T}_{\mathrm{T}}\right)$ and of the subsystem with parallel kinematics $\left(\mathrm{O}_{\mathrm{M} 2}\right)$.

Postprocessing calculations for subsystem with serial kinematics, in basic system $\mathrm{O}_{\mathrm{R}}$, are described in this chapter. Kinematic model and important tags are shown in Figure 5. Coordinate system of the machine $\left(\mathrm{O}_{\mathrm{M}}\right)$ is added in this model. In this system, the basic system, $\mathrm{O}_{\mathrm{R}}$, is established, with its origin in the center of the rotation of both tables. Coordinate system of the program is $\mathrm{O}_{\mathrm{W}}$ and it is connected with the workpiece (WP).

Further discussion refers to the case in which the coordinate system of the workpiece, used in programming, is parallel with the systems of the machine. For zero point (G55, for example, in Figure 5), only one translation is required, for correct
operation of the machine according to interpreted program, with preparation of the fixture and blank material on the table made in such a way. Workpiece coordinate system (OW) does not match with the basic coordinate system of the machine, and this is denoted by vector $\overrightarrow{r_{R}}$ in OW system, in which given program is valid. Coordinate system of the tool (T) has its origin in $\mathrm{O}_{\mathrm{T}}$.


Figure 4 - The concept of 5-axis reconfigurable machine tool, type $H 5 D$, with structure WCBVXYZT


Figure 5-The first kinematic model of reconfigurable 5-axis machine tools, type H5D
By programming of this machine using CAD/CAM software, path of the tool relative to the workpiece is created and then stored in CLF, as a data set of six elements: three for the vector of the tool tip position in coordinate system of the program ( $\vec{r}_{T}$ in $\left.\mathrm{O}_{\mathrm{W}}\right)$ and three for orientation of the tool center line ( $\vec{a}$ in $\mathrm{O}_{\mathrm{W}}$ ), for each pose of the machine along tool path, described by command GOTO/ $\overrightarrow{r_{T}}, \vec{a}$, according to APT format.

Coordinate systems in these calculations are:
Coordinate system of the machine $O_{M}\left(X_{M}, Y_{M}, Z_{M}\right)$ and table $O_{R}\left(X_{R}, Y_{R}, Z_{R}\right)$, coordinate system of the workpiece $O_{W}\left(X_{W}, Y_{W}, Z_{W}\right)$, and coordinate system of the tool $O_{T} \approx T_{T}\left(X_{T}, Y_{T}, Z_{T}\right)$. Coordinates of the vectors used for orientation of the tool $\left(\overrightarrow{a_{0}}, \vec{a}\right)$, vectors $\left(\overrightarrow{r_{T 0}}, \overrightarrow{r_{T}}, \overrightarrow{r_{R}}, \vec{p}, \vec{x}\right)$ and translation matrix for $\vec{v}\left[\mathrm{~T}_{\mathrm{r}}(\vec{v})\right]$ are:
$\overrightarrow{a_{0}}=\left\{\begin{array}{l}0 \\ 0 \\ 1 \\ 0\end{array}\right\} ; \quad \vec{a}=\left\{\begin{array}{l}a_{x} \\ a_{y} \\ a_{z} \\ 0\end{array}\right\} ; \overrightarrow{r_{T 0}}=\left\{\begin{array}{l}0 \\ 0 \\ 0 \\ 1\end{array}\right\} ; \overrightarrow{r_{T}}=\left\{\begin{array}{c}x_{T} \\ y_{T} \\ z_{T} \\ 1\end{array}\right\} ;$
$\overrightarrow{r_{R}}=\left\{\begin{array}{c}x_{R} \\ y_{R} \\ z_{R} \\ 1\end{array}\right\} ; \vec{p}=\left\{\begin{array}{c}p_{x} \\ p_{y} \\ p_{z} \\ 1\end{array}\right\} ; \vec{x}=\left\{\begin{array}{c}X \\ Y \\ Z \\ 1\end{array}\right\} ;\left[\mathrm{T}_{\mathrm{r}}(\vec{v})\right]=\left[\begin{array}{cccc}1 & 0 & 0 & v_{x} \\ 0 & 1 & 0 & v_{y} \\ 0 & 0 & 1 & v_{z} \\ 0 & 0 & 0 & 1\end{array}\right]$.
Other: $\vec{v}=\left\{v_{x}, v_{y}, v_{z}\right\}^{\mathrm{T}}$ is arbitrary given vector of translation, T is tool, W or WP is workpiece.

