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TWO-STAGE MODEL OF EXPLOSIVE PROPULSION OF METAL CYLINDER

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Abstract: *The paper considers acceleration of cylindrical metal liner by expanding detonation products. A new analytical model of liner motion is developed for the cases of axisymmetric (head-on) and grazing (side-on) detonation of the explosive charge. Suggested model relies on the two-stage regime of cylinder motion: (i) the first stage is the consequence of detonation wave-metallic liner interaction; as the result initial velocity is imparted to the liner, (ii) the second stage is gas-dynamic push of the liner governed by detonation product expansion, similarly to the Gurney approach. Results of the analytical model are validated by comparison with available experimental data.*

Key words: *explosive propulsion, detonation, shock waves, plasticity, analytical model*

1. INTRODUCTION

Acceleration of a cylindrical metal liner under the action of impulsive internal loading brought about by explosive detonation is of great importance primarily in the analysis of high-explosive warhead mechanisms. Detonation of an explosive charge generates gaseous detonation products of extremely high pressure (~ 20 ÷40 GPa) causing rapid expansion of a cylindrical metal liner. The goal of the present research is modeling of the cylinder motion until the onset of fragmentation process.

Gurney [1] formulated the classical model based on energy balance, analyzed in detail by Kennedy [2]. This model is still in wide use as a method for calculation of the final velocity of metallic liner, i.e. the initial velocity of generated fragments. However, Gurney's model does not take into account cylinder deformation and fails to describe the evolution of the cylinder motion. Moreover, the model has certain limitations and requires experimental determination of the Gurney energy [3].

Numerous physically based models are suggested that characterize acceleration, deformation and fragmentation of metallic cylinder due to the action of detonation products [4], [5], [6]. These models employ different concepts of material behavior and detonation product expansion. A new comprehensive analytical model of cylinder expansion under the action of detonation products will be briefly presented.

2. CYLINDER ACCELERATION MODEL

The following assumptions will be used for modeling of the motion of metallic liner caused by explosive detonation: (i) detonation wave is one-dimensional and in steady-state, (ii) explosive material is instantaneously transformed into detonation products, and (iii) reverberations of the shock wave in the tube are neglected.

Two stages of the cylinder acceleration process will be considered: (i) the first stage considers the interaction of the detonation wave and the liner; the result is almost immediate impart of the initial velocity to the liner, (ii) the second stage is related to the cylinder motion under the pressure of expanding detonation products, analogously to the Gurney model.

In addition, two types of detonation will also be analyzed: (i) axisymmetric detonation of cylindrical explosive charge that provides simultaneous onset of motion of the entire cylinder after detonation of complete explosive charge; cylindrical detonation wave is formed and head-on interaction occurs between the detonation wave and the cylinder wall (Fig. 1a), (ii) in the case of grazing (side-on) detonation, only the part of the cylinder traversed by the detonation wave is set into motion, while the detonation wave is orthogonal to the cylinder axis (Fig.1b).

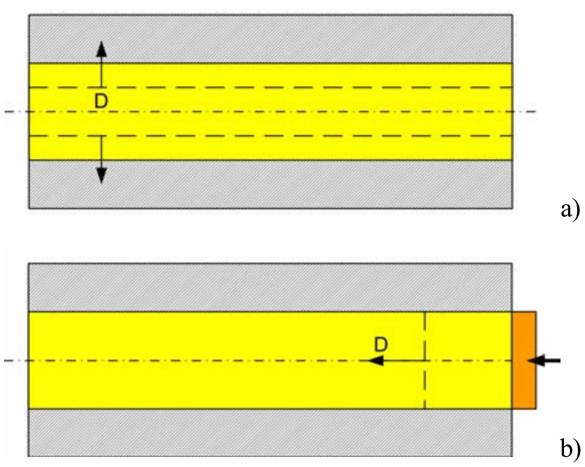


Figure 1. a) Axisymmetric detonation of explosive charge (head-on interaction of detonation wave and liner), b) grazing detonation (tangent, side-on, interaction of detonation wave and liner)

