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Modelling and Analysis of 2-axis Reconfigurable Parallel Mechanism MOMA with Translatory Actuated Joints

GORAN V. VASILIĆ, University of Belgrade,
Faculty of Mechanical Engineering, Belgrade
SAŠA T. ŽIVANOVIĆ, University of Belgrade,
Faculty of Mechanical Engineering, Belgrade

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Modelling and analysis of a 2-axis reconfigurable parallel mechanism is shown in this paper. Generalized model for solving the inverse and direct kinematic problem is presented. Generalized equations showing the solution of kinematic problems of parallel mechanism applied for any configuration of reconfigurable 2-axis parallel mechanism are realized. Secluded characteristical mechanism configurations are shown as the realization examples. Workspace is determined for chosen configurations and possible singular mechanism positions are analysed.

Key words: *2-axis reconfigurable wokparallel mechanism, inverse and direct kinematic problem, workspace, singularities*

1. INTRODUCTION

Reconfigurable 2-axis parallel mechanism – MOMA belongs to a generation of reconfigurable technological module [1] which can exist individually or in a combination with other mechanisms in order to build new machine tools. Acronym MOMA presents Modular machine tool (MOdular MAchine Tool) with open architecture for control on basis of reconfigurable 2-axis parallel mechanism with actuated translatory joints and legs with constant length.

MOMA is established as a modular system on whose bases reconfiguring both hardware and software part of the system can be performed. Only the part reffering to managing generalized model for solving inverse and direct kinematic problem, the analysis of workspace and singularities for chosen characteristic configurations of reconfigurable parallel mechanism, shown by Figure 1.

Parallel mechanism, which is considered in this paper, is produced by generalizing parallel mechanism 2D TeMoPaM (Technological module with parallel mechanism) [3]-[5], on a model of parallel mechanisms with similar configuration such as Trijoint [6], two parallelogram mechanism [7], Specht [8] and redundant driven plane parallel mechanism [9].

The studies also give specific attention to reconfigurable machine tools and manufacturing system whose configuration can easily and quickly be changed [10]. Characteristic of reconfigurable systems is customization or adjustment to the current production needs in economically accepted way [11]. Developed reconfigurable 2-axis parallel mechanism allows two reconfigurable approaches: (i) changing module on a machine and (ii) using integrated functions of reconfigurability on a machine [12].

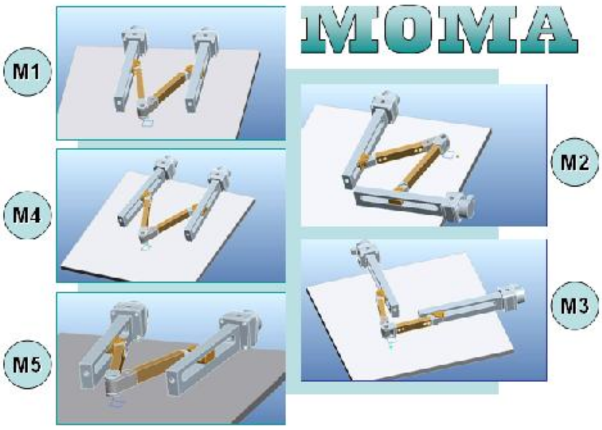

Figure 1 – CAD models of the five basic types M1-M5 reconfigurable 2-axis parallel mechanism [2]

The expected main result of configuring is one configuration of MOMA machine, chosen by some criterion. In order to achieve the main result it is neccessary to cover the configuration path from geometrical and kinematic models, through Jacobian inverse kinematics and singularity analysis, workspace analysis, optimization of some machine elements (for example, length of legs) [13], obtaining the virtual

Author's address: Goran Vasilić, University of Belgrade, Faculty of Mechanical Engineering, Belgrade, Kraljice Marije 16

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prototype, virtual prototype simulations, to the configuration of machine hardware on basis of available module fund and operation testing for the final verification of configurated new machine tool [14].

