RAPID EXPANSION OF METALLIC CYLINDER DRIVEN BY INTERNAL EXPLOSIVE DETONATION

Predrag Elek[1], Slobodan Jaramaz[2]

Faculty of Mechanical Engineering, University of Belgrade
Kraljice Marije 16, 11120 Belgrade 35
[1]e-mail: pelek@mas.bg.ac.rs
[2]e-mail: sjaramaz@mas.bg.ac.rs

Abstract. The paper considers acceleration of a cylindrical liner caused by expanding products of internal detonation. A new model of liner motion is developed for the cases of axisymmetric (head-on) and grazing (side-on) detonation of the explosive charge. Suggested model relies on the two-stage regime of cylinder motion: (i) the first stage is the consequence of detonation wave – metallic liner interaction; as a result the initial velocity is imparted to the liner, (ii) the second stage is gas-dynamic push of the liner governed by detonation product expansion, similarly to the Gurney model. The model completely describes the cylinder motion in the cases of axisymmetric and quasi-steady grazing detonation. Comparison with extensive database of experimental results shows that proposed model provides very good description of measured data in the domain relevant for terminal ballistics applications.

1. Introduction

Behavior of cylindric metal liner under the action of impulsive internal loading brought about by explosive detonation is of great importance primarily in the analysis of high-explosive warhead mechanisms, as well as from the aspect of structural integrity and safety of vessels with high-energy materials. Detonation of an explosive charge generates gaseous detonation products of extremely high pressure (~20 GPa) causing rapid expansion of cylindric metal liner. The goal is modeling of the cylinder motion and determination of its stress-strain state until the onset of fragmentation process.

Gurney [1] formulated the classical model based on energy balance, analyzed in detail by Kennedy [2]. This model is still in wide use as a method for calculation of the final velocity of metallic liner, i.e. the initial velocity of generated fragments. However, Gurney's model does not take into account cylinder deformation and fails to describe the evolution of liner motion. Moreover, the model has certain limitations and requires experimental determination of Gurney energy [3].

Numerous physically based models are suggested that characterize acceleration, deformation and fragmentation of metallic cylinder due to the action of detonation products [4], [5], [6]. These models employ different concepts of material behavior (elastic, ideally plastic, viscoplastic) and detonation product expansion.

A new comprehensive analytical model of cylinder expansion under the action of detonation products will be briefly presented.

2. Cylinder acceleration model

The following assumptions will be used for modeling of the motion of metallic liner caused by explosive detonation: (i) detonation wave is one-dimensional and in steady-state, (ii) explosive material is instantaneously transformed into detonation products, and (iii) cylinder material is incompressible.

Two stages of cylinder acceleration process will be considered: (i) the first stage implies the interaction of detonation wave and liner material; the result is virtually instant impart of the initial velocity to the cylindric liner, (ii) the second stage relates to the cylinder motion under the pressure of expanding detonation products, analogously to the Gurney model. In addition, two types of detonation will also be analyzed: (i) axisymmetric detonation of cylindric explosive charge that, in accordance to adopted assumptions, provides simultaneous onset of motion of the entire cylinder after detonation of complete explosive charge; cylindrical detonation wave is formed and head on interaction occurs between the detonation wave and the cylinder wall (Fig. 1a), (ii) in the case of grazing (side on) detonation, only the part of cylinder traversed by detonation wave is set into motion, while the detonation wave is orthogonal to the cylinder axis (Fig 1b).

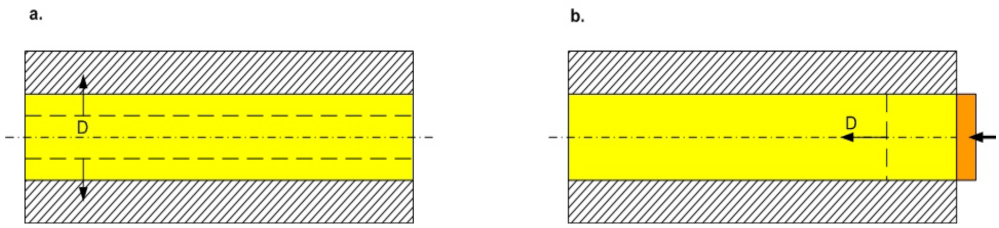


Figure 1. (a) Axisymmetric detonation of explosive charge (head on interaction of detonation wave and liner), and (b) grazing detonation (tangent, side on, interaction of detonation wave and liner)

2.1. Initial velocity of cylinder

The analysis of available experimental results (Fig. 2) shows that cylinder in the initial stage of motion has extremely high acceleration, i.e. in very short time interval reaches high velocity. This fact, as well as notable oscillatory character of liner velocity, indicates important effect of shock waves formed in the cylinder by the impulsive action of detonation wave. Backofen and Weickert [7], [8] analyzed numerous experimental data and introduced mentioned concept of two-stage acceleration of liner propelled by detonation products. Moreover, they suggested the empirical formula for calculation of the initial liner velocity v_i.

An analytical approach to the problem of interaction of detonation wave and metallic liner, resulting with the initial liner velocity v_i, based on impedance matching technique [10] will be presented here.

Normal interaction of detonation wave and liner. Normal (head on) interaction of the plane detonation wave and metallic liner is depicted in Fig. 3.
Hugoniot shock adiabat of detonation products in velocity-pressure (u-p) coordinate system can be written in the form:

$$p = \frac{\gamma + 1}{2} \rho_0 u^2 + (\gamma - 1) \rho_0 Q \quad (1)$$

where ρ_0 is explosive density, γ is polytrophic coefficient of detonation products, and Q is detonation heat.

