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DYNAMICS OF THE LIGHTNING CHANNEL USING GENERALIZED TRAVELING CURRENT SOURCE RETURN STROKE MODEL

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Abstract: Characteristics of the lightning channel corona sheath surrounding a thin channel core are examined using the generalized lightning traveling current source (GTCS) return stroke model. The return stroke process is modeled with positive charge coming from the channel core discharging negative leader charge in the corona sheath. The corona sheath model that predicts charge motion in the sheath is used to derive sheath radius vs. time expressions during the return stroke. According to the corona sheath model, previously pro-posed in [1] and [2], it consists of two zones, zone 1 (surrounding the channel core with net positive charge) and zone 2 (surrounding zone 1 with negative charge). We adopted the assumption of a constant electric field inside zone 1 of the corona sheath observed in laboratory experimental research of corona discharges. We examined the influence of different magnitudes of the breakdown electric field at the boundaries of both zones on the dynamics of the return stroke. The new channel discharge function, calculated from the close electric field waveform measured at 15m from the channel [7], is used. The calculations have shown that the maximum radii of zones 1 and 2 significiaantly decrease compared to the previous results given in [9]. Similar conclusion holds for the velocities of zone 1 and 2 boundaries. It is concluded that the new channel discharge function gives the results for the time dependence of both the radii and their velocities in a good agreement with the theoretical predictions.

Keywords: electrostatic plasma confinement, lightning return stroke.

INTRODUCTION

In an improved model of the lightning channel corona sheath proposed in [1] two slightly different models for the channel sheath are postulated, the model with the exponential charge decay in zone 2, ($R^-_{out} = const$) and the model with the shrinkage of zone 2 ($R^-_{out} \neq const$) depicted in Fig.1. It has been shown in [1] that both models give very similar results if the charge decay constant (describing the diffusion of negative charges from zone 2 to zone 1) is of the order of hundreds of microseconds. Both models can be viewed as generalizations of the model proposed earlier in [2].

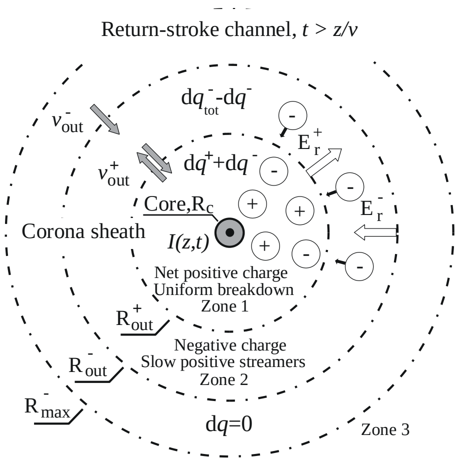

Fig.1 - *Horizontal cross section of the lightning channel containing the channel core and the corona sheath during the return stroke stage; the longitudinal infinitesimal length of the channel is dz. The magnitude of the electric field E^+_r within and at the boundary of zone 1 is assumed to be constant during the return stroke. Adopted from [3].*

CORONA SHEATH RADIUS OF ZONE 2

The expression of the corona sheath radius of zone 2 is the same as derived in [3, 5] since it represents the outer boundary of the charged corona containing whole leader charge i.e.

$$R^-_{out} = 2B\, f(u)\,, \qquad B = q^+_0 \,/\, (4\pi\varepsilon_0 \left|E^-_r\right|) \quad , \tag{1}$$

where $f(u) = 1 - f^+(u)$ is the channel discharging function defined in the GTCS model, $f^+(u)$ is the channel charging function (with positive charge), $u = t - z/v$ is the generalized time, [5]. Taking the time derivative of (1), one obtains the velocity of outer corona sheath shrinkage during the return stroke.

$$v^-_{out} = -2B(df^+/du)\,. \tag{2}$$

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CORONA SHEATH RADIUS OF ZONE 1