2. DESCRIPTION OF RECONFIGURABLE 2-AXIS PARALLEL MECHANISM

CAD model of basic conception of reconfigurable 2-axis parallel mechanism, type M1, is shown in Figure 2. Parallel mechanism consists of two identical driving translatory axis DA1 and DA2 which provide translational motion of sliders S1 and S2 along the guide. The maximum stroke of driving axis is h=200 mm. Legs L1 and L2 are tied to the sliders with the help of rotating jonts. Rotating joints are not actuated. Legs are also mutually tied by rotating joint and this bond between legs is at the same time a moving platform of parallel mechanism P. Legs of parallel mechanism have constant length.

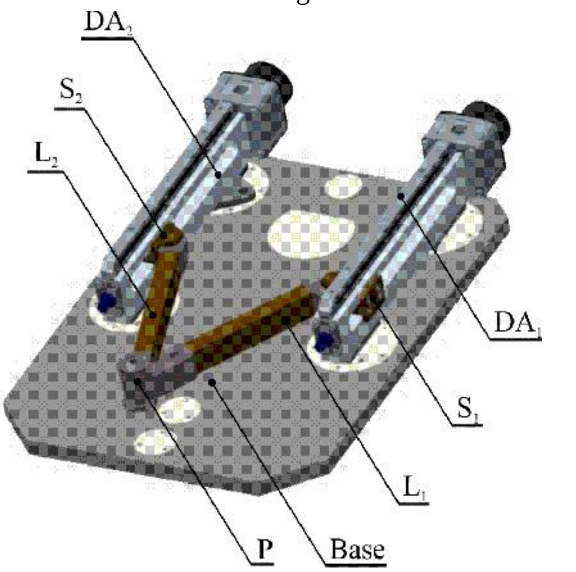


Figure 2 – CAD model of basic conception of 2-axis reconfigurable parallel mechanism, type M1

Parallel mechanism consists of series of modules and reconfigurable of mechanism from Figure 2 is in a fact that mutual position of some modules can be changed (for example, driving axis) and some modules are part of family of elements with different dimensions (for example, legs of mechanisam). Configuration of parallel mechanism can easily and quickly be changed by the construction program [2,14]. Every possible parallel mechanism configuration is defined by the construction program and some of possible parallel mechanism configurations for further analysis are shown in this paper in Figure 3.

Parallel mechanism configurations differ from one another by the lengths of legs l_i and by the orientation of driving axis defined by the angles α_i (Figure 3a). During the change of parallel mechanism configuration, driving axis DA_1 and DA_2 rotate around reference points R_1 and R_2.

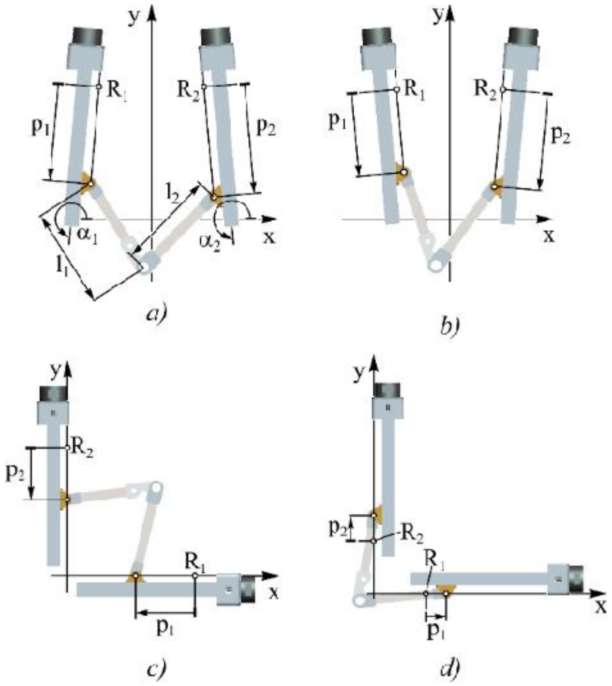


Figure 3 – Configurations of reconfigurable 2-axis parallel mechanism for analysis with basic parameters display