Postprocessing calculations can be, for each of these poses, described in this way: from its reference position, using rotations of tables, machine makes direction of programmed unit vector of the tool axis $(\vec{a})$ on $\overrightarrow{a_{0}}$, the only one available on the machine. Then, using translatory axes, it governs the tool tip ( $\mathrm{T}_{\mathrm{T}}$ ) to its inclined programmed position, after already realized rotations of tables. Tables are rotating first for -C , and then for -B .

Let $\operatorname{Tr}(\vec{v})$ denote a translation for vector $\vec{v}$ and $\operatorname{Rot}(\vec{o}, \varphi)$ denote rotation for angle $\varphi$ around unit vector $\vec{o}$ in a positive direction, where $\vec{o}$ is either unit vector $\vec{i}$ of axes $X_{M} \sim X_{\mathrm{R}}$ of the machine, or unit vector $\vec{j}$ of the axes $\mathrm{Y}_{\mathrm{M}} \sim \mathrm{Y}_{\mathrm{R}}$, or unit vector $\vec{k}$ of axes $\mathrm{Z}_{\mathrm{M}} \sim \mathrm{Z}_{\mathrm{R}}$.

Axis of table rotations on machine H5D, are parallel to Z and Y axis in coordinate systems of this machine. Angle of rotation has negative sign if this rotation is movement of the workpiece (with table). According to standard, angle $A$ is rotation around the unit vector $\vec{i}$, angle $B$ around the unit vector $\vec{j}$ and angle $C$ around unit vector $\vec{k}$. Rotations of the table and then translations of the tool can be formalized as direct kinematics of the machine using the following equations, respectively:

$$
\begin{align*}
& \operatorname{Tr}\left(\vec{r}_{R}\right) \cdot \operatorname{Rot}(\vec{k},-C) \cdot \operatorname{Rot}(\vec{j},-B) \cdot \operatorname{Tr}(\vec{p}) \cdot \vec{a}_{0}=\vec{a} \\
& \operatorname{Tr}\left(\vec{r}_{R}\right) \cdot \operatorname{Rot}(\vec{k},-C) \cdot \operatorname{Rot}(\vec{j},-B) \cdot \operatorname{Tr}(\vec{p}) \cdot \vec{r}_{T 0}=\vec{r}_{T} \tag{1}
\end{align*}
$$

Form of the translation matrix $\operatorname{Tr}(\vec{v})$ for arbitrary vector $\vec{v}$ is already shown. Besides, rotation matrices around machine axes have ordinary form, inherent to homogenous coordinates.

In equation (1) $\vec{p}$ denotes required movement of the tool, relative to the workpiece in $\mathrm{O}_{\mathrm{R}}$ system of the machine, while real required motions of machine axis, which will be written in a program, can be described by vector:

$$
\vec{g}=\left\{\begin{array}{llll}
X & Y & Z & 1
\end{array}\right\}^{T} .
$$

The second solution is a pair of angles of rotary axis. In our case these are angles C and B. Solutions of equation (1) need to separately derived. Solution for inverse geometric problem, obtained in this way, will be implemented in the postprocessor.

However, in this case such formalism is used for checking of just interpreted G code in the control system with open architecture, configured for observed machine, one from Figure 3. With substitutions sC for $\sin (C)$, cC for $\cos (C)$, sB for $\sin (B)$, cB for $\cos (B)$, both of these solutions of inverse geometric problem can be written in compact form . The first part of the solution can be written as:

$$
\begin{align*}
& B=\arccos \left(a_{z}\right), B \in(0, \pi), B \neq 0  \tag{2}\\
& C=\operatorname{atan} 2\left(\frac{\frac{a_{y}}{\mathrm{sB}}}{-\frac{a_{x}}{\mathrm{sB}}}\right), C \in[-\pi, \pi] . \tag{3}
\end{align*}
$$

