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ПРВ СИМПОЗИУМ ОД ОБЛАСТА НА ЕКСПЛОЗИВНИ МАТЕРИИ, ВООРУЖУВАЊЕ И ВОЕНА ТЕХНОЛОГИЈА, ОХРИД, 25 -28, СЕПТЕМВРИ 2002

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PERFORATION OF MULTI-LAYERED METALLIC TARGETS BY KINETIC PENETRATORS

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Summary

In the paper it is considered problem of perforation of multi-layered metallic targets by deformable cylindrical flat-ended penetrators. Analytical model for perforation of multi-layered targets is created on the basis of complex, multiphase, phenomenological model of deformation waves. Two variants of model involve cases of spaced and jointed layers of target. Computer program based on developed model enables determination of residual penetrator’s velocity as well as ballistic limit velocity. Double-layered target with constant total thickness is specially investigated in order to define optimal ratio of layers’ thickness that provides maximum target resistance. Results of analytical model are in good correspondence to experimental data. These results also enable formulation of few conclusions that can have practical importance.

Keywords: Perforation, multi-layered target, ballistic limit velocity, analytical model, experimental investigation

1. Introduction

In the paper it is considered penetration/perforation of multi-layered metallic targets by cylindrical penetrators – simulators of HE projectile fragments. Ideal impact conditions are assumed (normal impact of cylindrical flat-ended penetrator into immovable multi-layered target with plane and homogenous layers) at impact velocities less than 1200 m/s. It is also presumed that *plug formation* is dominant phenomenon in penetration process.

This class of penetration occurs at consideration of aircrafts, non-combat and light-armored vehicles attacked by HE projectiles. Its investigation is important from the aspect of efficiency and optimal projectile design, as well as from the point of view of vulnerability and design of ballistic protection.

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Adopted analytical approach to penetration modeling provide clear insight in physics of process and dominated phenomena, and also ensure efficient parametric studies and solutions with satisfactory accuracy.

Consideration is essentially different in cases of separated (spaced) and jointed (in contact) layers (Fig. 1.). Modeling of penetration of multi-layer targets is very complicated and basic problem is determination of influence of successive layer on penetration process of current layer. It is presented simplified approach, which is completely based on application of analytical model for penetration of monolithic targets.

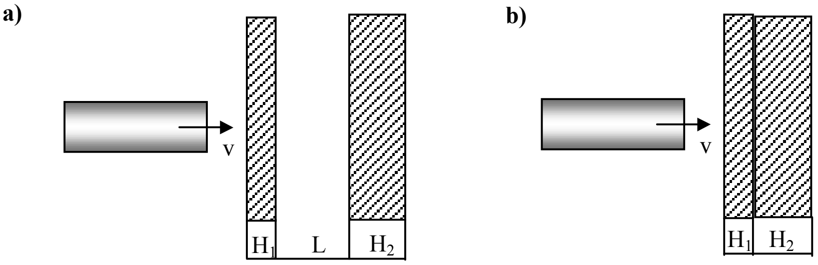

Fig. 1. Cases of penetration of multi-layer target with (a) spaced and (b) jointed layers

2. Model of deformation waves

It has been given very comprehensive reviews of different analytical appraoches to the modeling of penetration process for single targets, e.g. in [1]. Analysis of relevant models [2] showed that *model of deformation waves* (phenomenological model) *with deformable penetrator* represents the best analytical solution for penetration of single thin metallic plates.

This theoretical approach [3] considers motion of both primary and secondary target zone and introduces very complex penetration mechanisms. Basically, it is a multi-phase model in which compression and shear of target material has a key role. Due to impulsive nature of the process, it is regarded that these stresses have a wave form.