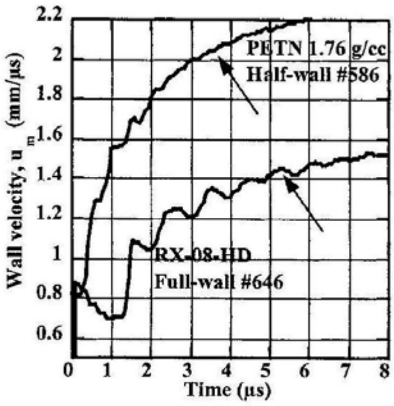


Figure 2. Typical rapid increase of cylinder velocity at the very beginning of motion; experimentally obtained diagram from [9]

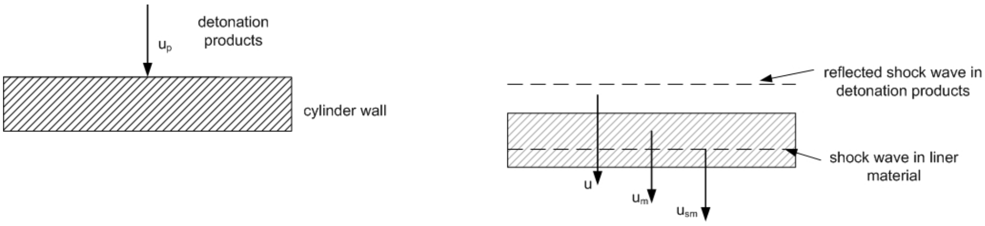


Figure 3. Normal (head on) interaction of detonation wave and solid obstacle; characteristic velocities of the process are indicated

When the detonation wave interacts with metallic liner, it reflects back, and Hugoniot adiabat of reflected wave has the form [10]:

$$p = \frac{\gamma + 1}{2} \rho_0 (2u_p - u)^2 + (\gamma - 1) \rho_0 Q \,, \quad (2)$$

where u_p is material velocity of detonation products at the moment of encounter with the obstacle. The shock adiabat of cylinder material is defined by the relation:

$$p_m = \rho_m u_m (c_m + s_m u_m) \,, \quad (3)$$

where u_m and p_m are velocity and pressure in the part of the cylinder encompassed by shock wave, while c_m and s_m are the equation of state parameters for the considered material.
The continuity condition between two considered media (gaseous detonation products and cylindric metal liner) implies that the velocities and pressures in the shock wave zone should be equal:

$$u = u_m, \quad p = p_m \,. \quad (4)$$

Equating the right hand sides of Eqs. (2) and (3), and using condition (4), quadratic equation emerges that enable determination of the unknown liner velocity u_m:

$$\left(\frac{\gamma+1}{2}\rho_0 - s_m\rho_m\right)u_m^2 - \left(2(\gamma+1)\rho_0 u_p + c_m\rho_m\right)u_m + 2(\gamma+1)\rho_0 u_p^2 + (\gamma-1)\rho_0 Q = 0\ . \quad (5)$$

If we introduce well-known relations for the velocity of detonation products and the detonation heat:

$$u_p = \frac{D}{\gamma+1}, \quad Q = \frac{D^2}{2(\gamma^2-1)} \quad (6)$$

the previous quadratic equation is easily solved. The obtained velocity u_m is at the same time the initial velocity of cylinder generated by the effect of detonation wave

$$v_i = u_m\ . \quad (7)$$

In order to simplify the analytical treatment of the problem, we will assume the effect of shock waves is dominant only at the onset of cylinder motion, i.e. the subsequent oscillatory motion produced by reverberations of shock waves in the cylinder can be neglected comparing to the motion under the action of rapidly expanding detonation products.

It should be emphasized that previous assumption restricts the domain of possible application of the model. The influence of reflected shock waves is dominant in the case of liner with thin walls, i.e. if the ratio of masses of liner and explosive charge $M/C<1$. In applications of explosive propulsion relevant to weapon system design, the metallic liner mass is principally significantly higher that the mass of explosive charge.

Initial velocity in the case of grazing detonation. It is experimentally proved [8] that the initial velocity of the outer (free) surface of liner, as a result of the action of shock wave, is about two times lower in the case of grazing detonation comparing to the case of axisymmetric detonation. Regarding the relation between the initial velocities of inner and outer surface of the cylinder, it can be concluded that the same ratio is also valid for the initial velocities of inner surface of cylinder:

$$\left(v_i\right)_{\text{side-on}} \approx \frac{1}{2}\left(v_i\right)_{\text{head-on}}\ . \quad (8)$$

The initial velocity of the inner surface of cylinder can be determined in a way similar to the case of axisymmetric detonation. In this case, the pressure generated in the cylindric liner is balanced with the pressure in the rarefaction Taylor wave of detonation products [11], [12]. It can be easily shown [13] that the pressure in rarefaction wave for one-dimensional model is defined by the relation:

$$p = \frac{\rho_0 D^2}{\gamma+1}\left[1 - \frac{\gamma^2-1}{2\gamma}\frac{u}{D}\right]^{\frac{2\gamma}{\gamma-1}}\ . \quad (9)$$

Analogously to the procedure for the case of head on interaction, numerical solution of the equation

$$\rho_m u(c_m + s_m u) = \frac{\rho_0 D^2}{\gamma+1}\left[1 - \frac{\gamma^2-1}{2\gamma}\frac{u}{D}\right]^{\frac{2\gamma}{\gamma-1}} \quad (10)$$

provides the velocity $u=u_0$ that is equal to the initial liner velocity $\left(v_i\right)_{\text{side-on}} = u_0$.