2.1. Initial velocity of the cylinder

An analysis of the available experimental data and the results of numerical simulations (e.g. [7]) shows that cylinder has extremely high acceleration in the initial stage of motion, i.e. it reaches high velocity in a very short time interval. This fact, as well as notable oscillatory character of liner velocity, indicates important effect of shock waves formed in the cylinder by the impulsive action of the detonation wave. Backofen and Weickert [8], [9] analyzed numerous experimental data and introduced mentioned concept of two-stage acceleration of liner propelled by detonation products. Moreover, they suggested the empirical formula for calculation of the initial liner velocity v_i.

An analytical approach to the problem of the interaction of the detonation wave and metallic liner, based on impedance matching technique [10] will be presented here.

Axisymmetric detonation (head-on interaction). Normal (head-on) interaction of the plane detonation wave and metallic liner is depicted in Fig.2.

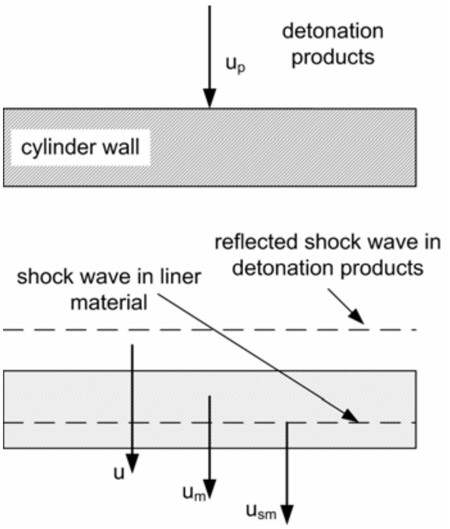


Figure 2. Normal (head-on) interaction of detonation wave and solid obstacle; characteristic velocities of the process are indicated

When the detonation wave interacts with metallic liner, it reflects back, and the Hugoniot adiabat of the reflected wave in velocity-pressure (*u*-*p*) coordinate system has the form [10]:

$$p = \frac{\gamma+1}{2}\rho_0(2u_p - u)^2 + (\gamma-1)\rho_0 Q\,, \qquad (1)$$

where ρ_0 is explosive density, γ is the polytropic coefficient of detonation products, u_p is the material velocity of detonation products at the moment of encounter with the obstacle, and Q is the detonation heat. The shock adiabat of cylinder material is defined by the relation:

$$p_m = \rho_m u_m (c_m + s_m u_m)\,, \qquad (2)$$

where u_m and p_m are velocity and pressure in the part of the cylinder encompassed by shock wave, while c_m and s_m are the equation of state parameters for the considered material.

The continuity condition between two considered media (gaseous detonation products and cylindrical metal liner) implies that the velocities and pressures in the shock wave zone should be equal:

$$u = u_m, \quad p = p_m\,. \qquad (3)$$

Equating the right hand sides of Eqs. (1) and (2), and using condition (3), the quadratic equation emerges that enable determination of the unknown liner velocity u_m. If we introduce well-known relations for the velocity of detonation products and the detonation heat:

$$u_p = \frac{D}{\gamma+1}, \quad Q = \frac{D^2}{2(\gamma^2-1)} \qquad (4)$$

the previous quadratic equation can be easily solved. The obtained velocity u_m is at the same time the initial velocity of the cylinder generated by the effect of detonation wave

$$(v_i)_{\text{head-on}} = u_m\,. \qquad (5)$$

In order to simplify the analytical treatment of the problem, we will assume the effect of shock waves is dominant only at the onset of cylinder motion, i.e. the subsequent oscillatory motion produced by reverberations of shock waves in the cylinder can be neglected comparing to the motion under the action of rapidly expanding detonation products.