Normal and shear stresses σ and τ in target material under impulsive loading are defined by constitutive equations

$$\sigma = \frac{1}{1-\varepsilon}\left[\sigma_y - B\ln(1-\varepsilon)\right], \qquad \tau = \tau_y + \frac{1}{3}B\gamma \tag{1}$$

where B is coefficient of target material strengthen, σ_y and τ_y are quasi-static normal and shear yield stress, ε and γ are relative normal deformation (engineering strain) and shear angle. The real, increased value of normal stress due to *constrained deformation* of target material is determined by equation $\sigma_C = K\sigma$. Because of discontinuity of velocity and deformation in the compression zone and undeformed zone, propagation of compression wave is modeled by concept of shock wave. Typical configuration of deformation zones as well as geometry of penetrator/target system is shown in Fig. 2a. Zone 1 is represented by rigid penetrator; zone 2 consists of deformed plug portion – plastic shock wave passed by this zone and its velocity is equal to the penetrator velocity during the whole process; zone 4 represents undeformed part of plug – its length decreases and its velocity increases to the value of velocity in zone 2, i.e. to the completion of plug formation. Zone 3 is formed by secondary (outer) target zone and its motion is a result of shear stresses. Mass of zone 3 is increased during the process due to propagation of shear plastic hinge; its velocity first increases and then decreases to the final arrestment. Zone 5 is consisted of immovable remainder of target. It is assumed that all zones behave as rigid bodies, in the sense that all points of the same zone (in defined moment) have the same velocities. Application of momentum law and relation between displacement and deformation gives equations

$$c_p = \frac{v_1 - v_4}{\varepsilon} + v_4 \,, \; \varepsilon(\sigma_c - \sigma_{yc}) = \rho(v_1 - v_4)^2 \tag{2}$$

which enable determination of unknown deformation ε, velocity of plastic shock wave c_p and stresses σ and σ_c.

In the phenomenological penetration model, first phase last from initial contact to the moment in which zones 3 and 4 start to move with different velocities. Equations of motion in first phase have a form:

$$\begin{aligned} \dot{v}_1 &= -\frac{\sigma_c A + \rho A(v_1 - v_4)^2 + 2\pi R \tau_q (x_2 - x_1)}{m + \rho A(x_2 - x_1)}, \qquad v_1 = v_2, \\ \dot{v}_4 &= \frac{\rho A(v_1 - v_4)^2 + 2\pi R \tau_q (x_2 - x_1) - 2\pi r H(\tau_y + \rho c_s v_3)}{\rho \pi r^2 H - \rho A(x_2 - x_1)}, \qquad v_3 = v_4 \end{aligned} \tag{3}$$

where m, A and R are penetrator's mass, cross-section area and radius, H is target thickness, r and c_s are position and velocity of hinge and τ_q is dynamic shear stress defined by (1).

In the second phase of penetration there are no new physical processes, which would essentailly change equations of motion. This phase terminates with arrestment of compression shock wave ($v_1=v_4$), so penetrator and plug start to move as a single body.

In the final phase of penetration, there is only shear force, so process is simply modeled. Perforation is completed when the platic fracture occurs, i.e. when condition $x_1=x_3+H$ is fullfiled.

It should be observed that it can exist more complicated "scenarios" of process, which are similarily modeled and which are in detail operated in computer program.

Previous analysis imply undeformable (rigid) penetrator; however, it is possible to generalize model to the case of *deformable penetrator* [4]. If the constitutive equation (1) is also valid for penetrator material, then by application of momentum law on part of target and penetrator subjected to the deformation in the moment of initial contact, it can be derived that real, reduced initial velocity of penetration has a value:

$$v' = \frac{v_0}{1 + \sqrt{q}}, \qquad q = \frac{\rho(\sigma_y + B)}{\rho_p(\sigma_{yp} + B_p)} \tag{4}$$

where q is the parametar dependent on characteristics of materials of target and penetrator (index p). Typical configuration of zones at penetration by deformable penetrator is given in Fig. 2b. New zone 6 is represented by undeformed portion of penetrator and defined by position of plastic compressive shock wave in penetrator (with velocity c_t). Equations of motion, conditions for phases termination and phase flow of penetration process are determined analogous to the model with rigid penetrator.