2.2. Acceleration of the cylinder by gas push process

Let us consider the second stage of the cylinder acceleration. The metallic cylinder of known density ρ_m and geometry (Fig. 4) is considered. The cylinder is assumed to be long enough to neglect the end effects, i.e. the axial outflow of detonation products. The simpler case of axisymmetric detonation will be analyzed first. The cylinder motion starts when the entire explosive charge is detonated. It is shown that due to the action of detonation wave, the cylinder receives the initial velocity v_i. One-dimensional model of the cylinder motion is considered, shock wave effects are neglected, and Gurney's postulate of detonation products homogeneity is adopted.

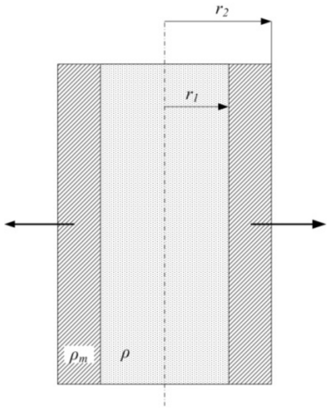


Figure 4. Geometric model of axisymmetric propulsion of metallic cylinder

Mass conservation law. Since the cylinder material is incompressible, the mass conservation law yields:

$$r_2^2 - r_1^2 = r_{20}^2 - r_{10}^2 = w^2 = \text{const.}, \quad r^2 - r_1^2 = r_0^2 - r_{10}^2 \tag{11}$$

where r_1 and r_2 are the radial positions of inner and outer cylinder surface, r_{10} and r_{20} are the corresponding initial cylinder dimensions, while $r \in [r_1, r_2]$ is the Lagrange coordinate of an arbitrary cylinder point, and $r_0 \in [r_{10}, r_{20}]$ is its initial value. The continuity equation can be also written in the form:

$$\frac{\partial v}{\partial r} + \frac{v}{r} = 0\,, \tag{12}$$

where v is the cylinder velocity. Integration of Eq. (13) gives:

$$vr = v_1 r_1 = c\,, \tag{13}$$

where $c=c(t)$ is the function of time t only. It is clear that determination of the position r_1 and the velocity v_1 of the inner cylinder surface enables computation of the position and velocity of any cylinder point using Eqs. (11) and (13).

Application of Lagrange's equation to the motion of the cylinder. Following the idea formulated by Flis [14], the motion of the cylinder due to the rapid expansion of detonation products is modeled by the Lagrange's equation in the form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_1}\right)-\frac{\partial L}{\partial r_1}=Q\,. \tag{14}$$

In the previous equation L is Lagrange's function

$$L=T-U\,, \tag{15}$$

T is the total kinetic energy of the system, U – the total potential of conservative forces, Q – the total non-conservative generalized force, and the position of the inner surface of cylinder r_1 is adopted as the generalized coordinate.

The total kinetic energy of the system consists of the kinetic energy of the cylinder T_M and the kinetic energy of detonation products T_{DP}:

$$T=T_M+T_{DP}\,. \tag{16}$$

The kinetic energy of the cylinder per unit length can be written as:

$$T_M=\int_V \frac{v^2}{2}\mathrm{d}m=\frac{M}{2}v_1^2\frac{r_1^2}{w^2}\ln\left(1+\frac{w^2}{r_1^2}\right), \tag{17}$$

where M is the cylinder mass per unit length.

The kinetic energy of the gaseous detonation products, having in mind the adopted assumption of their homogeneity ($\partial \rho/\partial r$=0), depends on the detonation products velocity profile. The power law for the detonation products is assumed:

$$v_{DP}=v_1\left(\frac{r}{r_1}\right)^{\alpha}, \quad r\in[0, r_1], \tag{18}$$

where $v_1=\dot{r}_1$ is the inner cylinder surface velocity, and α is an exponent. It can be shown [15] that the linear change of the detonation products velocity (α=1), which is also applied in the original Gurney's concept [1], corresponds to the hypothesis of homogenous detonation products. The kinetic energy of detonation products can now be written as:

$$T_{DP}=\frac{C}{2(\alpha+1)}v_1^2\,, \tag{19}$$

where C is the mass of explosive charge per unit length.

The total potential U of conservative forces is equal to the internal energy of detonation products E

$$U=E\,. \tag{20}$$

The potential derivative can be expressed in the form:

$$\frac{\partial U}{\partial r_1}=\frac{\partial E}{\partial V_{DP}}\frac{\partial V_{DP}}{\partial r_1}=-2\pi r_1 p\,, \tag{21}$$

where V_{DP} is the volume occupied by detonation products, and p is their current pressure.

The work of non-conservative forces is in fact the work of the forces that resist cylinder deformation. Normal radial and circular stresses have the dominant role in the cylinder deformation. Different approaches (e.g. [5], [6], and [16]) demonstrated that the distribution of radial stress was approximately linear:

$$\sigma_r(r)=-p\frac{r_2-r}{r_2-r_1}\,. \tag{22}$$

Tresca's hypothesis of the material plastic flow at the maximum shear stress is employed:

$$\sigma_\theta - \sigma_r = \sigma_y . \tag{23}$$

Regarding the fact that the detonation products pressures are extremely high – significantly greater than the yield stress of conventional materials used in explosive propulsion – the viscoplastic model for dynamic yield stress is applied [4]:

$$\sigma_y = \sigma_0 + \mu\dot{\varepsilon} = \sigma_0 - \mu\frac{\partial v}{\partial r} , \tag{24}$$

where σ_0 is the quasi-static flow stress, μ is dynamic viscosity, and $\dot{\varepsilon}$ is strain rate. Using Eqs. (22), (23) and (24), the deformation works of radial and circular stresses W_r and W_θ, and corresponding generalized forces can be calculated. The sum of these forces is the total non-conservative generalized force that can be written in the form:

$$Q = -\frac{\sigma_0 r_1}{2}\ln\left(1+\frac{w^2}{r_1^2}\right) - \frac{\mu v_1}{2}\frac{w^2}{w^2+r_1^2} + (2\pi-1)pr_1\left[\frac{r_1^2+w^2+r_1\sqrt{w^2+r_1^2}}{2w^2}\ln\left(1+\frac{w^2}{r_1^2}\right)-1\right]. \tag{25}$$