The second part of the solution, for positions of translatory axes of the machine $(X, Y, Z)$, after rearranging, usually can be written in the following form:

$$
\begin{align*}
& X=x_{R}+p_{x} \\
& p_{x}=\left(x_{T}-x_{R}\right) \cdot c B \cdot c C-\left(y_{T}-y_{R}\right) \cdot c B \cdot s C+\left(z_{T}-z_{R}\right) \cdot s B \\
& \quad Y=y_{R}+p_{y}, \quad p_{y}=\left(x_{T}-x_{R}\right) \cdot s C+\left(y_{T}-y_{R}\right) \cdot c C \\
& Z=z_{R}+p_{z}, \\
& p_{z}=-\left(x_{T}-x_{R}\right) \cdot s B \cdot c C+\left(y_{T}-y_{R}\right) \cdot s B \cdot s C+\left(z_{T}-z_{R}\right) \cdot c B \tag{4}
\end{align*}
$$

### 3.2Checking of postprocessing calculations

Verification of the postprocessing calculations is shown in Figure 6. Centre of the tool holder is denoted by Main spindle. Distance between Main spindle and TT is tool length, here $l=100 \mathrm{~mm}$. Then we observe the case when $\vec{r}_{R}=0$, when $\mathrm{O}_{\mathrm{W}}$ is set exactly in $\mathrm{O}_{\mathrm{R}}$. Solid line shows programmed pose of the machine in the system $\mathrm{O}_{\mathrm{R}}=\mathrm{O}_{\mathrm{W}}$, based on data from program in $\mathrm{O}_{\mathrm{W}}$ system. Dashed line shows the pose of the machine after execution of the movement
according to solutions (2)-(4), for special case: $\overrightarrow{r_{R}}=[0,0,0]^{T}, \overrightarrow{r_{T}}=[25,50,75,1]^{T}$ and
$\overrightarrow{a_{T}}=\left[3^{0,5} / 3,3^{0,5} / 3,3^{0,5} / 3,1\right]^{\mathrm{T}}$.
Results are as follows:
$B=54.736^{\circ}, \mathrm{C}=135.0^{\circ}, \mathrm{X}=30.619, \mathrm{Y}=-17.678$,
$\mathrm{Z}=86.602$, $\mathrm{px}=30.619$, $\mathrm{py}=-17.678, \mathrm{pz}=86.602$.
Overlapping of $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ with px , py , pz , respectively, is a consequence of overlapped coordinate systems OW and OR.

The first and second checking for machine


Figure 6 - Checking solving by the first kinematic model of reconfigurable 5-axis machine tool, type H5D
The second kind of checking are postprocessing calculations in a control system. It has to be conducted in two parts. The first includes checking shown in Figure 6: whether the programmed unit vector of the tool axis $\left(\overrightarrow{a_{T}}\right)$ after rotations for C , and then for B from (2) and (3), overlaps with $\overrightarrow{a_{0}}$, and whether translatory axes of the machine are in positions $\left[\begin{array}{llll}X & Y & Z & 1\end{array}\right]^{T}$, according to (4). In the second check, calculations of required positions of driving axis $p_{1}$ and $p_{2}$ of the machine $\mathrm{M}_{2}$ (Figure 7) with parallel kinematics, are performed.

This is the second required kinematic model of the machine of H5D type. Solutions for inverse and direct geometry were obtained using this model for subsystem with parallel kinematics. This solution will be implemented in open architecture control system execution of required corrections, which are consequences of machine kinematics. After that, planning of the tool path becomes possible, and afterwards interpolations in real time. Model shown in Figure 7 is general for two-axis mechanism, which is, among modules for configuring of machines (Figure 2), denoted as 2-axis parallel mechanism $\mathrm{M}_{2}$.

a) the structure of the model

$$
\begin{aligned}
& \overrightarrow{r_{R 1}}=\left\{\begin{array}{c}
x_{R 1} \\
y_{R 1}=0 \\
1
\end{array}\right\} ; \overrightarrow{r_{R 2}}=\left\{\begin{array}{c}
x_{R 2} \\
y_{R 2}=0 \\
1
\end{array}\right\} ; \overrightarrow{r_{P}}=\left\{\begin{array}{c}
x_{P} \\
y_{P} \\
1
\end{array}\right\} ; \\
& \overrightarrow{r_{B 1}}=\left\{\begin{array}{c}
x_{B 1} \\
y_{B 1} \\
1
\end{array}\right\},\left|\overrightarrow{r_{B 1}}\right|=p_{1} ; \overrightarrow{r_{B 2}}=\left\{\begin{array}{c}
x_{B 2} \\
y_{B 2} \\
1
\end{array}\right\},\left|\overrightarrow{r_{B 2}}\right|=p_{2} ; \\
& \overrightarrow{r_{l 1}}=\left\{\begin{array}{c}
x_{l 1} \\
y_{l 1} \\
1
\end{array}\right\}, \overrightarrow{r_{l 2}}=\left\{\begin{array}{c}
x_{l 2} \\
y_{l 2} \\
1
\end{array}\right\},\left|\overrightarrow{r_{l 1}}\right|=l=l,\left|\overrightarrow{r_{l 2}}\right|=l_{2}=l,
\end{aligned}
$$