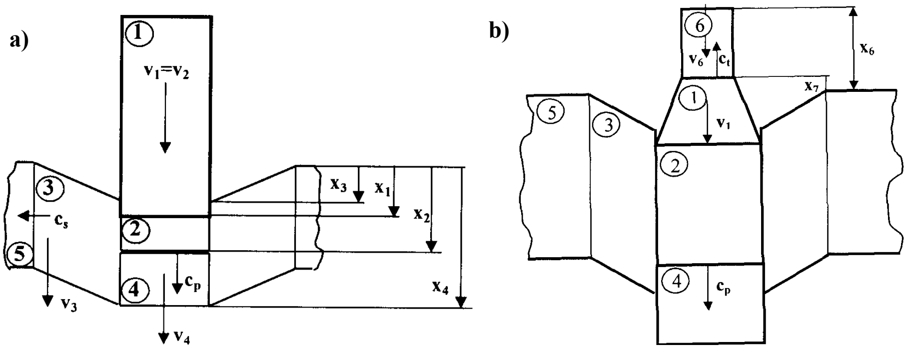

Fig. 2. Geometric configuration of penetrator-target system in the case of (a) rigid and (b) deformable penetrator

In order to obtain better compatibility between theoretical and experimental results, specially for targets of greater thickness, it is necessary to modify presented model. It can be achieved by correction of value of parameter K which takes into acount increase of stress σ in target material, due to presence of surrounding material that constraints plug compression. According [3] in considered model is adopted that this parameter represents characteristic of material, which has constant value K=[1.7 ÷ 2.0]. However, it is clear that value of parameter K depends also on conditions of compression, particulary on relative deformation ε, which is experimentally verified. As this dependence is not generally known, modification is refered on application of constant, average value of K during the entire process. Considering the fact that at thin plates average value of relative deformation is primarily determined by penetrator's impact velocity v_0, it is adopted functional dependence of parameter K on this velocity in the form [2]:

$$K = \begin{cases} K_0 + \dfrac{K_1 - K_0}{c_{cr}} v_0, & 0 < v_0 \le c_{cr} \\ K_1, & c_{cr} \le v_0. \end{cases}, \qquad c_{cr}^2 = \lim_{\varepsilon \to 0} \frac{1}{\rho} \frac{\partial \sigma_C}{\partial \varepsilon} = \frac{K_1}{\rho}(\sigma_y + B) \qquad (5)$$

In the last equation c_{cr} is the maximum velocity of compression wave in the target material in the moment of impact; K_0 and K_1 are minimum and maximum value of parameter K, which corresponds to zero impact velocity and impact velocity equal to c_{cr}, respectively. So, value of parameter K first linearly increases with increase of impact velocity from zero to c_{cr}, and then for higher impact velocity, it is constant.

On the basis of presented modified model, it is developed computer programs MODELRIGID and MODELDEF for simulation of penetration process in cases of rigid and deformable penetrator. It is shown [2] that these programs provide results that are in very good accordance to experimental data for monolithic targets.

3. Perforation of multi-layer target with spaced layers

This is, from the point of modeling, significantly simpler case of penetration. By target with spaced layers, it is understood target in which successive layer has no influence on the perforation of current layer. So, perforation of current layer is finished before effective mass of penetrator (which includes masses of plugs from previous layers) reach next layer. If H_i denotes thickness of i-th layer, and L_i distance between i-th and $(i+1)$-st layer of target, it can be conditionally regarded that layers are spaced if it is satisfied

$$L_i \geq \sum_{i=1}^{n} H_i \qquad i = \overline{1,n\text{-}1} \tag{6}$$

where n is total number of target layers. This condition is fulfilled if the distance between layers is greater than length of overall previously formed plug (it is consider that plug length is approximately equal to the thickness of corresponding layer). Under these conditions perforation of multi-layer target can be regarded as a process, which represents simple superposition of penetrations of n single layers. Residual penetrator velocity from current layer is its striking velocity for successive layer, while the mass of penetrator is increased for the corresponding plug's mass. So, it can be written