Substitution of the kinetic energies, Eqs. (17), (19), the potential, Eqs. (20), (21), and the generalized force (25) in the Lagrange's equation (14) leads to the final differential equation of motion of the inner cylinder surface:

$$\begin{aligned}\left[\frac{M}{C}\ln\left(1+\frac{w^2}{r_1^2}\right)\frac{r_1^2}{w^2}+\frac{1}{\alpha+1}\right]\dot{v}_1 &= \frac{2p}{\rho_0}\frac{r_1}{r_{10}^2}\left[1-\frac{2\pi-1}{2\pi}\left(\frac{r_1^2+w^2+r_1\sqrt{r_1^2+w^2}}{2w^2}\ln\left(1+\frac{w^2}{r_1^2}\right)-1\right)\right]+ \\ &+\frac{M}{C}\left[\frac{v_1}{w^2+r_1^2}\left(r_1v_1-\frac{\mu}{2\pi\rho_m}\right)-\frac{r_1}{w^2}\ln\left(1+\frac{w^2}{r_1^2}\right)\left(v_1^2+\frac{\sigma_0}{2\pi\rho_m}\right)\right]\end{aligned} \tag{26}$$

where M/C is the ratio of the cylinder mass to the explosive charge mass:

$$\frac{M}{C} = \frac{\rho_m w^2}{\rho_0 r_{10}^2} . \tag{27}$$

Detonation products pressure. In order to solve the equation of motion (26), the detonation products pressure $p=p(r_1)$ must be defined. Two approaches are possible: (i) the polytrophic expansion law for detonation products can be used, or (ii) application of an empirically based equation of state for detonation products.

Based on the first approach, the pressure p is determined under the condition that the polytrophic expansion starts from the Chapman-Jouget state [17], [3]. Using the results of the elementary detonation theory [18], the detonation products pressure can be determined as a function of the inner cylinder surface position:

$$p(r_1) = \frac{1}{\gamma+1}\left(\frac{\gamma}{\gamma+1}\right)^{\gamma}\rho_0 D^2\left(\frac{r_{10}}{r_1}\right)^{2\gamma} . \tag{28}$$

The values of parameter γ are usually in the interval [2.7, 3.0], and if experimental results lacks the most common approximation is $\gamma\approx 3$.

The second approach, based on the empirically established equation of state of detonation product, provides the more reliable results. Because of simplicity and data availability, Jones-Wilkins-Lee (JWL) equation of state is frequently used in practice:

$$p(V) = A\exp\left(-R_1 V\right) + B\exp\left(-R_2 V\right) + CV^{-(1+\omega)} \tag{29}$$

where V is the detonation products expansion factor

$$V = \frac{\rho_0}{\rho} = \left(\frac{r_1}{r_{10}}\right)^2, \tag{30}$$

while A, B, C, R_1, R_2 and ω are the experimentally determined parameters. The handbook [19] is comprehensive and reliable source of data for the JWL equation of state of different explosives.

Substituting the pressure (Eq. (28) or (29)) in the equation of the cylinder motion (26), an ordinary differential equation of the second order emerges. This equation can be easily solved by numerical methods. Off course, the appropriate initial conditions are applied:

$$\left(r_1\right)_{t=0} = r_{10}, \quad \left(\dot{r}_1\right)_{t=0} = v_i\,. \tag{31}$$

Second stage in the case of grazing detonation. In this case, the modeling of the cylinder motion implies determination of the cylinder velocity as a vector: the velocity magnitude should be calculated, as well as the direction of velocity vector. Regarding the velocity magnitude, basically the presented model for axisymmetric detonation can be used. If the lateral outflow is negligible (which is true for a slender cylinder), an elementary cylinder part is thought to be propelled by the corresponding elemental detonation products (Fig. 5). Therefore, in the second, gas-dynamic phase of the cylinder motion, it is insignificant whether this elementary mass of gaseous detonation products emanates from an axisymmetric or grazing detonation [15]. Hence, the proposed model of gas-push process based on the Lagrange's equation will be also applied in this case.