P is platform,
$\mathrm{B}_{1,2}$ is the base of driving axes, respectively, the drive axes. $\varphi_{1}=\varphi_{2}=3 \pi / 2$ are the angles of driving axes.
b) tags in the model

## Figure 7 - The second kinematic model of 5-axis

 reconfigurable machine tool, type H5DThis mechanism, embedded in such a way, via its platform P, shown in Figure 4, is used for driving of one passive translatory joint, for simultaneous movement of X and Y axis of the serial part of the machine. Data for the second postprocessing calculation are required positions of X and Y axis of the machine. Based on these positions it is possible to calculate positions $p_{1}$ and $p_{2}$ of driving axes $\mathrm{B}_{1}$ and $\mathrm{B}_{2}$, respectively.

Model shown in Figure 7 has a coordinate system which is suitable for (i) deriving of solutions of inverse and direct kinematic problem of this subsystem, but also for further compensations inherent for control systems for machines with parallel kinematics, and also for machines with hybrid kinematics (there are two rotary and two linearparallel diving axis) and (ii) for embedding of subsystem in a whole system

Subsystem $\mathrm{M}_{2}$ is put in the machine H5D by simple mapping, shown also in Figure 7: $\mathrm{X}_{\mathrm{M} 2} \equiv-\mathrm{Y}_{\mathrm{M}}$ and $Y_{M 2} \equiv X_{M}$, where $\left(X_{M}, Y_{M}\right)$ are axes of the active coordinate system of the machine (either with origin in $\mathrm{O}_{\mathrm{R}}$, as a centre of rotations of tables, or with origin in $O_{M}$, as a general case). Transformation from $X_{M}$ axis to $\mathrm{Y}_{\mathrm{M} 2}$ axis assumes just translation, and from $\mathrm{Y}_{\mathrm{M}}$ to $\mathrm{X}_{\mathrm{M} 2}$ just with altering of the sign (by rotation for $\pi$ ). Because of this, for module $\mathrm{M}_{2}$, model from Figure 7 will be translated in a third and fourth quadrant, Figure 4. This means that the general model is adapting to module $\mathrm{M}_{2}$ in the following way: $x_{R 1}=-x_{R 2}, \varphi_{1}=\varphi_{2}=3 \pi / 2, l_{1}=l_{2}=l$.

According to tags from Figure 7 following sums are made:

$$
\begin{align*}
& \vec{r}_{R 1}+\vec{r}_{B 1}+\vec{r}_{l 1}=\vec{r}_{P}, \text { for drive axes } \mathrm{p}_{1} \text { and } \\
& \vec{r}_{R 2}+\vec{r}_{B 2}+\vec{r}_{l 2}=\vec{r}_{P}, \text { for drive axes } \mathrm{p}_{2} . \tag{5}
\end{align*}
$$

Solution of the inverse geometric problem of module $\mathrm{M}_{2}$ are positions of driving axes $\left(p_{1}, p_{2}\right)$, which satisfy equations (5), all for the position of the platform P given by vector $\overrightarrow{r_{P}}$. Calculation of this position is based on existing solution of the inverse geometric problem, given in (4), based on a program which controls operation of the machine. Under conditions given in Figure 7 ( $l_{1}=l_{2}=l$ and $y_{R 1}=y_{R 2}=0$ ), one can obtain solution of the inverse geometric problem for $\mathrm{M}_{2}$ :

$$
\begin{align*}
& p_{1}=\left(x_{P}-x_{R 1}\right) \cdot c \varphi_{1}+y_{P} \cdot s \varphi_{1}-\sqrt{t_{1}} \\
& p_{2}=\left(x_{P}-x_{R 2}\right) \cdot c \varphi_{2}+y_{P} \cdot s \varphi_{2}-\sqrt{t_{2}} \tag{6}
\end{align*}
$$