$$\begin{aligned} &v_{s1} = v_0, \quad (v_r)_i = (v_s)_{i+1}, \\ &m_1 = m, \quad m_{i+1} = m_i + (m_{pl})_i, \quad i = \overline{1,n\text{-}1} \end{aligned} \tag{7}$$

where index s is related to striking (impact), and index r to residual velocity of penetrator. Regarding penetrator's diameter, it can be considered that due to strengthen, its overall deformation (and therefore increase of its diameter) is completely accomplished during the penetration of first layer. If initial diameter of penetrator d is increased to the value d_d during the penetration of first layer, then can be written

$$d_1 = d, \quad d_i = d_d, \quad i = \overline{2,n} \tag{8}$$

Exposed concept, based on conditions (7) and (8), enables determination of all relevant parameters of penetration of multi-layer target, using model for monolithic target. In the Fig. 3a it is presented flow-chart on which computer program is based. Values I and O represents vectors of input and output values (velocity, mass and diameter), and MODELDEF/MODELRIGID are functions that performs calculation of penetration parameters for monolithic target on the basis of modified model of deformation waves with deformable/rigid penetrator.

4. Penetration of multi-layer target with jointed layers

If the target layers are in direct contact (L_i=0), it is the case of jointed layers that undergo to more complex analysis. Namely, successive target layer significantly influences to the penetration of current layer by increasing resistance to penetrator progression. It is clear that this influence is manifested by constrained deformation and motion of both primary (plug) and secondary zone of current layer. Effect of successive layer is not easy to determine; so, for exact treatment of phenomena influenced by next layer, it would probably be necessary to create quite new analytical model. However, adopted approach implies adaptation of existing model (for monolithic targets) to the case of layered target.

Considering deformation of penetrator, modified model of deformation waves regards practically its instant deformation in the moment of impact into target, so penetrator is regarded as a rigid body in further penetration process. However, deformation of penetrator is actually occurred continually during the all time of penetration. Having in mind this continuity of penetrator's deformation and fact that in the case of jointed layers penetrator is permanently loaded (in contrast to variant of spaced layers), it can be accepted assumption that deformation of penetrator is greater than in the case of spaced layers. In this way, effect of increased resistance influenced by successive layer can be taken into account by means of increased penetrator’s diameter. Since the model can’t treat this increase as a continual, it can be presupposed series of penetrator’s deformation every time when terminates perforation of current and starts penetration of successive layer. Hence, analysis is reduced on previous case of target with spaced layers, involving hypothesis of deformable penetrator at impact in each successive layer, which approximately takes into account influence of next layer. Therefore, equation (7) still remain and assumption (8) gets the form

$$(d_i)_d = d_{i+1} \tag{9}$$

where d_i and $(d_i)_d$ are related to values of penetrator’s diameter before and after the perforation of i-th layer of target.

By multiple application of program for penetration of monolithic targets (MODELDEF), it can be determined parameters of penetration process, according to flow-chart, Fig. 3b.

