EMW TRANSFORMATION IN WEAKLY NONLINEAR PLASMA WHEN TRANSVERSE DC MAGNETIC FIELD IS SUDDENLY SWITCHED OFF

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1. INTRODUCTION

Linear transformation of plane, monochromatic electromagnetic wave (EMW) in plasma when external static magnetic field is suddenly switched off is considered, in radio approximation, in [1] (longitudinal propagation) and [2] (transverse propagation). The same problem in weakly nonlinear plasma (longitudinal propagation) is considered in [3].

In linear theory it was shown, in both cases of propagation, that when angular frequency of source wave had specific values, most of the energy of the source wave was converted to the energy of new created wiggler magnetic field in isotropic plasma medium. Wiggler magnetic field has application for generation of coherent radiation in Free Electron Laser.

In the case of transverse propagation [2] it was shown that EMW of an elliptic polarization, with angular frequency ω_0, was transformed in transverse wave modes with angular frequency ω_e and ω_P, where

$$\omega_E = \sqrt{\omega_P^2 + \eta_P^2 \omega_0^2}; \quad \omega_P = \sqrt{q^2 N_0 / \varepsilon_0 m}; \quad \eta_P = \sqrt{1 - \frac{\omega_P^2}{\omega_0^2 (1 + \omega_B^2 / (\omega_P^2 - \omega_0^2))}}; \quad \omega_B = \frac{q B_0}{m}, \quad (1)$$

where N_0 is electron plasma density, q and m are charge and mass of an electron, ε_0 is permitivity of free space and B_0 is magnitude of external static magnetic field. In addition to these wave modes there are purely space-varying components of magnetic and velocity fields with wave number $k_P = \eta_P k_0$ (k_0 is the wave number in free space). The existence of an electric field component in direction of propagation couses perturbation of the electron plasma density.

Transformation of the source wave in weakly nonlinear plasma was analyzed by the perturbation theory of the second order, in radio approximation.

2. PROBLEM FORMULATION AND CLOSED FORM SOLUTION

For t<0 elliptic polarized source electromagnetic (EM) plane wave, with angular frequency ω_0 and wave number $\vec{k}_P = k_P \cdot \vec{z}$, is propagating in magnetized plasma ($\vec{B}_0 = B_0 \cdot \vec{y}$) in positive z direction. At t=0 external magnetic field is suddenly switched off. Plasma becomes isotropic medium and supports, in linear theory, wave modes analyzed in [2].

EM fields, electron velocity and electron plasma density fields in magnetized plasma have to satisfy two Maxwell's equations, equation of motion for the electronic fluid and continuity equation:

$$\frac{\partial n_0}{\partial t} + \nabla(n_0 \vec{u}_0) = 0, \quad (2) \quad ; \quad rot\,\vec{h}_0 - \varepsilon_0 \frac{\partial \vec{e}_0}{\partial t} + n_0 q \vec{u}_0 = 0, \quad (4)$$

$$rot\,\vec{e}_0 + \mu_0 \frac{\partial \vec{h}_0}{\partial t} = 0, \quad (3) \quad ; \quad \frac{d(n_0 m \vec{u}_0)}{dt} = -n_0 q\left[\vec{e}_0 + \mu_0 \vec{u}_0 \times \vec{h}_0 + \vec{u}_0 \times \vec{B}_0\right] \quad (5)$$

In weakly nonlinear plasma EM, velocity and electron density fields could be expressed as

$$\vec{e}_0 = \vec{e}_{01} + \vec{e}_{02} + \ldots;\ \vec{h}_0 = \vec{h}_{01} + \vec{h}_{02} + \ldots;\ \vec{u}_0 = \vec{u}_{01} + \vec{u}_{02} + \ldots;\ n_0 = N_0 + n_{01} + n_{02} + \ldots. \quad (6)$$

Substituting the values for EM, velocity and electron density fields (6) in Eqs. (2)-(5) one obtains two separate linear systems for the fields ($\vec{e}_0 = (e_{0x1} + e_{0x2})\vec{x} + (e_{0z1} + e_{0z2})\vec{z}$; $\vec{h}_0 = (h_{0y1} + h_{0y2})\vec{y}$; $\vec{u}_0 = (u_{0x1} + u_{0x2})\vec{x} + (u_{0z1} + u_{0z2})\vec{z}$; $n_0 = N_0 + n_{01} + n_{02}$) in the following form:

$$\frac{\partial n_{01}}{\partial t} + N_0 \nabla \vec{u}_{01} = 0, \quad (2a) \quad ; \quad \frac{\partial n_{02}}{\partial t} + N_0 \nabla \vec{u}_{02} + n_{01} \nabla \vec{u}_{01} = 0, \quad (2b)$$