In (6) following substitutions were used:
$t_{l}=l^{2}-\left(\left(x_{P}-x_{R I}\right) \cdot s \varphi_{I}\right)^{2}+\left(x_{P}-x_{R I}\right) \cdot y_{P} \cdot s 2 \varphi_{I}-\left(y_{P} \cdot c \varphi_{I}\right)^{2}$
$t_{2}=l^{2}-\left(\left(x_{P}-x_{R 2}\right) \cdot s \varphi_{2}\right)^{2}+\left(x_{P}-x_{R 2}\right) \cdot y_{P} \cdot s 2 \varphi_{2}-\left(y_{P} \cdot c \varphi_{2}\right)^{2}$
$c \varphi_{I}=\cos \left(\varphi_{1}\right), s \varphi_{1}=\sin \left(\varphi_{1}\right), s 2 \varphi_{1}=\sin \left(2 \varphi_{1}\right)$
$c \varphi_{2}=\cos \left(\varphi_{2}\right), s \varphi_{2}=\sin \left(\varphi_{2}\right), s 2 \varphi_{2}=\sin \left(2 \varphi_{2}\right)$
Solving of direct geometric problem starts from system of equations (5), written in algebraic form. By subtracting the second equation from the first one, the following equation is obtained:

$$
s_{1} \cdot x_{P}+s_{2} \cdot y_{P}=s_{3},
$$

where:

$$
\begin{aligned}
s_{1}= & 2\left(x_{R 2}+p_{2} \cdot \mathrm{c} \varphi_{2}-x_{R 1}-p_{1} \cdot \mathrm{c} \varphi_{1}\right), \\
s_{2}= & 2\left(p_{2} \cdot \mathrm{~s} \varphi_{2}-p_{1} \cdot \mathrm{~s} \varphi_{1}\right), \\
s_{3}= & x_{R 2}^{2}-x_{R 1}^{2}+p_{2}^{2}-p_{1}^{2}- \\
& 2 x_{R 1} \cdot p_{1} \cdot \mathrm{c} \varphi_{1}+2 x_{R 2} \cdot p_{2} \cdot \mathrm{c} \varphi_{2} .
\end{aligned}
$$

From $s_{3}=s_{1} \cdot x_{P}+s_{2} \cdot y_{P}$ the elimination of either the first unknown variable $\left(x_{P}\right)$, or the second unknown variable $\left(y_{P}\right)$ can be made. It was shown that elimination of unknown variable $y_{P}$ becomes unspecified for $x_{P}=0$. Therefore, elimination of $x_{P}$ is needed:

$$
x_{P}=\frac{s_{3}-s_{2} \cdot y_{P}}{s_{1}} .
$$

New substitutions are introduced:

$$
s_{4}=\frac{s_{3}}{s_{1}} \text { and } s_{5}=-\frac{s_{2}}{s_{1}} .
$$

Then:

$$
\begin{equation*}
x_{P}=s_{4}+s_{5} \cdot y_{P} \tag{7}
\end{equation*}
$$

With new substitutions:
$s_{6}=1+s_{5}^{2}$,
$s_{7}=2\left(s_{4}-x_{R 1}\right) \cdot s_{5}-2 s_{5} \cdot p_{1} \cdot c \varphi_{1}-2 p_{1} \cdot s \varphi_{1}$ and
$s_{8}=\left(s_{4}-x_{R I}\right)^{2}-2\left(s_{4}-x_{R I}\right) \cdot p_{1} \cdot c \varphi_{1}+p_{I}^{2}-l^{2}$
The second part of the solution of direct geometric problem is obtained in a form:

$$
\begin{equation*}
y_{P}=\frac{-s_{7}-\sqrt{s_{7}^{2}-4 s_{6} s_{8}}}{2 s_{6}} \tag{8}
\end{equation*}
$$