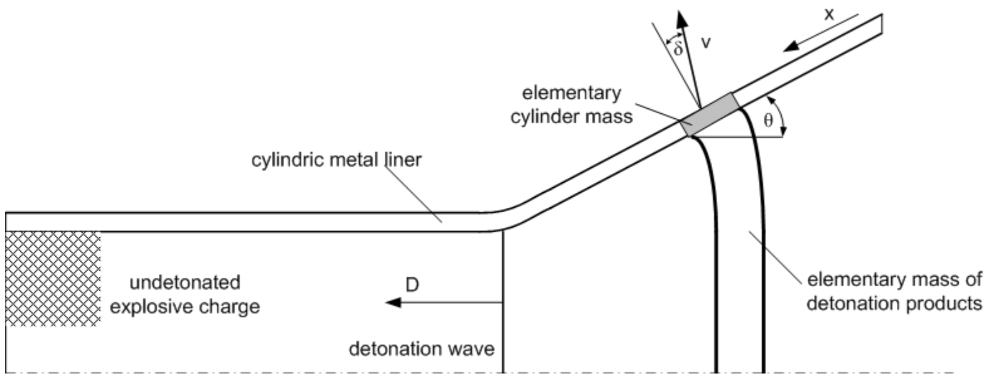


Figure 5. Grazing detonation: the elementary cylinder mass propelled by the corresponding elementary mass of the gaseous detonation products

The cylinder acceleration process by the grazing detonation wave can be considered quasi-steady – the velocity of any elementary cylinder part has the same time history. If the position of detonation wave is defined by the coordinate x, the cylinder velocity in the case of grazing detonation can be expressed as:

$$v_s(t,x) = \begin{cases} v\left(t - \dfrac{x}{D}\right), & x \le Dt \\ 0, & x > Dt \end{cases} \tag{32}$$

The velocity direction is defined by the angle δ between the velocity vector and the line normal to the cylinder axis. By the classical Taylor model [20] of the cylinder motion by grazing detonation this angle is determined by:

$$\sin\delta = \frac{V}{2D}\,. \tag{33}$$

The analysis of liner kinematics [21], [22] shows that the angle of liner rotation θ, and the angle of velocity vector δ can be numerically determined from the system of equations:

$$\dot{\delta} = \frac{\dot{v}}{v}\,\mathrm{tg}(\theta - \delta), \qquad \dot{\theta} = -\frac{\partial v}{\partial x}\cos(\theta - \delta)\,. \tag{34}$$

In the case of small rotation angles, the previous system of equation can be simplified to the Taylor equation (33) and θ=2δ.

3. Comparison with experimental results and discussion

In order to investigate validity of the suggested model computational results are compared with the available experimental data for axisymmetric, as well as for grazing detonation.

3.1. Comparison with experimental data for axisymmetric detonation

The results of presented one-dimensional model of the cylinder motion under the action of gaseous products of axisymmetric detonation are analyzed through comparison with the experimental data from [23]. Motion of steel cylinder after electric initiation by the bundle of copper wires is observed by high-speed digital camera. The characteristics of the explosive and cylinder used, which are at the same time the input data for the model, are shown in Table 1.

Table 1. Characteristics of explosive and metallic cylinder

Explosive		**Cylinder** (r_{10}=14 mm, r_{20}=17 mm)	
pentrit (PETN)		stainless steel JIS SUS 304	
density ρ_0 (kg/m^3)	950	density ρ_m (kg/m^3)	7850
detonation velocity D (m/s)	5420	yield stress σ_0 (MPa)	340
polytrophic constant γ_{CJ}	2.57	dynamic viscosity μ (kPas)	3.0

Comparison of the experimentally determined radial displacement of the outer cylinder surface and the model results is shown in Fig. 6. Cylinder acceleration process is obviously impulsive and initial acceleration is of the order of 10^8 m/s^2. Good correspondence between experimental and model results is noted, until the moment of fragmentation onset (~15 μs) accompanied with longitudinal cracks on the outer cylinder surface. From that moment, the cylinder loses structural integrity and the analyzed model can not be applied. Fragmentation process completes after $t_f \approx 18$μs when a massive leakage of detonation products is observed. The outer cylinder radius in the moment of fragmentation completion is r_{2f}=30 mm, and the corresponding cylinder strain is ε_f=0.76.

The time history of cylinder velocity is presented in Fig. 7. As can be seen from the diagram, the outer cylinder surface velocity calculated by the Gurney formula is expectedly higher than the measured fragmentation velocity, whereas the model result is slightly lower than the experimental value.

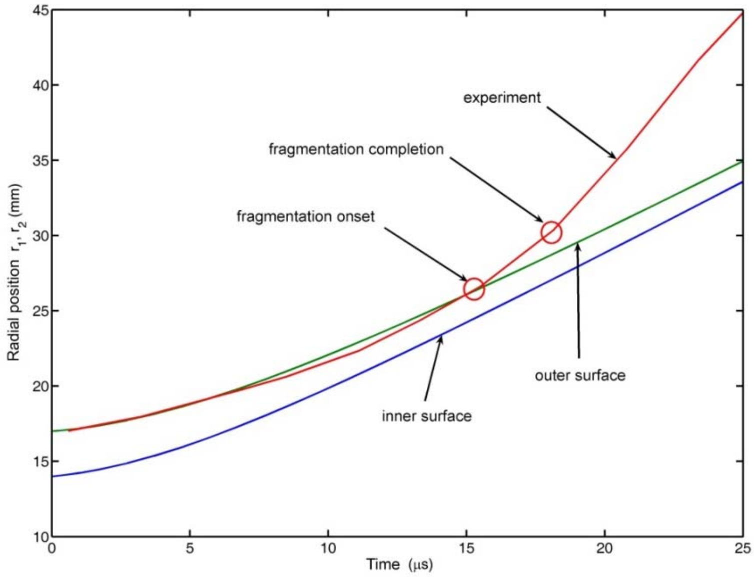


Figure 6. Radial position of inner and outer cylinder surface under the action of detonation products – comparison of the experimental data [23] and the model prediction