$$rot\,\vec{e}_{01} + \mu_0 \frac{\partial \vec{h}_{01}}{\partial t} = 0, \quad (3a) \quad ; \quad rot\,\vec{e}_{02} + \mu_0 \frac{\partial \vec{h}_{02}}{\partial t} = 0, \quad (3b)$$

$$rot\,\vec{h}_{01} - \varepsilon_0 \frac{\partial \vec{e}_{01}}{\partial t} + N_0 q \vec{u}_{01} = 0, \quad (4a) \quad ; \quad rot\,\vec{h}_{02} - \varepsilon_0 \frac{\partial \vec{e}_{02}}{\partial t} + N_0 q \vec{u}_{02} = -n_{01} q \vec{u}_{01}, \quad (4b)$$

$$\frac{\partial \vec{v}_{01}}{\partial t} + \frac{q}{m}\vec{e}_{01} + \frac{q}{m}\vec{u}_{01} \times \vec{B}_0 = 0, \quad (5a);$$

$$\frac{\partial \vec{u}_{02}}{\partial t} + \frac{q}{m}\vec{e}_{02} + \frac{q}{m}\vec{u}_{02} \times \vec{B}_0 = -(\vec{u}_{01}\nabla)\vec{u}_{01} - \frac{q}{m n_0} n_{01}\vec{e}_{01} - \frac{q}{m}\vec{u}_{01} \times (\mu_0 \vec{h}_{01} + \frac{n_{01}}{N_0}\vec{B}_0) - \frac{\vec{u}_{01}}{N_0}\frac{\partial n_{01}}{\partial t} - \frac{n_{01}}{N_0}\frac{\partial \vec{u}_{01}}{\partial t}. \quad (5b)$$

The solution for the first order fields perturbation are given in [2].

For solving the system of partial differential equations (2b)-(5b) with corresponding initial conditions ($\vec{e}_{02}(z, t = -\infty) = \vec{h}_{02}(z, t = -\infty) = ; \vec{u}_{02}(z, t = -\infty) = 0$) we have applied Laplace transform in time ($L\{g(z,t)\} = \int_0^{+\infty} g(z,t)\exp(-st)dt = G(z,s)$) and as the plasma is unbounded Fourier transform in space ($F\{G(z,s)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} G(z,s)\exp(-jkz)dz = G(k,s)$).

After solving the linear system of algebraic equations in k and s domains and implementing inverse Laplace and Fourier transform, EM field components of second order in magnetized plasma could be obtained in the following form:

$$e_{02x}(z,t) = E_{02x}^{0} + \sum_{i=1}^{2}\left[E_{02x}^{i}\cos(\omega_{i\alpha}t) + E_{02x}^{2i+1}\cos(\omega_{i\beta}t - 2k_P z) + E_{02x}^{2i+2}\cos(\omega_{i\beta}t + 2k_P z)\right] + E_{02x}^{7}\cos(2\omega_0 t - 2k_P z), \tag{7}$$

$$e_{02z}(z,t) = \sum_{i=1}^{2}\left[E_{02z}^{i}\sin(\omega_{i\alpha}t) + E_{02z}^{2i+1}\sin(\omega_{i\beta}t - 2k_P z) + E_{02z}^{2i+2}\sin(\omega_{i\beta}t + 2k_P z)\right] + E_{02z}^{7}\sin(2\omega_0 t - 2k_P z), \tag{7a}$$

$$h_{02y}(z,t) = -H_{02y}^{0}\cos(2k_P z) + \sum_{i=1}^{2}\left[H_{02y}^{2i-1}\cos(\omega_{i\beta}t - 2k_P z) - H_{02y}^{2i}\cos(\omega_{i\beta}t + 2k_P z)\right] + H_{02y}^{5}\cos(2\omega_0 t - 2k_P z), \tag{8}$$

where

$$\omega_{1,2} = \sqrt{a(k^2) \pm \sqrt{a^2(k^2) - b(k^2)}}\,;\, 2a(k^2) = 2\omega_P^2 + \omega_B^2 + k^2c^2;\, b(k^2) = \omega_P^4 + (\omega_P^2 + \omega_B^2)k^2c^2;$$

$$\omega_{1,2\alpha} = \omega_{1,2}(k=0);\, \omega_{1,2\beta} = \omega_{1,2}(k = 2k_P). \tag{9}$$

Amplitudes of excited EM fields are determined in the closed form.