Applying the Gauss' law on the elementary cylinder whose length is dz with the top and bottom faces of radius R_{out}^+, the electric field on the boundary of zone 1 can be obtained as the superposition of the fields created by the positive charges $q_0^+ f^+$ (coming from the core)

$$E_{1r}^+ = q_0^+ f^+ /(2\pi\varepsilon_0 R_{out}^+) \ , \qquad (3)$$

and the field created by the negative (leader) charges within zone 1, which generate the field $E_{2r}^- = -q_0^+ /(2\pi\varepsilon_0 R_{out}^-)$, [3]. It follows

$$q_0^+ f^+ /(2\pi\varepsilon_0 R_{out}^+) - q_0^+ /(2\pi\varepsilon_0 R_{out}^-) = E_r^+ . \qquad (4)$$

Solving (4) one obtains

$$R_{out}^+ = q_0^+ f^+ /(2\pi\varepsilon_0 E_r^+ + q_0^+ / R_{out}^-) \ , \qquad (5)$$

where R_{out}^- is given by (1). At the time onset $f^+(u=0)=0$ [3], from (5) it follows $R_{out}^+ = 0$. At the end of the discharge $f^+(u \to \infty) \to 1$, from (1) one obtains $R_{out}^- \to 0$, and from (5) it follows $R_{out}^+ \to 0$. It can be concluded that R_{out}^+ has at least one peak which represents simultaneously the zero value of the expansion velocity of the boundary of zone 1. From (1) and (5) it follows

$$R_{out}^+ = 2B[f^+(1-f^+)]/[\xi(1-f^+)+1] \ , \quad \xi = E_r^+ / \left|E_r^-\right| . (6)$$

From (5) it follows the expansion velocity of the boundary of zone 1 ($v_{out}^+ = dR_{out}^+ / du$)

$$v_{out}^+ = 2B\frac{df^+}{du}\left[\xi\left(1-f^+\right)^2 + 1 - 2f^+\right] / \left[\xi\left(1-f^+\right)+1\right]^2 . (7)$$

The maximum value of R_{out}^+ can be easily obtained from (7) ($v_{out}^+ = 0$). It follows

$$f_0^+ = \left(\xi + 1 - \sqrt{\xi+1}\right) / \xi \ . \qquad (8)$$

The quotient of maxima of radii of zone 2 (1) and zone 1 (6) is

$$R_{\max}^- / R_{\max}^+ = \left[\xi\left(1-f_0^+\right)+1\right] / \left[f_0^+\left(1-f_0^+\right)\right] \ . \qquad (9)$$

The quotient of the velocities of the boundaries of zone 1 and 2 ((2) and (7), respectively) is

$$\left|v_{out}^-\right| / v_{out}^+ = \left[\xi\left(1-f^+\right)+1\right]^2 / \left[\xi\left(1-f^+\right)^2 + 1 - 2f^+\right] . (10)$$

At the time onset $f^+(u=0)=0$, from (10) it follows

$$\left|v_{out}^-\right| / v_{out}^+ \Big|_{u=0} = \xi + 1 \ . \qquad (11)$$

If $\xi = 1$ (that is $E_r^+ = \left|E_r^-\right|$), from (8), (9) and (11) it follows $f_0^+ = 0.586$, $R_{\max}^- / R_{\max}^+ = 5.83$ and $\left|v_{out}^-\right| / v_{out}^+ \Big|_{u=0} = 2$, respectively, [3].

THE NEW CHANNEL DISCHARGE FUNCTION

Total vertical electric field waveform (Fig.2, with the field direction pointing downwards to the ground) measured at the horizontal distance of 15 m from the channel core [7] is approximated with the function

$$E = E_{m2}\left(t/\tau_{12}\right)^n / [1 + \left(t/\tau_{12}\right)^n] - E_{m1} \ , \qquad (12)$$

where $E_{m1} = 0.128\,\mathrm{MV/m}$, $E_{m2} = 0.148\,\mathrm{MV/m}$, $n = 1.863$ and $\tau_{12} = 0.268\,\mu\mathrm{s}$.