In Figure 3a and 3b angles α_i are defined by the statement $\alpha_i=3\pi/2+\gamma_i$. For Figure 3a configuration angles γ_i have values γ_1=-5° and γ_2=+5°, for Figure 3b configuration γ_1=+5° and γ_2=-5°. For parallel mechanism configurations in Figure 3c and Figure 3d the basis of mechanism changes in a way that orientation angles of operating axis for parallel mechanism configuration in Figure 3c have value $\alpha_1=\pi$ and $\alpha_2=3\pi/2$ whilst in Figure 3d configuration have value α_1=0° and $\alpha_2=\pi/2$. In Figure 3 are also shown internal coordinates of parallel mechanism p_i and reference points R_i which are essential for further analysis of reconfigurable parallel mechanism.

3. GEOMETRIC MODEL OF PARALLEL MECHANISM AND SOLVING IKP AND DKP

Generalized geometric model of reconfigurable parallel mechanism with two degrees of freedom which will be used for solving inverse kinematics problem (IKP) and direct kinematics problem (DKP) is shown in Figure 4.

Two coordinate systems are adopted for this geometric model and those are stationary coordinate system {B} connected to a base and motional coordinate system connected to the platform of parallel mechanism. Vectors defined by the geometry of parallel mechanism shown in Figure 4 are:

- ${}^B\mathbf{p}_{Ri} = [x_{Ri}\ y_{Ri}]^T$ - Position vector of reference points R in relation to stationary coordinate system {B};
- ${}^B\mathbf{p}_{OP} = [x_P\ y_P]^T$ - Position vector of platform of parallel mechanism P in relation to stationary coordinate system {B};
- ${}^B\mathbf{a}_i = [a_{xi}\ a_{yi}]^T$- Unit vectors defined by the orientation of parallel mechanism guides. At this moment the general form of unit vectors ${}^B\mathbf{a}_i$ is present, but in next chapter it will be precisely defined according to the parallel mechanism configuration
- $p_i{}^B\mathbf{a}_i$- Vectors of the internal coordinates, whilst p_i is a scalar controlled by the actuators;
- $l_i{}^B\mathbf{z}_i$-Vectors defined by the position, orientation and the length of legs. Vectors $\mathbf{z}_i$ are unit vectors defined by orientation of legs. Values of l_i presents length of legs.

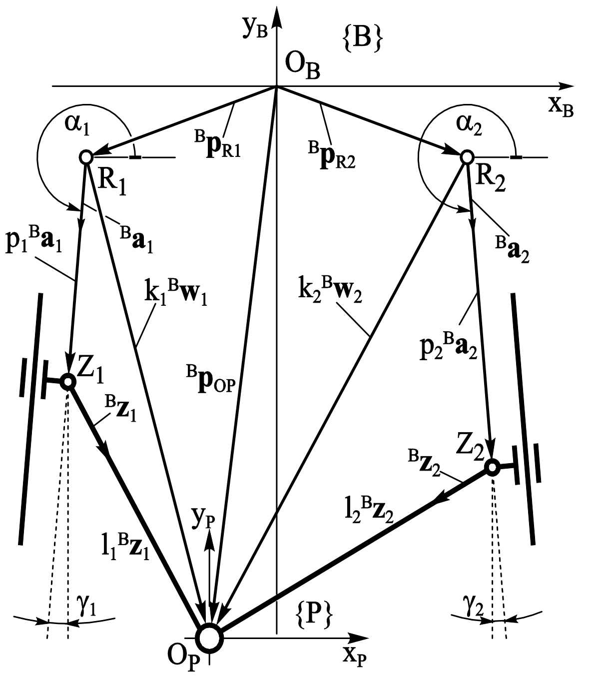


Figure 4 – Generalized geometric model of parallel mechanism MOMA

According to the geometric model in Figure 4 the following vector equations can be written:

$$k_i{}^B\boldsymbol{w}_i = {}^B\boldsymbol{p}_{OP} - {}^B\boldsymbol{p}_{Ri} \tag{1}$$

$$l_i{}^B\boldsymbol{z}_i = k_i{}^B\boldsymbol{w}_i - p_i\boldsymbol{a}_i \tag{2}$$