It should be emphasized that previous assumption restricts the domain of possible application of the model. The influence of reflected shock waves is dominant in the case of liner with thin walls, i.e. for the ratio of masses of liner and explosive charge M/C<1. In applications of explosive propulsion relevant to weapon system design, the metallic liner mass is generally significantly higher that the mass of explosive charge.

Grazing (side-on) detonation. It is experimentally proved [9] that the initial velocity of the outer (free) surface of liner, as a result of the action of shock wave, is about two times lower in the case of grazing detonation comparing to the case of axisymmetric detonation. Regarding the relation between the initial velocities of inner and outer surface of the cylinder, it can be concluded that the same ratio is also valid for the initial velocities of inner surface of cylinder:

$$\left(v_{\mathrm{i}}\right)_{\text{side-on}} \approx \frac{1}{2}\left(v_{\mathrm{i}}\right)_{\text{head-on}} . \tag{6}$$

The initial velocity of the inner surface of cylinder can be determined in a way similar to the case of axisymmetric detonation. In this case, the pressure generated in the cylindrical liner is balanced with the pressure in the rarefaction Taylor wave of detonation products [11], [12]. It can be easily shown [13] that the pressure in rarefaction wave for one-dimensional model is defined by the relation:

$$p = \frac{\rho_0 D^2}{\gamma + 1}\left[1 - \frac{\gamma^2 - 1}{2\gamma}\frac{u}{D}\right]^{\frac{2\gamma}{\gamma-1}} . \tag{7}$$

Analogously to the procedure in the case of head-on interaction, numerical solution of the equation

$$\rho_{\mathrm{m}} u (c_{\mathrm{m}} + s_{\mathrm{m}} u) = \frac{\rho_0 D^2}{\gamma + 1}\left[1 - \frac{\gamma^2 - 1}{2\gamma}\frac{u}{D}\right]^{\frac{2\gamma}{\gamma-1}} , \tag{8}$$

provides the velocity $u=u_0$ that is equal to the initial liner velocity $\left(v_{\mathrm{i}}\right)_{\text{side-on}} = u_0$.

2.2. Acceleration of the cylinder by gas push process

In the second stage of the cylinder acceleration, the metallic cylinder of known density ρ_{m} and geometry is considered. The cylinder is assumed to be long enough to neglect the end effects, i.e. the axial outflow of detonation products.

Axisymmetric detonation case. The cylinder motion starts when the entire explosive charge is detonated. It is shown that due to the action of detonation wave, the cylinder receives the initial velocity v_{i}. One-dimensional model of the cylinder motion is considered, shock wave effects are now neglected, and Gurney's postulate of detonation products homogeneity is adopted.

Mass conservation law. Since the cylinder material is incompressible in the absence of shock waves, the mass conservation law yields:

$$r_2^2 - r_1^2 = r_{20}^2 - r_{10}^2 = w^2 = \text{const.}, \quad r^2 - r_1^2 = r_0^2 - r_{10}^2 \tag{9}$$

where r_1 and r_2 are the radial positions of inner and outer cylinder surface, r_{10} and r_{20} are the corresponding initial cylinder dimensions, while $r \in [r_1, r_2]$ is the Lagrange coordinate of an arbitrary cylinder point, and $r_0 \in [r_{10}, r_{20}]$ is its initial value. The continuity equation can be also written in the form:

$$v r = v_1 r_1 = c , \tag{10}$$

where v is the cylinder velocity and $c=c(t)$ is the function of time t only. It is clear that determination of the position r_1 and the velocity v_1 of the inner cylinder surface enables computation of the position and velocity of any cylinder point using Eqs. (9) and (10).

Application of Lagrange's equation to the motion of the cylinder. Following the idea formulated by Flis [14], the motion of the cylinder due to the rapid expansion of detonation products is modeled by the Lagrange's equation in the form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_1}\right) - \frac{\partial L}{\partial r_1} = Q . \tag{11}$$

In the previous equation L is the Lagrange's function

$$L = T - U , \tag{12}$$

where T is the total kinetic energy of the system, U – the total potential of conservative forces, Q – the total non-conservative generalized force, and the position of the inner surface of cylinder r_1 is adopted as the generalized coordinate.