The variation of the channel discharge function with the channel height close to the ground (a few tens of meters) as well as the time delay of the current wave (z/v) can be neglected [5], that is $f(z, t - z/v) \approx f(t)$. Therefore, the calculation of the new channel discharge function from the electric field can be simplified. As a result, the channel discharge function is more accurate calculated compared to [9] and [5], since a strong, time-depending magnetic field certainly introduces some disturbances (in [9], the measurements of the horizontal electric field at only 10 cm distance from the channel core are used).

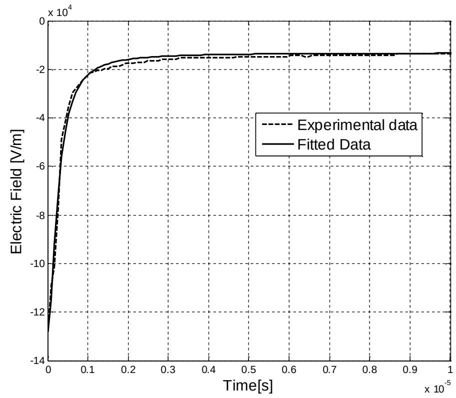

Fig.2 - *The experimental data and fitted curve (12) of close vertical electric field component at 15m [7]. Note that the field direction is pointing downwards to the ground.*

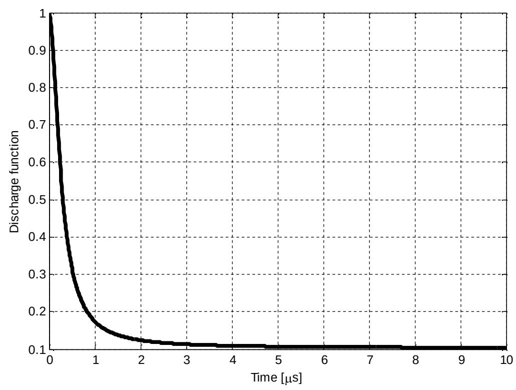

Fig.3 - *Calculation of the new channel discharge function based on the close electric field measurements at 15m [7], Fig.2. The channel discharge function is given by (16).*

Since in the GTCS model the elementary transferred charge can be expressed as [5]

$$dq(z,t) = q_{tot}^{-}(z)\, f(z, t - z/v)dz \approx f(t) q_{tot}^{-}(z)dz, \quad (13)$$

where $q_{tot}^{-}(z)$ is the initial (negative) line charge density along the channel. Using (13) the total vertical electric field component can be expressed as [5]

$$E(t) = -f(t)E_0 \ , \quad E_0 = \frac{1}{2\pi\varepsilon_0} \int_0^{\infty} \frac{z}{R^3} \left| q_{tot}^{-}(z) \right| dz \, , \quad (14)$$

where $R = (z^2 + r^2)^{1/2}$. The upper limit in integral (14) reaches a high value and it is set to infinity. Comparing (14) and (12) it follows straightforwardly

$$E_0 = E_{m1} . \quad (15)$$

The channel discharge function can be expressed as

$$f(t) = -E(t) / E_{m1} . \quad (16)$$

Using (16) the channel discharge function is calculated and presented in Fig.3.

CHANNEL CORONA SHEATH DYNAMICS

In Figs. 4 and 5 the radii of the corona sheaths of zones 2 and 1 versus time are calculated according to (1) and (6), respectively. In Figs. 6 and 7 the velocities of the corona sheath expansions of zones 2 and 1 versus time are calculated according to (2) and (7), respectively.

The quotient of the field magnitudes $\xi = E_r^+ / \left| E_r^- \right|$ is used as a parameter, Tables I and II.