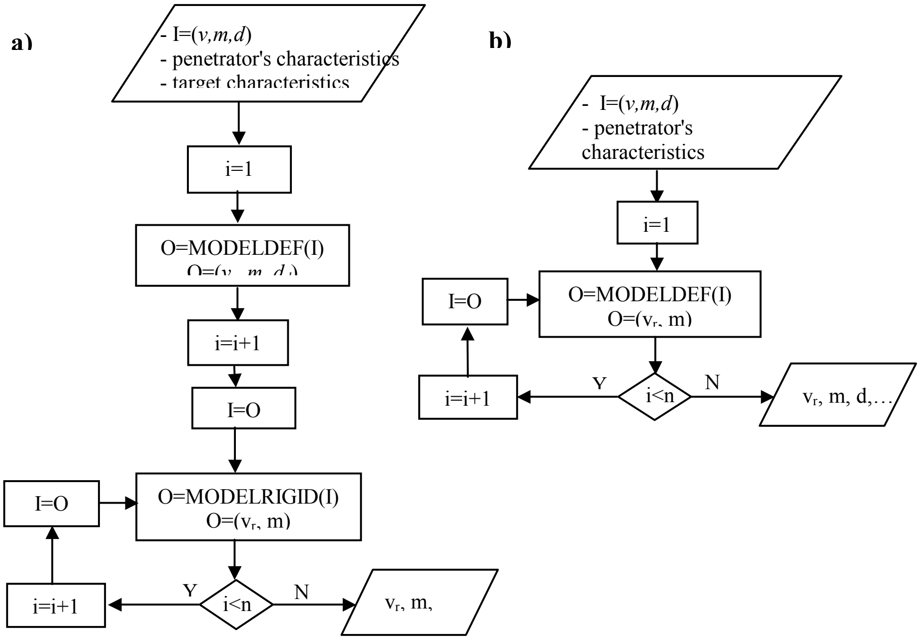

Fig. 3. Flow-charts for determination of penetration parameters in the case of layered target with (a) spaced and (b) jointed layers

Case of separated layers (L_i>0) that not satisfy condition (6) (layers are very close each to other) is not considered. Since the influence of succesive layer in this case is manifested only in certain resistance to plug motion, it can be assumed that this case can be considered by model with spaced layers, rather than with model for penetration of targets with jointed layers.

5. Analysis of results and discussion

Experimental investigation of penetrability [5,2] is accomplished in ballistic tunnel of Institute for Military Engineering. It was performed perforation of double-layered steel jointed plates by standard steel penetrator (simulator of fragments). The basic characteristic of penetrator and plates are presented in Table 1. It were executed two series of shootings on the targets formed by combination of order of plates with thickness 1.25 mm and 2.20 mm. Average values of measured impact and residual velocities are shown in Table 2.

Analytical model for penetration of multi-layer targets, based on simple application of modified model of deformation waves, is tested with prior objective to determine its qualitative characteristics. Regarding complexity of investigated phenomena and approximate nature of model, it will be consider only the fundamental parameters of penetration process – penetrator's residual velocity and *ballistic limit velocity* (miminum perforation velocity).

Table 1. Basic characteristics of penetrator and plates used in experiment

PENETRATOR		PLATE	
Material (steel)	Č.1731	Material (steel)	Č.0146
Hardness (HB)	243	Hardness (HB)	80
Mass (g)	1.411	Normal yield stress (MPa)	320
Maximum diameter (mm)	5.71	Transfersal dimensions (mm)	400 x 400

Table 2. Impact and residual velocities at penetration of double-layered target

Thickness of target layers (mm)	Impact velocity (m/s)	Residual velocity (m/s)
1.25+2.20	915	405
2.20+1.25	923	508

Residual penetrator's velocity as a function of its impact velocity for cases of jointed and spaced layers is shown in Fig. 4.

In the case of jointed layers (Fig. 4a) it is shown that target with thinner layer (1.25 mm), which is in front of thicker layer (2.20 mm), is more resistant, i.e. obtained residual velocities are lower. It is noticable good correspondence of computed and experimental results. It is remaked significant difference between ballistic limit velocities, considering that target has equal total thickness (3.45 mm); with increasing impact velocities, difference between residual velocities gradually decrease.

Residual velocities in variant of spaced layers are, of course, greater than in the case of jointed layers (Fig. 4b), which is expected regarding lower resistance of this structure. Relatively small difference of residual velocities indicates that order of layers, if they are separated, hasn't significant influence on target resistance. Absence of experimental verification of model results for the case of spaced layers, necessitate to reserve in respect to last conclusion.

Undoubtedly, comparison of resistance of monolithic and multi-layer target with equal total thickness (and masses) is important and interesting. The question is whether is possible to form multi-layer target of increased resistance by suitable separation of monolithic target thickness. This will be examined on the example of double-layered target (with jointed layers that enables greater resistance) with total thickness 3.45 mm. Fig. 5 presents residual penetrator velocity and ballistic limit velocity as a functions of relative thickness of first layer of the target. Both quantities presents measures of target resistance – the greater ballistic limit and the lower residual velocity determinate the more resistant target.