By squaring the equation (2) quadrating equation of this form is obtained

$$l_i^2 = p_i^2 - 2p_i\left({}^B\boldsymbol{a}_i k_i{}^B\boldsymbol{w}_i\right) + \left(k_i{}^B\boldsymbol{w}_i\right)^2 \tag{3}$$

On the basis of equation (3) the solution of inverse kinematic problem in general form is obtained

$$p_i = \left({}^B\boldsymbol{a}_i k_i{}^B\boldsymbol{w}_i\right) \pm \sqrt{\left({}^B\boldsymbol{a}_i k_i{}^B\boldsymbol{w}_i\right)^2 - \left(k_i{}^B\boldsymbol{w}_i\right)^2 + l_i^2} \tag{4}$$

By the exchange of parameters and known values of parallel mechanism in equation (3), implicit equations (5) and (6) used for further analysis of parallel mechanism are obtained

$$\begin{aligned} &p_1^2 - 2p_1\left[a_{x1}\left(x_P - x_{R1}\right) + a_{y1}\left(y_P - y_{R1}\right)\right] + \\ &+\left(x_P - x_{R1}\right)^2 + \left(y_P - y_{R1}\right)^2 - l_1^2 = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} &p_2^2 - 2p_2\left[a_{x2}\left(x_P - x_{R2}\right) + a_{y2}\left(y_P - y_{R2}\right)\right] + \\ &+\left(x_P - x_{R2}\right)^2 + \left(y_P - y_{R2}\right)^2 - l_2^2 = 0 \end{aligned} \tag{6}$$

For further analysis of parallel mechanism equation (5) is going to be observed as implicit function f_1 whilst equation (6) is going to be observed as implicit function f_2.

I) The solution of inverse kinematic problem

If vector ${}^B\mathbf{p}_{OP}$ is known regarding platform coordinates of parallel mechanism in stationary coordinate system {B}, solving the equations (5) and (6) by internal coordinates p_i, equations which present the solution of inverse kinematic problem are obtained

$$p_1 = B_1 - \sqrt{B_1^2 - C_1} \tag{7}$$

$$p_2 = B_2 - \sqrt{B_2^2 - C_2} \tag{8}$$

where

$$\begin{aligned} B_i &= a_{xi}\left(x_P - x_{Ri}\right) + a_{yi}\left(y_P - y_{Ri}\right) \\ C_i &= \left(x_P - x_{Ri}\right)^2 + \left(y_P - y_{Ri}\right)^2 - l_i^2 \end{aligned} \tag{9}$$

II) The solution of direct kinematic problem

By solving equation system (5) and (6) by the external coordinates of the parallel mechanism x_P and y_P, the solution of direct kinematic problem is obtained. The equation (10.a) is valid for Figure 2a, 2b and 2d configurations, and equation (10b) is valid for Figure 2c configuration. The equation (11) is valid for all configurations.

$$y_P = \frac{-v_{10} - \sqrt{v_{10}^2 - 4v_9v_{11}}}{2v_9} \tag{10.a}$$

$$y_P = \frac{-v_{10} + \sqrt{v_{10}^2 - 4v_9v_{11}}}{2v_9} \tag{10.b}$$

$$x_P = v_7 + y_Pv_8 \tag{11}$$

The introduced changes in solving direct kinematic problem are given in equations (12).