From Figs.4 and 5 it can be concluded that the radius of the corona sheath surface of zone 2 (R_{out}^-) strongly depends on the breakdown electric field $\left| E_r^- \right|$ whereas the radius of zone 1 shows only weak dependence on the breakdown electric field E_r^+. Both radii decrease with the increase of the corresponding field magnitude. The velocities of the corona sheath surfaces of zones 1 and 2 strongly depend on the breakdown electric fields $\left| E_r^- \right|$ and E_r^+, Figs. 6 and 7. They also decrease with the increase of the corresponding field magnitude. It is obvious that the velocities are much less (at least three orders of magnitude) then the return stroke velocity [5]. However, if $\xi = 1$ (that is $E_r^+ = \left| E_r^- \right|$) from (8), (9) and (11) it follows $f_0^+ = 0.586$ ($v_{out}^+ = 0$). If $\xi << 1$ (that is $E_r^+ << \left| E_r^- \right|$), it follows $f_0^+ \approx 0.5$ ($v_{out}^+ = 0$). Since the channel charging function f^+ is monotonically rising function [5], the decreasing of ξ decreases slightly the rise time of reaching the maximum radius of corona sheath of zone 1. This behaviour can be seen in Figs. 3 and 5. Simultaneously, the quotient of maximum radii $R_{\max}^- / R_{\max}^+$ slightly decreases with the decreasing of ξ (if $\xi = 1$, $f_0^+ = 0.586$ one obtains

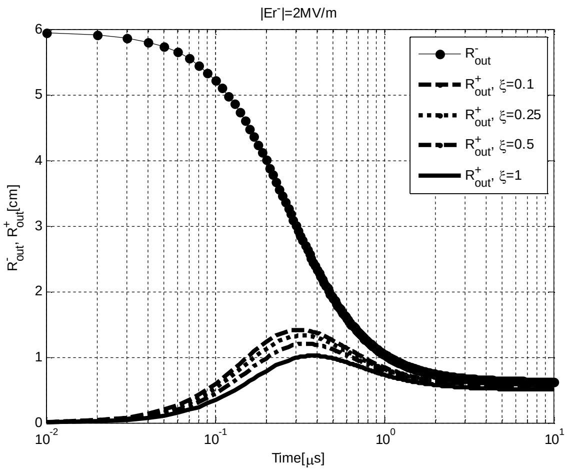

Fig.4 - *The radii of the corona sheaths of zones 2 (R_{out}^-) and 1 (R_{out}^+) versus time ((1) and (6), respectively) according to the GTCS model. The positive (E_r^+) and the negative ($\left| E_r^- \right|$) breakdown electric are assumed to be constant during the return stroke, Table I.*

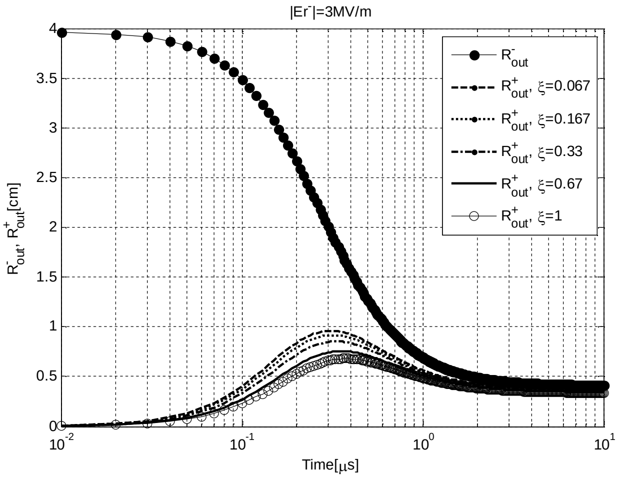

Fig.5 - *The radii of the corona sheaths of zones 2 (R_{out}^-) and 1 (R_{out}^+) versus time ((1) and (6), respectively) according to the GTCS model. The positive (E_r^+) and the negative ($\left| E_r^- \right|$) breakdown electric are assumed to be constant during the return stroke, Table II.*

Table I

The magnitude of the electric fields on the outer surfaces of zones 1 and 2 for $\left| E_r^- \right| = 2\,\mathrm{MV/m}$.

$\left\| E_r^- \right\|$ [MV/m]	2	2	2	2
E_r^+ [MV/m]	0.2	0.5	1	2
$\xi = E_r^+ / \left\| E_r^- \right\|$	0.1	0.25	0.5	1

Table II

The magnitude of the electric fields on the outer surfaces of zones 1 and 2 for $\left| E_r^- \right| = 3\,\mathrm{MV/m}$.