The total kinetic energy of the system consists of the kinetic energy of the cylinder T_{M} and the kinetic energy of detonation products T_{DP}:

$$T = T_{\mathrm{M}} + T_{\mathrm{DP}} . \tag{13}$$

The kinetic energy of the cylinder per unit length can be written as [15]:

$$T_{\mathrm{M}} = \int_V \frac{v^2}{2}\mathrm{d}m = \frac{M}{2} v_1^2 \frac{r_1^2}{w^2} \ln\left(1 + \frac{w^2}{r_1^2}\right) , \tag{14}$$

where M is the cylinder mass per unit length.

The kinetic energy of the gaseous detonation products, having in mind the adopted assumption of their homogeneity ($\partial\rho/\partial r=0$), depends on the detonation products velocity profile. It can be shown [16] that the linear change of the detonation products velocity, which is also applied in the original Gurney's concept [1], corresponds to the hypothesis of homogenous detonation products. The kinetic energy of detonation products can now be written as:

$$T_{DP} = \frac{1}{4} C v_1^2 , \tag{15}$$

where C is the mass of explosive charge per unit length. The total potential U of conservative forces is equal to the internal energy of detonation products $U=E$. The potential derivative can be expressed in the form:

$$\frac{\partial U}{\partial r_1} = \frac{\partial E}{\partial V_{DP}}\frac{\partial V_{DP}}{\partial r_1} = -2\pi r_1 p , \tag{16}$$

where V_{DP} is the volume occupied by detonation products, and p is their current pressure.

The work of non-conservative forces is in fact the work of the forces that resist cylinder deformation. Considering that the circular stress is dominant, the total non-conservative generalized force that can be written in the form:

$$Q = \frac{C\sigma_f}{\rho_0}\left[\left(\frac{r_{20}}{r_{10}}\right)^2 - 1\right]\frac{1}{\sqrt{w^2 + r_1^2}} . \qquad (17)$$

Flow stress of cylinder material σ_f is assumed to be constant.

Substitution of the kinetic energies, Eqs. (14) and (15), the potential, Eq. (16), and the generalized force, Eq. (17), in the Lagrange equation (11) leads to the final differential equation of motion of the inner cylinder surface:

$$\left[\frac{M}{C}\frac{r_1^2}{w^2}\ln\left(1+\frac{w^2}{r_1^2}\right)+\frac{1}{2}\right]a_1 + \frac{\sigma_f}{\rho_0}\frac{w^2}{r_{10}^2}\frac{1}{\sqrt{w^2+r_1^2}} = \frac{2r_1}{\rho_0 r_{10}^2}p(r_1) - \frac{M}{C}\frac{v_1^2}{r_1}\left[\frac{r_1^2}{w^2}\ln\left(1+\frac{w^2}{r_1^2}\right) - \frac{r_1^2}{w^2+r_1^2}\right] \qquad (18)$$

where the ratio of the cylinder mass to the explosive charge mass is defined as:

$$\frac{M}{C} = \frac{\rho_m w^2}{\rho_0 r_{10}^2} . \qquad (19)$$

Detonation products pressure. In order to solve the equation of motion (19), the detonation products' pressure $p=p(r_1)$ must be defined. The pressure p is determined under the condition that the polytropic expansion starts from the Chapman-Jouget state [18], [3]. Using the results of the elementary detonation theory [19], the detonation products pressure is determined as the function of the inner cylinder surface position:

$$p(r_1) = \frac{1}{\gamma+1}\left(\frac{\gamma}{\gamma+1}\right)^{\gamma}\rho_0 D^2\left(\frac{r_{10}}{r_1}\right)^{2\gamma} . \qquad (20)$$

The values of parameter γ are usually in the interval [2.7, 3.0], and if experimental results lacks the most common approximation is $\gamma \approx 3$.