$$v_1 = 2(p_1 a_{x1} + x_{R1})$$
$$v_2 = 2(p_1 a_{y1} + y_{R1})$$
$$v_3 = p_1^2 + 2p_1\left(a_{x1}x_{R1} + a_{y1}y_{R1}\right) - l_1^2 + x_{R1}^2 + y_{R1}^2$$
$$v_4 = 2(p_2 a_{x2} + x_{R2})$$
$$v_5 = 2(p_2 a_{y2} + y_{R2})$$
$$v_6 = p_2^2 + 2p_2\left(a_{x2}x_{R2} + a_{y2}y_{R2}\right) - l_2^2 + x_{R2}^2 + y_{R2}^2$$
$$v_7 = \left(v_6 - v_3\right)/\left(v_4 - v_1\right)$$
$$v_8 = \left(v_2 - v_5\right)/\left(v_4 - v_1\right)$$
$$v_9 = 1 + v_8^2$$
$$v_{10} = 2v_7 v_8 - v_1 v_8 - v_2$$
$$v_{11} = v_7^2 - v_1 v_7 + v_3 \qquad (12)$$

4. THE ANALYSIS OF WORKSPACE

Before the workspace analysis itself, it is neccessary to complete derived equations by defining unit vector ${}^B\mathbf{a}_i$. According to Fig.4 listed unit vectors are in form ${}^B\mathbf{a}_i=[a_{xi}\ \ a_{yi}]^T$, but they can be also written the other way

$${}^B\boldsymbol{a}_i = T^{-1}\left[\cos\sim_1 \quad \sin\sim_1\right]^T \qquad (13)$$

Matrix T is the transformation matrix and it is introduced to generalize derived equations and to make them valid for all parallel mechanism configurations. For parallel mechanism configurations in Figure 3a and 3b transformation matrix is

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (14)$$

Transformation matrix (14) is unit matrix and it doesn't change unit vectors ${}^B\mathbf{a}_i$, so their form is ${}^B\mathbf{a}_i=[\cos\alpha_i\ \sin\alpha_i]^T$.

For mechanism configurations in Fig. 3c and 3d angles α_1=225° and α_2=315° are adopted. The angle β is the angle which all the unit vectors are rotated in relation to the axis of stationary coordinate system {B}. Introducing transformation matrix (15) for parallel mechanism configuration in Figure 3c and transformation matrix (16) for mechanism configuration in Figure 3d, unit vectors ${}^B\mathbf{a}_i$ are taken into direction of axis coordinate system {B} by transformation.

Described transformations are shown in Figure 5.

$$T = -\begin{bmatrix} \cos & -\sin \\ \sin & \cos \end{bmatrix} \qquad (15)$$

$$T = \begin{bmatrix} \cos & -\sin \\ \sin & \cos \end{bmatrix} \qquad (16)$$

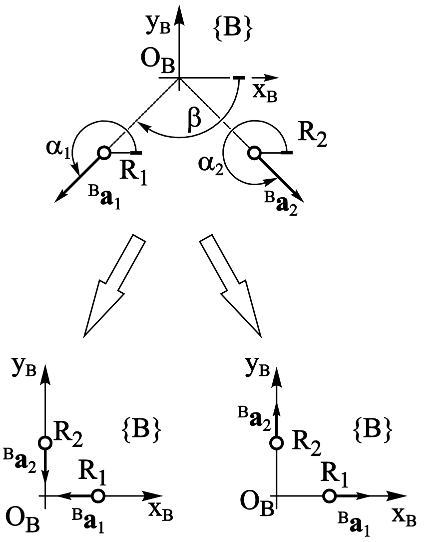


Figure 5 – Transformation of unit vector ${}^B\boldsymbol{a}_i$

For the analysis of operating space of parallel mechanism configuration shown in Figure 3, the following parameters are adopted:

- The legs of the same lengths were used for all configurations $l_1=l_2$=195mm;
- For parallel mechanism configuration in Figure 3a and 3b the coordinates of reference points in coordinate system {B} are x_{R1}=-100 mm, x_{R2}=100 mm, $y_{R1}= y_{R2}$=0mm;
- For parallel mechanism configuration in Figure 3c the coordinates of reference points in coordinate system {B} are x_{R1}=250mm, $y_{R1}=x_{R2}$=0 mm, y_{R2}=250mm;
- For parallel mechanism configuration in Figure 3d the coordinates of reference points in coordinate system are {B} are x_{R1}=117 mm, $y_{R1}=x_{R2}$=0 mm, y_{R2}=117mm;
- For parallel mechanism configuration in Figure 3a the angles that define the orientation of the guide are $\alpha_1=3\pi/2$-5° and $\alpha_2=3\pi/2$+5°;
- For parallel mechanism configuration in Figure 3b the angles that define the orientation of the guide are $\alpha_1=3\pi/2$+5° and $\alpha_2=3\pi/2$-5°.