$\left\| E_r^- \right\|$ [MV/m]	3	3	3	3	3
E_r^+ [MV/m]	0.2	0.5	1	2	3
$\xi = E_r^+ / \left\| E_r^- \right\|$	1/15	1/6	1/3	2/3	1

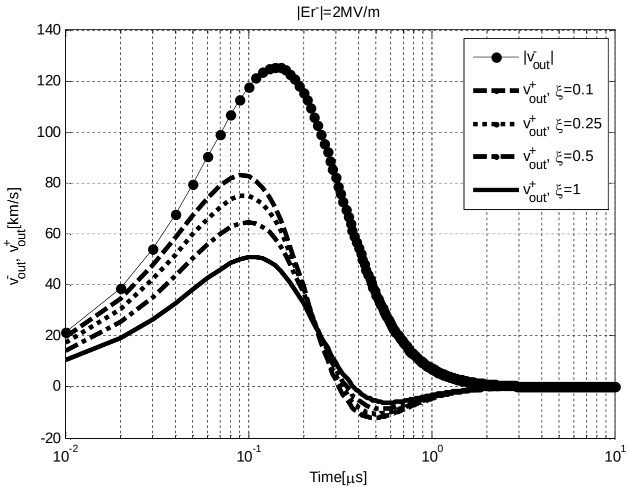

Fig.6 - *The velocities of the outer surfaces of zones 2 (v_{out}^{-}) and 1 (v_{out}^{+}) versus time ((2) and (7), respectively) according to the GTCS model. The positive (E_r^+) and the negative ($\left|E_r^-\right|$) breakdown electric field are assumed to be constant during the return stroke, Table I.*

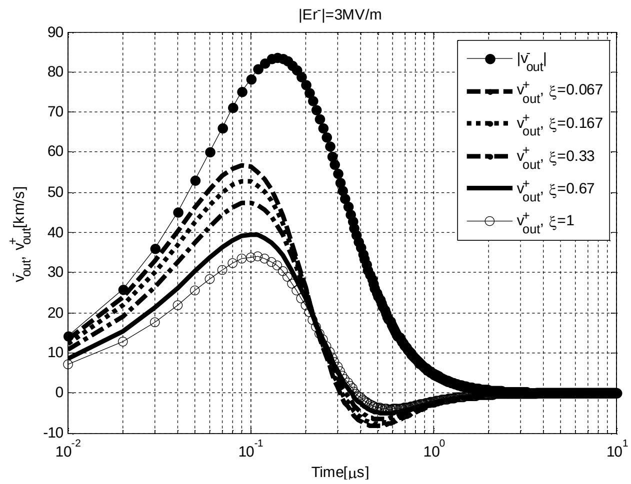

Fig.7 - *The velocities of the outer surfaces of zones 2 (v_{out}^{-}) and 1 (v_{out}^{+}) versus time ((2) and (7), respectively) according to the GTCS model. The positive (E_r^+) and the negative ($\left|E_r^-\right|$) breakdown electric field are assumed to be constant during the return stroke, Table II.*

$R_{\max}^{-} / R_{\max}^{+} = 5.83$, if $\xi << 1$, $f_0^+ \approx 0.5$ one obtains $R_{\max}^{-} / R_{\max}^{+} \approx 2(\xi + 2) < 5.83$).

CONCLUSION

We examined the influence of different magnitudes of the breakdown electric field on the boundaries of both zones on the dynamics of the return stroke. This approach can be viewed as the generalization of the corona sheath model given in the previous studies [1, 2, 3]. The calculations have shown that the radii of zones 1 and 2 decrease with the increasing of the magnitude of the breakdown electric field in the corresponding zone. Similar conclusion holds for the velocities of the boundaries of zones 1 and 2. However their velocities are much less (at least three orders of magnitude) then the return stroke velocity, [5]. Simultaneously, a slight decrease of the rise time in reaching the maximum of the corona sheath radius of zone 1 as well as the decrease of the time of zero crossing of the velocity are observed.

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