Solutions (6), (7) and (8) were implemented in the system that controls this machine. Checking was realized on the example of model with rotary/tilting table inside mechanical structure of the machine. Alternative approach assumes the rotary/tilting table in front of the machine structure that can imply smaller dimensions of the whole machine. Checking was realized for the following parameters in $\mathrm{M}_{2}: x_{R 2}=-x_{R 1}=340, y_{R 1}=y_{R 2}=0$,
$l=l_{1}=l_{2}=550, \varphi_{1}=\varphi_{2}=3 \pi / 2$.
Input data for checking of the whole postprocessing calculations for hybrid machine, in case where $O_{W} \equiv O_{R}$, were: $\mathrm{r}_{\mathrm{R}}=\left[\begin{array}{lll}0 & 0 & 0\end{array}\right], \mathrm{r}_{\mathrm{T}}=\left[\begin{array}{lll}50 & 100\end{array}\right.$ 50]. Orientation of the tool relative to the workpiece: $\mathrm{B}=54.736^{\circ}$ i $\mathrm{C}=135^{\circ}$, should be set using rotary tables and afterwards translate the tool, relative to $O_{R}$ : $\mathrm{X}=\mathrm{p}_{\mathrm{x}}=-20.412, \mathrm{Y}=\mathrm{p}_{\mathrm{y}}=-35.355, \mathrm{Z}=\mathrm{p}_{\mathrm{z}}=115.470$.

For the calculation of the position of the sliders p 1 and p 2 the input data are:

$$
\begin{aligned}
& x_{P}=-Y=35.355 \text { and } \\
& y_{P}=X-x_{s t}+x_{G V}=-20.412-1040+349.675
\end{aligned}
$$

were $\mathrm{x}_{\mathrm{st}}$ i $\mathrm{x}_{\mathrm{GV}}$ denote parameters for needed translations along $\mathrm{Y}_{\mathrm{M} 2}$ coordinate axis and they are taken from virtual prototype of the machine, Figure 8.

The results for the drive axes are:

$$
p_{1}=308.732 \text { and } p_{2}=252.816
$$

Checking is done on the virtual prototype machine, in the CAD/CAM environment, taking a position and orientation according to the calculated coordinates $\mathrm{p}_{1}, \mathrm{p}_{2}, \mathrm{Z}, \mathrm{B}, \mathrm{C}$, where the tool takes the correct position and orientation to the workpiece, as can be observed in Figure 8.


$$
\begin{aligned}
& X=p_{x}=-20.412, Y=p_{y}=-35.355, Z=p_{z}=115.470 \\
& B=54.736^{\circ}, C=135^{\circ}, p_{1}=308.732, p_{2}=252.816
\end{aligned}
$$

Figure 8-Checking result of postprocessing in the CAD virtual prototype
The shape and dimensions of the workspace by subsystem M2 are shown in Figure 9.

To work with rotary tables a part of the workspace is available; it is designated as a reserve for working with rotary tables: rotary tables are as a rule used for machining of workpieces with dimensions proportional to the dimensions of rotary table, in the middle of the workspace of the machine along axis $\mathrm{Y}_{\mathrm{M}}$.


Figure 9 - Workspace of module M2, based on the second kinematic model of 5-axis reconfigurable machine tool, type H5D

Figure 10 over the rectangular part of the workspace the change of Jacobian determinant is plotted. It can be seen that the determinant in terms of absolute value is close enough to the 1 .


Figure 10 - Jacobian determinant of module M2 according to the second kinematic model reconfigurable desktop 5 -axis machine tool, type $H 5 D$

## 4. ONE EXAMPLE OF VERIFICATION POSTPROCESSING

For verification we have selected machining of pyramid, where during machining the position of the worktable remains constant for long time and the control is easier. Parts of the program for these positions have been performed separately, in the regime of 3-axis machining, on the horizontal machining center LOLA HMC500, with two manually controlled rotary axes added, Figure 11. Positioning by two manually controlled rotary axes is achieved by using fixture, in which the first axis is blocked (Figure 11a), while the other two axes are used to realize the orientation of the workpiece, axes B and C . If we observe the chosen machine (Figure 11 b ) as a machine rotated for $-90^{\circ}$ around X axis (Figure 11c), conceptually the machine of S5D type is obtained (Figure 11d). It is here used for checking the configured postprocessor for vertical 5-axis machine with structure WXYZBCT.