Using the derived equations and adopted parameters of parallel mechanism for every mechanism configuration in Figure 3, workspaces shown in Figure 6 are obtained.

The obtained workspaces are attainable workspaces but because of the final dimensions of the elements of parallel mechanism, the legs of mechanism can be found only on the one side of the guides so the parallel mechanism platform can't be found in every point of the workspace. This part of the workspace in Figure 6b and 6d is textured by curve.

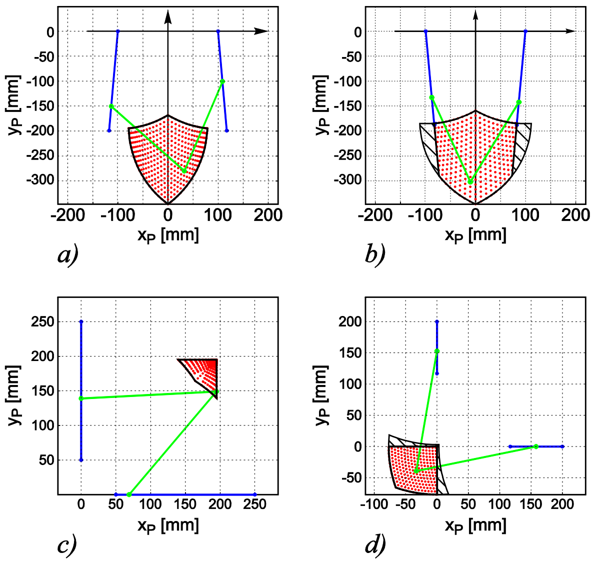


Figure 6 – The workspaces of different parallel mechanism configurations

5. JACOBIAN MATRIX AND SINGULARITY ANALYSIS

For the further analysis of reconfigurable parallel mechanism in Figure 3 Jacobian matrix of inverse kinematics is derived – the equations (17) and (18) and Jacobian matrix of direct kinematics given in equations (19) and (20).

$$J_P = \begin{bmatrix} \frac{\partial f_1}{\partial p_1} & \frac{\partial f_1}{\partial p_2} \\ \frac{\partial f_2}{\partial p_1} & \frac{\partial f_2}{\partial p_2} \end{bmatrix} \tag{17}$$

$$\frac{\partial f_i}{\partial p_i} = 2p_i - 2\left[a_{xi}\left(x_P - x_{Ri}\right) + a_{yi}\left(y_P - y_{Ri}\right)\right]$$

$$\frac{\partial f_i}{\partial p_j} = \frac{\partial f_j}{\partial p_i} = 0 \tag{18}$$

$$J_x = \begin{bmatrix} \frac{\partial f_1}{\partial x_P} & \frac{\partial f_1}{\partial y_P} \\ \frac{\partial f_2}{\partial x_P} & \frac{\partial f_2}{\partial y_P} \end{bmatrix} \tag{19}$$

$$\frac{\partial f_i}{\partial x_P} = 2\left(x_P - x_{Ri}\right) - 2p_i a_{xi}$$

$$\frac{\partial f_i}{\partial y_P} = 2\left(y_P - y_{Ri}\right) - 2p_i a_{yi} \tag{20}$$

On the basis of Jacobian matrix of inverse and direct kinematics, Jacobian matrix of parallel mechanism is derived and its general form is given in the equation (21).

$$J = J_P^{-1} \cdot J_x \tag{21}$$

In the equations (18) and (20) the components of the unit vectors are present, and for the analysis of specific parallel mechanism configurations derived transformation matrix (14), (15) and (16) are used. According to the equation (21) with the parallel mechanism parameters from the previous chapter, for every configuration in Figure 3 the valuations of determinant of the Jacobian matrix were calculated for every point of attainable workspace of the mechanism and the results are shown in the form of a diagram in Figure 7.