Substituting the pressure, Eq. (20), in the equation of the cylinder motion (18), an ordinary differential equation of the second order emerges. This equation can be easily solved by numerical methods, applying the appropriate initial conditions:

$$\left(r_1\right)_{t=0} = r_{10}, \quad \left(\dot{r}_1\right)_{t=0} = v_i . \qquad (21)$$

Case of grazing detonation. In this case, the modeling of the cylinder motion implies determination of the cylinder velocity as a vector: the velocity magnitude should be calculated, as well as the direction of velocity vector. Regarding the velocity magnitude, basically the presented model for axisymmetric detonation can be used. If the lateral outflow is negligible (which is true for a slender cylinder), an elementary cylinder part is thought to be propelled by the corresponding elemental detonation products (Fig. 3).

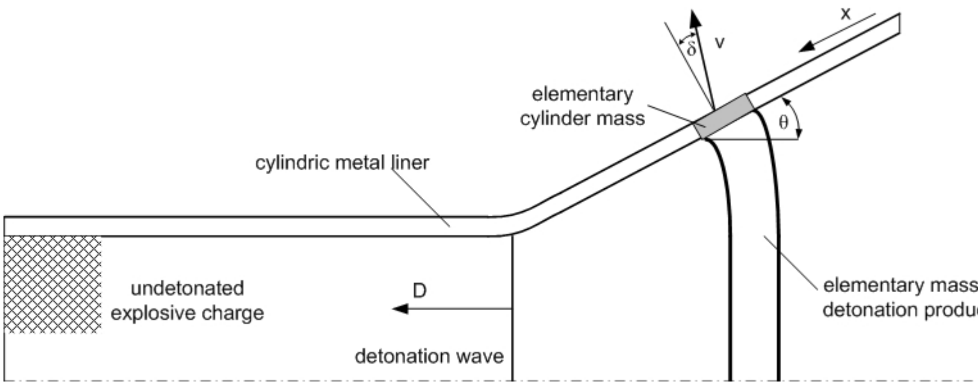


Figure 3. Grazing detonation: an elementary cylinder mass propelled by the corresponding elementary mass of the gaseous detonation products

Therefore, in the second, gas-dynamic phase of the cylinder motion, it is insignificant whether this elementary mass of gaseous detonation products emanates from an axisymmetric or grazing detonation [17]. Hence, the proposed model of gas-push process based on the Lagrange's equation will be also applied in this case.

The cylinder acceleration process by the grazing detonation wave can be considered quasi-steady – the velocity of any elementary cylinder part has the same time history. The velocity direction is defined by the angle δ between the velocity vector and the line normal to the cylinder axis. By the classical Taylor model [20] of the cylinder motion by grazing detonation this angle is determined by:

$$\sin\delta = \frac{V}{2D} . \qquad (22)$$

3. COMPARISON WITH EXPERIMENTAL RESULTS AND DISCUSSION

In order to investigate validity of the suggested model, computed results are compared with the available experimental data for both axisymmetric and grazing detonation.

3.1. Comparison with experimental data for axisymmetric detonation

The results of presented one-dimensional model of the cylinder motion under the action of gaseous products of axisymmetric detonation are analyzed through comparison with the experimental data from [21]. Motion of steel cylinder after electric initiation by the bundle of copper wires is observed by high-speed digital camera. The characteristics of the explosive and cylinder used, which are at the same time the input data for the model, are shown in Table 1.

Comparison of the experimentally determined radial displacement of the outer cylinder surface and the model results is shown in Fig. 4. Cylinder acceleration process is obviously impulsive and initial acceleration is of the order of 10^8 m/s^2. Good correspondence between experimental and model results is noted, until the moment of onset of fragmentation (~15 μs) accompanied with longitudinal cracks on the outer cylinder surface. From that moment, the cylinder loses structural integrity and the analyzed model can not be applied.