At t=0 external magnetic field is suddenly switched off. Applying the same mechanism as in magnetized plasma (equations are the same, but $B_0 = 0$) and taking into account initial conditions (the continuity of the fields at t=0) EM field component in isotropic plasma medium could be obtained in the following form:

$$e_{2x}(z,t) = -E_{2x}^{0}\sin(\omega_P t)\cdot t + \sum_{i=1}^{3}\left[E_{2x}^{i}\cos(\varphi_i t) + E_{2x}^{2i+2}\cos(\varphi_i t - 2k_P z) + E_{2x}^{2i+3}\cos(\varphi_i t + 2k_P z)\right] + E_{2x}^{10}\cos(\omega_{\beta i}t - 2k_P z) + E_{2x}^{11}\cos(\omega_{\beta i}t + 2k_P z), \tag{10}$$

$$e_{2z}(z,t) = E_{2z}^{0}\sin(2k_P z) + \sum_{i=1}^{4}\left[E_{2z}^{i}\cos(\xi_i t) + E_{2z}^{2i+3}\cos(\xi_i t - 2k_P z) + E_{2z}^{2i+4}\cos(\xi_i t + 2k_P z)\right], \tag{10a}$$

$$h_{2y}(z,t) = -H_{2y}^{0}\cos(2k_P z) + \sum_{i=1}^{3}\left[H_{2y}^{2i-1}\cos(\varphi_i t - 2k_P z) - H_{2y}^{2i+1}\cos(\varphi_i t + 2k_P z)\right] + H_{2y}^{7}\cos(\omega_{\beta i}t - 2k_P z) - H_{2y}^{8}\cos(\omega_{\beta i}t + 2k_P z), \tag{11}$$

where

$$\varphi_1 = \omega_P;\, \varphi_2 = \omega_E - \omega_P;\, \varphi_3 = \omega_E + \omega_P;\, \omega_\beta = \sqrt{\omega_P^2 + 4k_P^2c^2};\, \xi_1 = \omega_P;\, \xi_2 = 2\omega_P;\, \xi_3 = \omega_E;\, \xi_4 = 2\omega_E$$

Amplitudes of those EM fields are determined in the closed form, but due to very complex structure they are not presented in explicit form in this paper. After very lengthy algebraic manipulations it was shown that the amplitude of pure space-varying longitudinal electric field, obtained in nonlinear theory, could be presented in the form

$$E_{2z}^{0} = \frac{\Omega_0}{\Omega_e^2}\left(1 - \frac{\Omega_0^2}{\Omega_P^2}\right)\cdot\left[3 + 2\Omega_e^2\left(\eta_p^2 - 1\right)\right]\cdot E_2;\qquad \Omega_{0,e} = \omega_{0,e}/\omega_P;\quad E_2 = qE_0^2/4mc\,\omega_P. \tag{12}$$

First order fields perturbation are given in [2].

Normalized amplitude of pure space-varying longitudinal electric field, obtained in nonliner theory, versus angular frequency of the source wave is presented in Fig. 1.

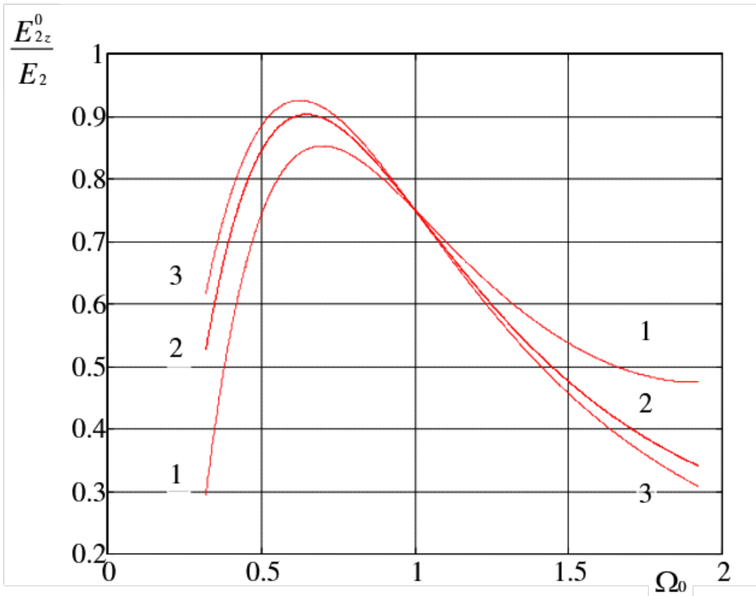

Fig. 1. Normalized amplitude of space-varying longitudinal electric component with wave number $2k_p$ versus Ω_0. ($\Omega_B=\sqrt{8}$ (1), 4 (2), 5 (3)).

3. CONCLUSION

The problem of EMW transformation in weakly nonlinear plasma when transverse DC magnetic field is suddenly switched off is solved in the closed form. When angular frequency of the source wave is near lower cut off frequency, efficiency of the excitation of new created space-varying longitudinal electric field could be controlled by either changing the angular frequency of source wave ω_0 or changing magnitude of external static magnetic field B_0.

LITERATURE

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