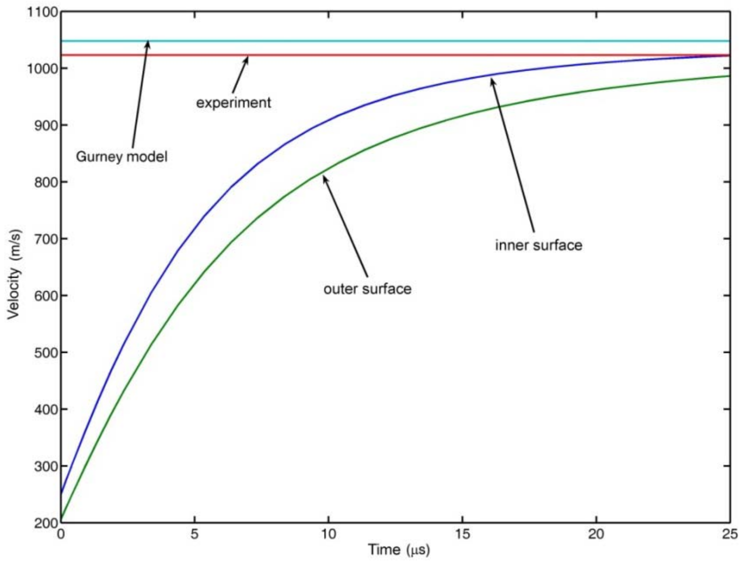


Figure 7. Time history of the expanding cylinder velocity – Gurney model, experimental data [23] and model results

3.2. Comparison with experimental data for grazing detonation

There are numerous studies related to the experimental investigation of the metallic cylinder expansion by grazing detonation. The most part of these studies deal with modeling of copper cylinder expansion under the action of detonation products of different explosive compositions („cylinder test"). Measurement of the cylinder displacement, i.e. identification of expansion dynamics, enables determination of equation of state of detonation products, the Gurney energy of explosive used, etc.

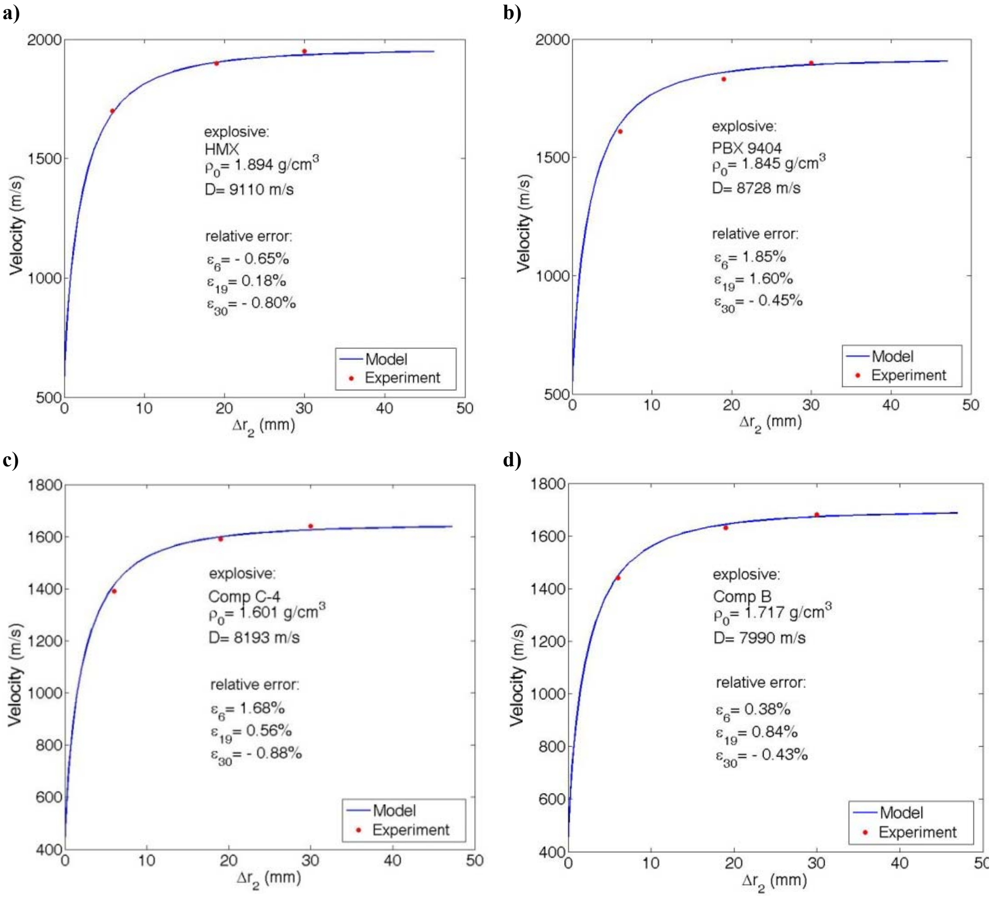
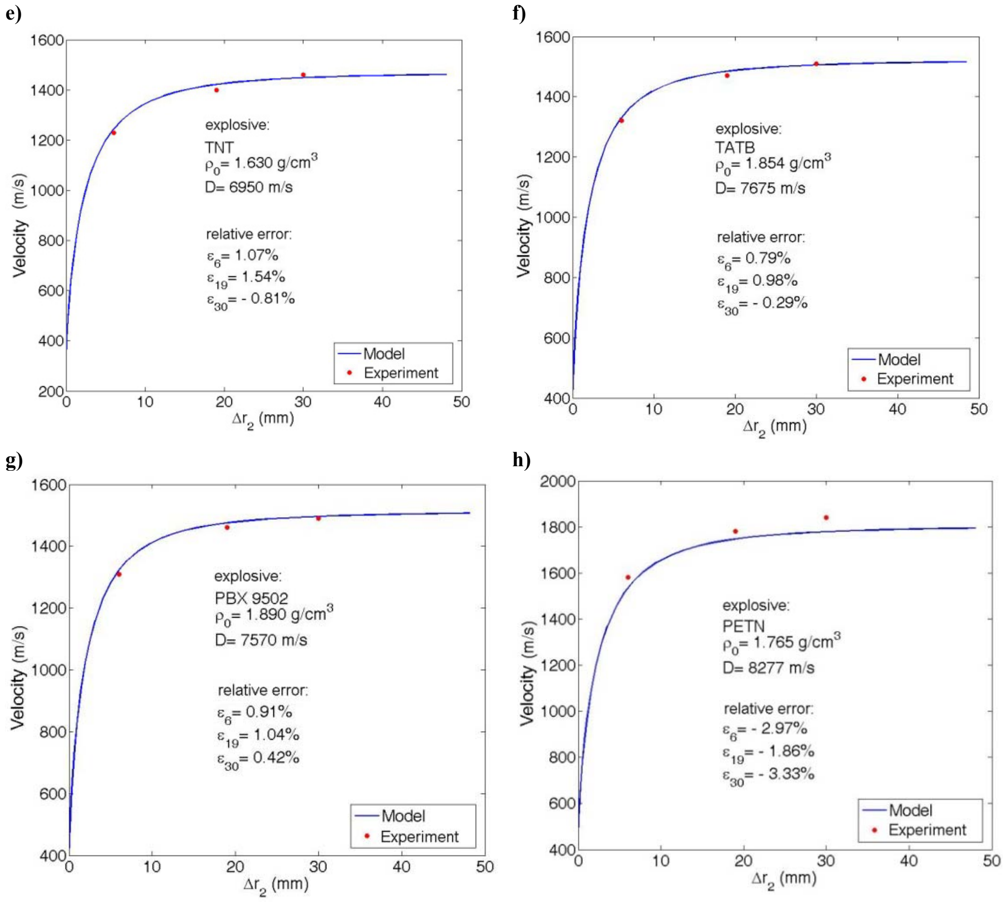