Table 1. Characteristics of explosive and metallic cylinder

Explosive	
pentrit (PETN)	
density ρ_0 (kg/m^3)	950
detonation velocity D (m/s)	5420
polytropic constant γ_{CJ}	2.57
Cylinder (r_{10}=14 mm, r_{20}=17 mm)	
stainless steel JIS SUS 304	
density ρ_m (kg/m^3)	7850
yield stress σ_0 (MPa)	340
dynamic viscosity μ (kPas)	3.0

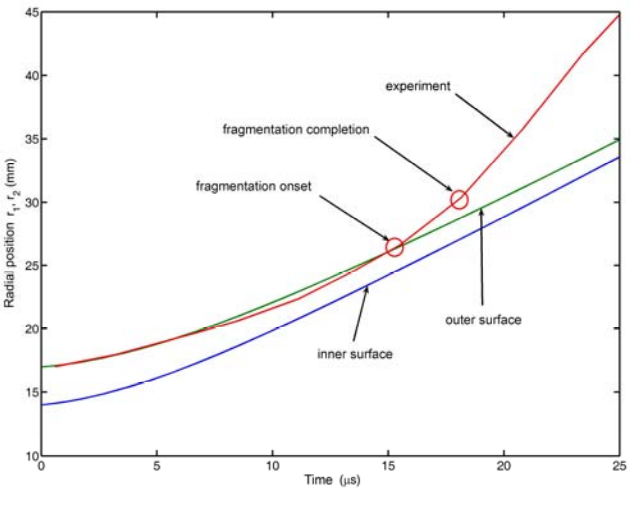


Figure 4. Radial position of inner and outer cylinder surface under the action of detonation products – comparison of the experimental data [21] and the model prediction

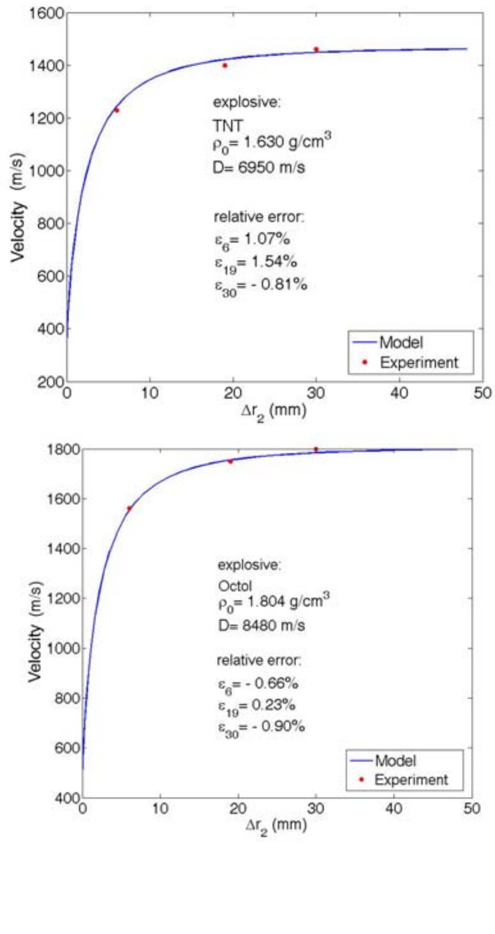
Fragmentation process completes after t_f≈18μs when a massive leakage of detonation products is observed. The outer cylinder radius in the moment of fragmentation completion is r_{2f}=30 mm, and the corresponding cylinder strain is ε_f=0.76.