Program for machining is prepared using CAD/CAM systems and the obtained $G$ code using configured postprocessor. Since the workpiece is four-sided pyramid, tool orientation is achieved using four separate tilting the workpiece for angles B and C which were obtained in the $G$ code, after postprocessing tool path from the CLF. First we have prepared the program for 3 -axis pre-machining when the orientation of worktable $\mathrm{B}=0$ and $\mathrm{C}=0$. Afterwards pre-machining continues with the machining of each of the four sides of the pyramid. After the completition of pre-machining, tool orientation is achieved by using tilting fixture for the calculated angles B and C , for machining the first side of the pyramid (C0. B-21.801).

d) The equivalent of machine tool, type S5D

Figure 11-LOLA HMC500 with attached fixture for workpiece orientation


Figure 12 - Checking the four position and orientation for the case of machining four-sided pyramid

After the completion of machining of each side of the pyramid, machine stops with command M0 and takes up a new tool orientation using tilting fixture for the calculated angles B and C . These tiltings for each side of pyramid are: (C0. B-21.801), (C90, B-21.801), (C180. B-21.801) and (C270. B-21801) and are presented as a frozen tool position and orientation according to the workpiece in Figure 12.

Machining of four-sided pyramid on the machine LOLAHMC500 is shown in Figure 13.


Figure 13 - Machining four-sided pyramid on the machine LOLA HMC500

At the beginning, blank workpiece was premachined in the programmed 3-axis milling operation in $\mathrm{z}=$ const tool paths, leaving allowance for finishing, Figure 13a. Figure 13b shows milling of the first side of the pyramid, with rotation of B-axis only (B21.801), where $C=0$. Milling of the last side of the pyramid with orientation setting $\mathrm{C}=270$. $\mathrm{B}=-21.801$, is shown in Figure 13c. Final result of milling of all four sides of the pyramid is shown in Figure 13d and Figure 13e. In this way postprocessing calculations
are verified through particular checking for four programmed orientations.

## 5. CONCLUSION

The main goals set out in this paper, were related to the development of one class of desktop reconfigurable machine tools, based on the concept of CNC open architecture control, and for machine tools with specific configurations.

The paper presents the initial concept for the development of desktop reconfigurable 5-axis machine tool. The system of assembly components is established in the form of configurator with the rules for using basic modules during synthesis of the machine structures in the building program for planned multifunctional reconfigurable desktop five axis machine tools. Among all the structures of machine tools planned in the building program one class of reconfigurable desktop machine, types S3D, S4D, S5D and H5D was selected.

Formalism for postprocessing calculations for machines of WCBVXYZT type is shown in this paper, with checking of calculations on a virtual prototype and with machining test on an available machine equipped with two-axis fixture for setting orientation of the workpiece.

The application of these machines is important for further research in the field of multi-axis machining and reconfigurable 5 -axis machine tools, as well as for education in programming, which is particularly important for educational institutions, in the acquisition of knowledge about the multifunctional reconfigurable machines tools.

Realization of the virtual five-axis machine which is integrated in a control system and system for programming will be considered in our future research. Also, the use of one kind of hybrid object programming of CNC machine tools, which is well known as STEP-NC, is planned, to the extent to which this method of programming will be applicable for future CNC units [7].

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## REZIME

JEDNA STONA REKONFIGURABILNA MAŠINA ALATKA SA HIBRIDNOM KINEMATIKOM

U ovom radu je predstavljena jedna stona rekonfigurabilna mašina alatka sa hibridnom kinematikom, za četiri tipa mašina alatki, sa opisom primenjenog mehanizma i uspostavljenim modularnim sistemom za njihovo konfigurisanje. $U$ radu je opisan i postprocesor za petoosnu obradu, primenjen na kinematičku strukturu mašine sa obrtno-nagibnim stolom, sa rotacijama (B,C). Takođe je pokazano izračunavanje pozicija aktuatora p1 i p2 kada mašina radi sa hibridnom kinematikom. Verifikacija postprocesora je ostvarena na virtuelnom prototipu u CAD/CAM okruženju i eksperimentalno na jednoj raspoloživoj troosnoj mašini, sa dodatim dvoosnim priborom. Eksperimentalni rezultati su potvrdili konfigurisani postprocesor, koji može da se koristi za programiranje mašina.

Ključne reči: konfigurisanje, rekonfigurabilna mašina alatka, postprocesor


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