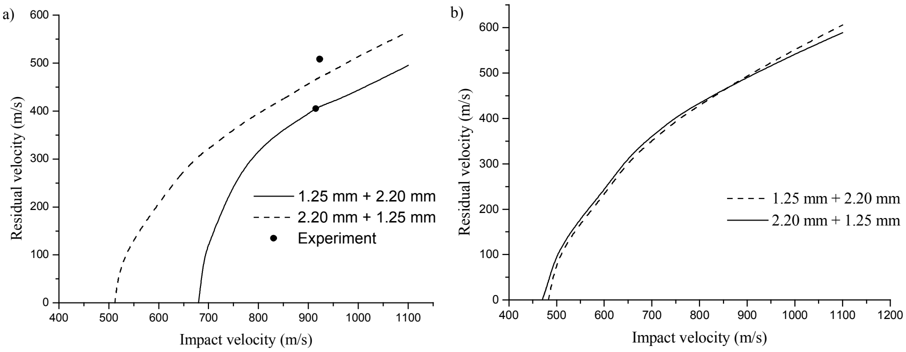

Fig. 4. Residual penetrator velocity as a fuction of impact velocity for double-layered target with (a) jointed and (b) spaced layers

In Fig. 5a it is noted that minimum residual velocity is obtained for small thickness of first layer (impact velocity 919 m/s corresponds to average value of simulator's velocities realized in experiment); this velocity gradually increase and reach maximum (that defines target of minimum resistance) at first layer thickness 60 ÷80% of total target thickness. After that, residual velocity decreases again.

For ballistic limit velocity model provides analogous results (Fig. 5b). This velocity has maximum at low thickness of first layer, and reaches minimum for targets with first layer thickness 40÷70 % of total thickness. Further increase of thickness of first layer causes increase of ballistic limit velocity.

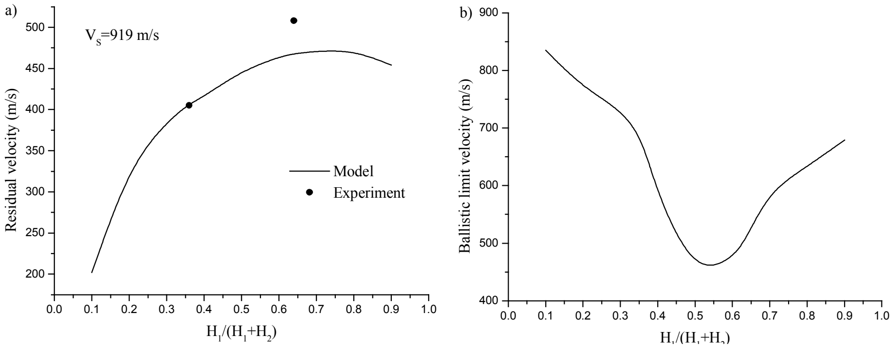

Fig. 5. Influence of relative thickness of first layer on (a) penetrator's residual velocity and (b) ballistic limit velocity (penetration of double-layer targets with jointed layers - striking velocity v_s=919 m/s, total thickness H_1+H_2=3.45 mm)

Both diagrams (which are qualitatively the same for different values of impact velocity and total target thickness) lead to conclusion that the greater resistance of double-layer target is obtained at low thickness of first layer – maximum resistance would be obtained in the case of zero thickness of first layer, which corresponds to monolithic target. Increase of first layer share in total target thickness causes decrease of its resistance and reaches minimum at ratio $H_1/(H_1+H_2)$=0.5÷0.8. For greater thickness of first layer resistance increases again.