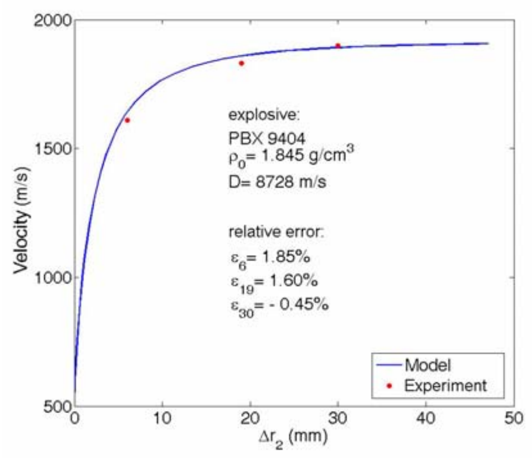
3.2. Comparison with experimental data for grazing detonation

There are numerous studies related to the experimental investigation of the metallic cylinder expansion by grazing detonation ("cylinder test"). Measurement of the cylinder displacement enables determination of detonation products' equation of state, the Gurney energy of explosive used, etc.

The study [22] is a comprehensive collection of experimental results related to the mentioned „cylinder test". The inner radius of the standard copper cylinder used is r_{10}=12.7 mm, the cylinder wall thickness is δ=2.60 mm, and the cylinder length is L=305 mm. The following handbook values of physical and mechanical properties of copper are used in the computer program based on the suggested model: density ρ=8945 kg/m^3, quasi-static yield stress σ_f=90 MPa, and parameters in the equation of state – c_m=3940 m/s and s_m=1.49. The cylinder expansion is described by the measured velocities in three characteristic positions, corresponding to the detonation products expansions of V=2, 7 and 11. These expansion factors coincide with the outer cylinder surface displacements of 6 mm, 19 mm, and 30 mm, respectively. The results of some of these experiments are compared to the model predictions in Fig. 5. The explosive type and its experimentally determined properties – density ρ_0 and detonation speed D are indicated on each diagram.

The relative errors of theoretically determined velocities compared to the measured values are also specified on the diagrams. As can be seen from the diagrams, the predicted velocities are in excellent accordance with experimental data. The relative error of velocity does not exceed 3% and is usually lower than 1%.





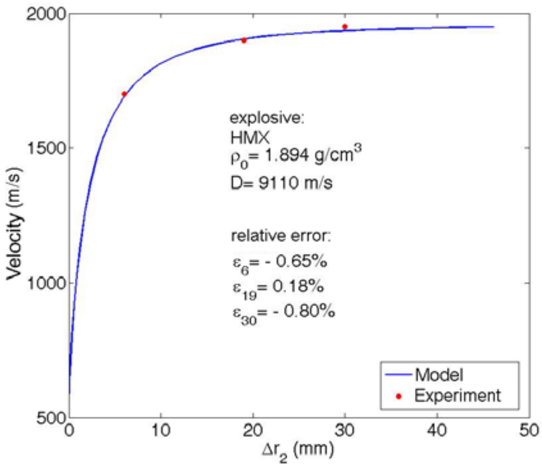


Figure 5. Comparison of cylinder test experimental results [22] with the prediction of theoretical model for TNT, Octol, PBX 9404 and HMX

4. CONCLUSION

Cylindrical liner motion under the action of expanding detonation products has been considered. A new cylinder acceleration model is suggested for the cases of axisymmetric and grazing detonation of explosive charge. The model implies two-stage character of the liner motion: (i) early interaction of the detonation wave and metallic liner results in impart of the initial velocity to the cylindrical liner, and (ii) the gas-push stage of rapid cylinder motion, similarly to the Gurney's model. The initial liner velocity is determined by impedance matching technique. The cylinder motion in the second stage is modeled by the Lagrange's equation.

Comparison with experimental results shows that the model predictions and experimental data are in good agreement in the domain that is of practical importance for explosive propulsion.

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References

[1] Gurney, R.W., The initial velocities of fragments from bombs, shells and grenades, US Army Ballistic Research Lab, BRL report 405, 1943.

[2] Kennedy, J.E., The Gurney model for explosive output for driving metal, in: Explosive Effects and Applications, J.A. Zukas and W.P. Walters (Eds.), Springer, New York, 2003.

[3] Danel, J.F., Kazandjian, L., "A few remarks about the Gurney energy of condensed explosives", Propellants, Explosives, Pyrotechnics, 29(5) (2004), 314-316.