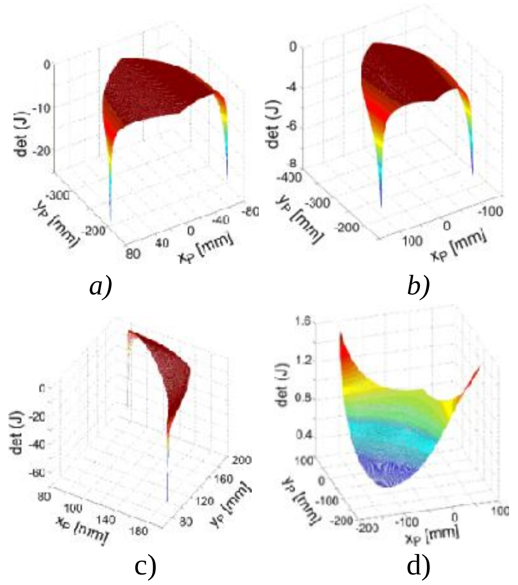


Figure 7 – The distribution detJ for different parallel mechanism configurations

Analysing the distribution det (J) and the diagram in Figure 7, singular positions of parallel mechanism configurations in Figure 3 are given. The singular parallel mechanism configurations in Figure 3a and 3b are shown in Figure 8a and 8b. Parallel mechanism configurations in Figure 3a and 3b don't have singularities of direct kinematics.

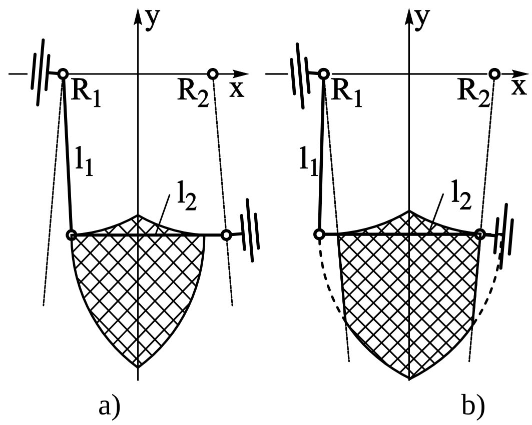
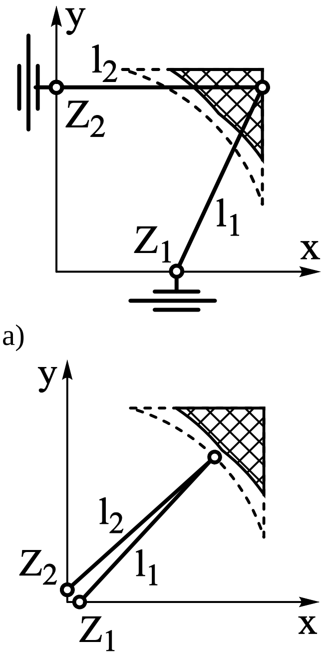


Figure 8 – Singular configurations: a) for mechanisms from Figure 3a; b) for mechanism from Figure 3b

Singular parallel mechanism configurations in Figure 3c are shown in Figure 9 and these are singularity of inverse kinematics and singularity of direct kinematics. Singularities of inverse kinematics appear when even one of the platform coordinates is equal to the legs lengths (Figure 9a). For the adopted parameters of parallel mechanism it is x_P=195[mm] and/or y_P=195[mm]. Singularities of direct kinematics appear when the cranks between legs and sliders are found in the origin point, or when their coordinates have values $x_{Z1} = x_{Z2} = y_{Z1} = y_{Z2} = 0$.