Figure 8. Comparison of the cylinder test experimental results [24] with the prediction of theoretical model. Change of the outer cylinder surface velocity as a function of its displacement is shown. The main properties of explosive compositions (density ρ_0 and detonation velocity D) are indicated along with the relative errors of theoretical results at displacements of 6, 19 and 30 mm. Explosive compositions: a) octogen (HMX), b) PBX-9404 (94/3/3 HMX/nitrocellulose/chloretyl phosphate), c) composition C-4 (91/9 RDX/organic binder), d) composition B (64/36 RDX/TNT)

The study [24] is a comprehensive collection of experimental results related to the mentioned „cylinder test". The inner radius of copper cylinder used is r_{10}=12.7 mm, the cylinder wall thickness is δ=2.54 mm, and the cylinder length is L=12d_{10}=305 mm. The following handbook values of physical and mechanical properties of copper are used in the computer program based on the suggested model: density ρ=8945 kg/m^3, quasi-static yield stress σ_0=75.5 MPa, dynamic viscosity μ=10^3 Pas, and parameters in the equation of state – c_m=3940 m/s and s_m=1.49. Measurement of the cylinder motion is performed by a high-speed camera in the streak mode. The cylinder expansion is described by the measured velocities in three characteristic positions, corresponding to the detonation products expansions of V=2, 7 and 11. These expansion factors coincide with the outer cylinder surface displacements of 6 mm, 19 mm, and 30 mm, respectively. The results of these experiments are compared to the model predictions in Fig. 8.



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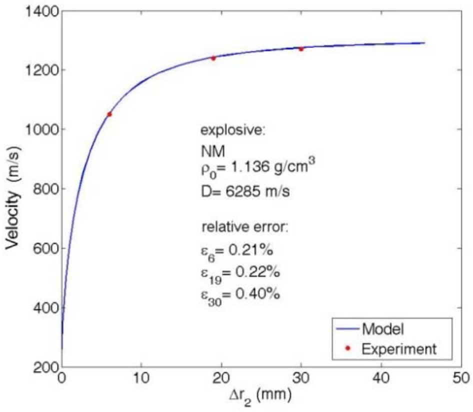

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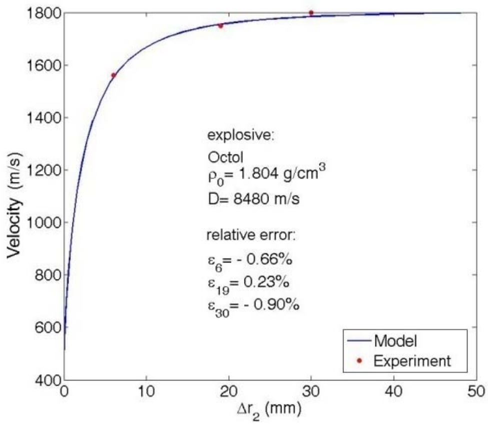

Figure 8. (continued) Comparison of the cylinder test experimental results [24] with the prediction of theoretical model. Explosive compositions: e) TNT, f) TATB, g) PBX-9502 (95/5 TATB/Kel-F800), h) pentrite, i) NM – nitromethane, j) octol (75/25 HMX/TNT).