Of course, model enables consideration of greater number of layers and their influence on resistance of multi-layer target. On the diagram (Fig. 6), it is presented residual and ballistic limit velocity as a functions of number of layers *n* with equal thickness (total target thickness is constant). With increase of number of layers to *n*=3 residual velocity increase, and for triple-layered target and for targets with more than three layers residual velocity is practically constant. Ballistic limit velocity rapidly drops at increase of number of layers up to three, and then its decrease is gradual. Therefore, results obtained by application of analytical model show the fact that increase of number of layers leads first to rapid and then to moderate decrease of multi-layer target resistance.

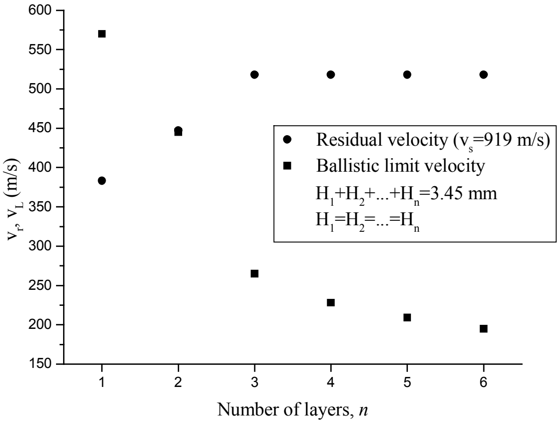

Fig. 6. Influence of number of layers n of multi-layer targets with equal total thickness on residual velocity and ballistic limit velocity

All qualitative observations metioned above are in accordance to published theoretical and experimental results [6,7,8]; however, they shold be proved by independent and more comprehensive experiments.

Conclusion

Analysis of penetration of multi-layer targets performed on the basis of the results obtained by simple analytical model can be reduced on next statements:

- Investigation of penetration of multi-layered targets is of great importance from the aspects of projectile and ballistic protection design. Analytical models represent efficient tools for penetration studies.
- Multi-layer targets with jointed layers, at the same penetration conditions, have greater resistance than targets with spaced layers. However, this conclusion need not to be valid if the penetrator's tip is not flat; it is possible that change of tip's shape during the penetration of first layer (e.g. transition from conical to flat shape) causes significant increase of resistance in the case of target with spaced layers.
- Monolithic target has greater resistance than any other multi-layer target of equivalent thickness, maid of the same material. Qualitative explanation is in the fact that, at penetration of monolithic plate, entire structure of target (i.e. secondary zone) resist to penetrator all the time of penetration and, therefore, gives greater resistance than individual layers of multi-layer target.

- Increase of number of layers of multi-layer target, at constant total thickness, causes decrease of target's resistance.
- Analysis of double-layer target penetration shows that maximum resistance can be obtained for very small (<20% of total thickness) or very high (>80% of total thickness) thickness of first layer.

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ПЕРФОРАЦИЈА НА ПОВЕЌЕСЛОЈНИ МЕТАЛНИ ЦЕЛИ СО КИНЕТИЧКИ ПЕНЕТРАТОРИ

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Резиме

Во трудот се согледува проблемот на перфорација на повеќеслојни метални цели со помош на деформабилен цилиндричен пенетратор со рамен врв. Аналитичкиот модел за перфорација на повеќеслојни цели е креиран врз основа на комплексен, повеќефазен, феноменолошки модел на деформирачки бранови. Двете варијанти на моделот опфаќаат случаеви на раздвоени и зближени слоеви на метата. Компјутерскиот програм заснован врз изградениот модел овозможува одредување на резидуалните брзини на пенетраторите, како и на брзината на балистичкиот лимит. Посебно е испитувана двослојна мета со константна вкупна густина со цел да се дефинира оптималниот однос на густините на слоевите што ќе овозможи максимална отпорност на метата. Резултатите од аналитичкиот модел се во добра кореспонденција со експерименталните податоци. Овие резултати, исто така овозможуваат формулирање на неколку заклучоци што можат да имаат практично значење.

Клучни зборови: перфорација, повеќеслојна мета, брзина на балистички лимит, аналитички модел, експериментални испитувања.