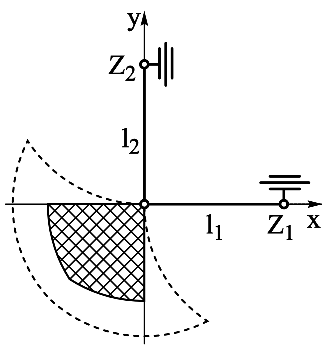
b)

Figure 9 – Singular configurations for mechanism from Figure 3c – a) Singularities of inverse kinematics; b) Singularities of direct kinematics

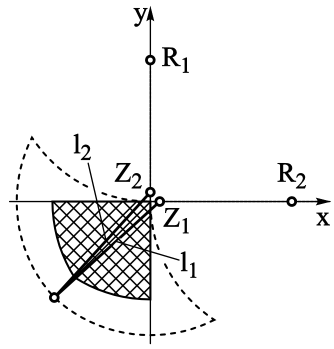
As in the previous case, mechanism in Figure 3d has the singularities of inverse kinematics (Figure 10a) and the singularities of direct kinematics (Figure 10b).

The singularity of inverse kinematics appears when the platform coordinates are $x_P=y_P=0$, whereas the singularity of direct kinematics appears when the crank coordinates between couplers and sliders have values $x_{Z1} = x_{Z2} = y_{Z1} = y_{Z2} = 0$.

The positions of parallel mechanism platform in singular mechanism positions are shown in Figure 8-10.



a)



b)

Figure 10 – Singular configurations for mechanism from Figure 3d: a) Singularities of inverse kinematics; b) Singularities of direct kinematics

But because of the geometry of component elements of the mechanism and also the whole assembly of mechhanism the parallel mechanism platform can't be found in every point of the workspace given on the basis of derived equations of kinematic problems.

In Figure 8-10 the workspace obtained by calculation is rounded by discontinuous lines where as the workspace for real physical model is textured and rounded by full line. According to that, parallel mechanism configuration in Figure 3b cannot take singular position shown in Figure 8b. Also the configurations in Figure 3c and 3d cannot take the positions where singularities of direct kinematics appear as shown in Figure 9b and 10b, respectively.

6. CONCLUSION

The main purpose of this paper is managing of generalized kinematic model for solving inverse and direct kinematic problem of reconfigurable 2-axis parallel mechanism. Derived general equations, with the change of certain parameters, are further appliable for management forming for any configuration of reconfigurable parallel mechanism, which the characteristic of integrated reconfigurability of managing this kind of machine is enabled by.

Before the physical realization of any parallel mechanism configuration, derived equations were used for analysis of wanted configuration. First, the workspace was analysed, and then the singularities of parallel mechanism. As it is shown in this paper, these kind of analyses can be done for any configuration and optimal configuration for user's needs can be obtained.

By conducting the analysis for four shown configurations it can be concluded that by the change of certain mechanism parameters, except the shape and size of the operating space, singularities can be influenced, or if the singular mechanism positions are known, they can be physically prevented and avoided.

Further plans for described parallel mechanism is the research of new configurations of reconfigurable mechanism and also the implementation of mechanism in hybrid reconfigurable mechanisms with three and more degrees of loose.

REMARK

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REZIME

MODELIRANJE I ANALIZA REKONFIGURABILNOG DVOOSNOG PARALELNOG MEHANIZMA MOMA SA OSNAŽENIM TRANSLATORNIM ZGLOBOVIMA

U ovom radu je prikazano modeliranje i analiza jednog rekonfigurabilnog dvoosnog paralelnog mehanizma. Predstavljen je generalizovani model za rešavanje inverznog i direktnog kinematičkog problema. Izvedene su uopštene jednačine koje predstavljaju rešenje kinematičkih problema paralelnog mehanizma koje važe za bilo koju konfiguraciju rekonfigurabilnog dvoosnog paralelnog mehanizma. Kao primeri realizacije pokazane su izdvojene karakteristične konfiguracije mehanizma. Za izdvojene konfiguracije određen je radni prostor i analizirani su mogući singularni položaji mehanizma.

Ključne reči: *rekonfigurabilni dvoosni paralelni mehanizam, inverzni i direktni kinematički problem, radni prostor, singulariteti*