[4] Tan, D., Sun, C., Wang, Y., "Acceleration and viscoplastic deformation of spherical and cylindrical casings under explosive loading", Propellants, Explosives, Pyrotechnics, 28(1) (2003) 43-47.

[5] Odintsov, V.A., Chudov, L.A., Rasshirenie i rasrushenie obolchek pod deistviem produktov detonacii, Problemi dinamiki uprugo-plasticheskih sred, 5, Mir, Moskva, 1975.

[6] Lloyd, R.M. Fragmentation warhead principles, in: Conventional Warhead Systems Physics and Engineering Design, AIAA, 1998.

[7] Reaugh, J.E., Souers, P.C., "A constant-density Gurney approach to the cylinder test", Propellants, Explosives, Pyrotechnics, 29(2) (2004), 124-128.

[8] Backofen, J.E., Weickert, C.A., "A 'Gurney' formula for forward projection from the end of an explosive charge", Proc. 14th Int. Symp. Ballistic, Quebec, 59-68, 1993.

[9] Backofen, J.E., Weickert, C., "Initial free-surface velocities imparted by grazing detonation waves", Shock Compression of Condensed Matter, M.D. Furnish, L.C. Chhabildas, R.S. Hixon (Eds.), American Institute of Physics, Part 2, 919-922, 1999.

[10] Meyers, M.A. Ch. 7. Shock wave attenuation, interaction, and reflection; Ch. 9. Explosive-material interactions; Ch. 10. Detonation, In: Dynamic behavior of materials, 180-201, 229-243, 244-270, Wiley Interscience, 1994.

[11] Kiselev, V.V., "Estimate of the parameters of metal plates accelerated by sliding detonation of charges of condensed HE in the initial phase of the process", Fizika Goreniya i Vzryva, 31(1) (1995) 138-142.

[12] Deribas, A.A., "Acceleration of metal plates by a tangential detonation wave", Journal of Applied Mechanics and Technical Physics, 41(5) (2000), 824-830.

[13] Davis, W.C., Shock waves; rarefaction waves; equations of state, In: Explosive Effects and Applications, J.A. Zukas, W.P. Walters (Eds.), Springer, New York, 2003.

[14] Flis, W.J., "A Lagrangian approach to modeling the acceleration of metal by explosives", Developments in Theoretical and Applied Mechanics, 17 (1994), 190-203.

[15] Elek, P., Modelling of dynamic fragmentation in terminal ballistics, Ph.D. dissertation, Faculty of

Mechanical Engineering, University of Belgrade, (in Serbian), 2008.

[16] Tucker, D.C., Prediction of the theoretical behavior and energy transfer when solids are subjected to explosive loading, Summary Report, AD0613697, University of Denver, 1965.

[17] Elek, P., Jaramaz, S., "Analytical model of metallic cylinder motion under the action of detonation products", (in Serbian), 2. Simpozijum OTEH, 3-5. October, Belgrade, 2007.

[18] Cooper, P.W. Introduction to detonation physics, In: Explosive Effects and Applications, J.A. Zukas, W.P. Walters (Eds.), Springer, New York, 2003.

[19] Ficket, W., Davis, W.C., Detonation: Theory and Experiment, Dover, 2000.

[20] Taylor, G.I., Scientific papers of Sir Geoffrey Ingram Taylor, G.K. Batchelor (Ed.), Vol 3. Aerodynamics and the Mechanics of Projectiles and Explosions, Cambridge University Press, 1963.

[21] Hiroe, T., Fujiwara, K., Abe, T., Yoshida, M., "Rapid expansion and fracture of metallic cylinders driven by explosive loads", 13. AIP Conf. of Shock Compression of Condensed Matter, 706, Portland, 465-468, 2004.

[22] Akst, I.B., Heat of detonation, cylinder test, and performance munitions, Ninth Symposium of Detonation, Portland, 1989.