The measured values of the radial velocity at three characteristic positions and theoretically determined velocity as a function of the outer cylinder surface displacement Δr_2 are shown for ten widely used explosive compositions. The explosive type and its experimentally determined properties – density ρ_0 and detonation speed D are indicated on each diagram. Having in mind that for the most explosive compositions with defined densities the parameters of JWL equation of state are not available, the version of the analytical model with pressure determined by polytrophic expansion is applied. The polytrophic constant γ is determined according to the empirical equation [25]:

$$\gamma = 1.8 + 0.6\rho_0 , \qquad (35)$$

where the explosive density is taken in g/cm^3. The relative errors of theoretically determined velocities compared to the measured values are also specified on the diagrams. As can be seen from the diagrams, the predicted velocities are in excellent accordance with experimental data. The relative error of velocity does not exceed 3% and is usually lower than 1%.

Comprehensive experimental-numerical study of the metallic cylinder acceleration by the explosive detonation is presented in [26] and [27]. Copper cylinder expansion by the grazing detonation of explosive composition PBX-9501 (95/5–HMX/binder) was thoroughly investigated. The cylinder motion was diagnosed by high-speed camera and Fabry-Perot interferometry. Comparison of experimentally determined and theoretically predicted cylinder velocities is shown in Fig. 9. Thickness of the cylinder is 5.08 mm (r_{10}=51.03mm, r_{20}=56.11 mm), while the metal-explosive ratio is M/C=1.02. Theoretical results calculated on the basis of polytrophic expansion (γ=const.) and the JWL equation of state are mutually very close. These results are in good accordance with experimental data, except in the initial stage of motion dominated by shock waves, which is abstracted by introduction of the initial velocity.

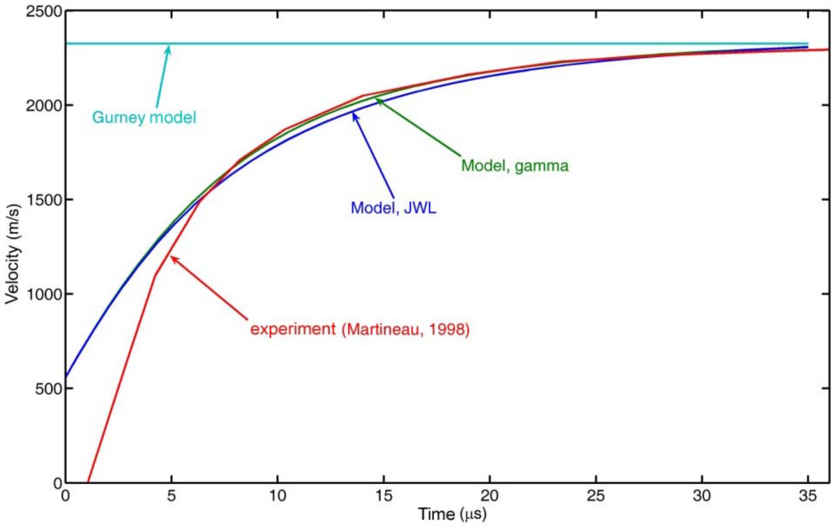

Figure 9. Copper cylinder velocity as a function of time: comparison of experimental results [26] with theoretical predictions for polytrophic expansion of detonation products and JWL equation of state

Experimentally and theoretically determined profiles of the outer cylinder surface 49.65 μs after initiation are presented in Fig. 10. Calculated results fits experimental data very well, except at the cylinder end, which is the effect of detonation product outflow that is ignored in the suggested model.

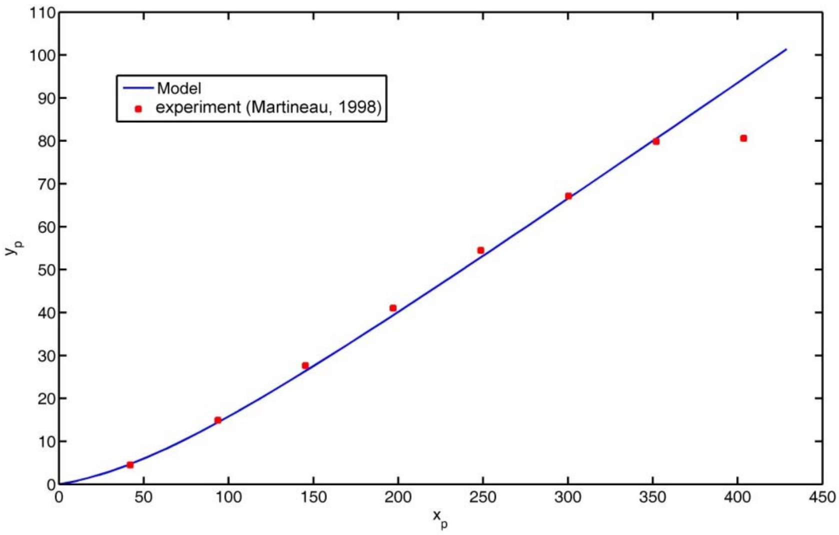

Figure 10. Outer cylinder surface profile: comparison of experimental data [26] with the results of suggested model

4. Conclusion

Cylindrical liner motion under the action of expanding detonation products has been considered in the paper. A new cylinder acceleration model is suggested for the cases of axisymmetric and grazing detonation of explosive charge. The model implies two-stage character of the liner motion: (i) early interaction of the detonation wave and metallic liner results in impart of the initial velocity to the cylindric liner, and (ii) the gas-push stage of rapid cylinder motion, similarly to the Gurney's model. The initial liner velocity is determined by impedance matching technique. The cylinder motion in the second stage is modeled by the Lagrange's equation, taking into account liner deformation, as well as detonation products pressure according to the polytrophic law or JWL equation of state.
Comparison with comprehensive database of available experimental results indicates the model predictions are in good accordance with experimental data in the domain that is of practical importance for explosive propulsion.

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