

# Generalized TCS Model with the Current Reflection at Ground and at the Upper End of the Lightning Channel

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**Abstract**— The generalized travelling current source model (GTCS) is extended to take into account the current reflections occurring at ground and at the upper end of the lightning channel. The ground reflection and the top reflection factors are overtaken from the extended TCS model. The current sources are placed along the activated length of the lightning channel and represented by the channel discharge function introduced earlier in the GTCS model. The multiple reflections originate current waves moving up and down on the lightning channel. The total current is composed by the source current according to the original GTCS model and the reflected currents. The current along the channel can be represented as two sums of the integrals of the current sources along the channel.

**Index Terms**—Atmospheric discharge, Lightning, Return stroke.

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## I. INTRODUCTION

During the return stroke process, the current flows through the lightning channel lowering the leader charge to earth. The lightning return stroke models which belong to “engineering” models commonly assume the lightning channel as a transmission line, where the current reflections are ignored at the terminations. In reality, the channel-base is terminated by the ground impedance and the current reflections occur at the transition from the plasma channel in air to ground. Current reflections are also expected at the upper end of the lightning channel acting like the open end of a transmission line. Objective of the paper is to extend the generalized traveling current source model in order to take into account both current reflections.

## II. THE GTCS MODEL WITH CURRENT REFLECTIONS

The case of the GTCS return stroke model [1, 2] is analyzed. The classical TCS model [3] can be derived as a special case of the GTCS model [1]. The GTCS model with the current reflections occurring at the ground and at the upper end of the lightning channel during the return stroke is shown in Fig.1. The ground reflection coefficient is defined as

$$\rho = i_{0/u} / i_{0/d}, \quad (1)$$

where  $i_{0/u}$  is the upward (reflected) and  $i_{0/d}$  is the downward moving current wave at the channel-base having the current reference directions along the z-axis. The channel top current reflection coefficient is [3]

$$R = -A, \quad A = (c - v)/(c + v), \quad (2)$$

where  $v$  is the return stroke velocity and  $c$  is the speed of the light. Since  $v \leq c$  from (2) one obtains

$$-1 < R < 0. \quad (3)$$

The values of the ground reflection coefficient is

determined by the soil characteristics and it spans the interval  $[-1,1]$ . For a good grounding system the ground reflections are taken into account with  $\rho > 0$ . In that case both current waves in (1) have the same direction. The total current at the ground level can be represented as the sum of the downward moving current wave and the upward moving current wave reflected from the ground, that is

$$i_0 = i_{0/d} + i_{0/u} = (1 + 1/\rho) i_{0/u} = (1 + \rho) i_{0/d}. \quad (4)$$

The current at the ground level can be determined either by knowing downward current or the upward current at the ground level. The GTCS model assumes that the current sources  $di_Q$  are homogeneously distributed over the activated part of the channel. In Fig.1, an infinitesimal current source at some height is depicted. According to the GTCS, the current of the source is given by

$$di_Q(u) = q_{tot}^-(\xi) \frac{\partial}{\partial u} f(u) d\xi, \quad (5)$$

where  $u = t - z/v$  is the generalized time,  $f(u)$  is the channel discharge function [1, 2], and  $q_{tot}^-$  is the negative leader line charge density deposited along the channel corona sheath prior to the return stroke. Note that  $di_Q > 0$  since the discharge of the negative charged channel is considered. The current pulse generated by the current source splits into two components. The downward moving current source component is

$$di_Q^{(d)} = p di_Q, \quad (6)$$

where  $p \in [0,1]$  is the current source split factor, Fig.1. It is directed upwards since  $p > 0$  i.e.  $di_Q^{(d)} > 0$ . The upward moving current source component is

$$di_Q^{(u)} = -(1-p) di_Q. \quad (7)$$

If  $p = 1$ , there is no upward moving current component i.e. one obtains the classical TCS model. The first current component  $i_{0/d}^{(d)}$  originates from the downward portion of the injected current pulse  $di_Q^{(d)}$  including all reflections from the ground and later from the upper end of the channel, Fig.1. The second component  $i_{0/d}^{(u)}$  is generated by the upward portion of injected current pulse  $di_Q^{(u)}$  including all reflections from the upper end of the channel and later from the ground. The total downward current at the channel-base is

$$i_{0/d} = i_{0/d}^{(d)} + i_{0/d}^{(u)}. \quad (8)$$

According to (4) the total channel-base current is

$$i_0 = (1 + \rho)(i_{0/d}^{(d)} + i_{0/d}^{(u)}). \quad (9)$$

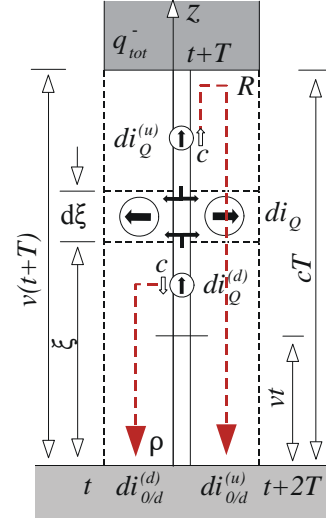


Fig. 1. The current reflections from the ground and from the upper end of the channel according to the GTCS model

### III. THE TOTAL CURRENT ALONG THE CHANNEL

According to [3] the downward current pulse  $i_{0/d}^{(d)}$  at the channel base generated by the current source component  $di_Q^{(d)}$  is

$$i_{0/d}^{(d)}(t) = p \sum_{v=0}^{N \rightarrow \infty} (R\rho)^v \int_0^{v A^v t} q_{tot}^-(\xi) \frac{\partial}{\partial u} f(u) d\xi, \quad (10)$$

where  $u = A^v t - \xi / v^*$ . The current generator pulse is defined by the initial line charge density (corona charge)  $q_{tot}^-$  along the channel and the channel discharge function  $f(u)$  where  $u \geq 0$  represents the generalized time [2, 3].

$$di_Q(u) = q_{tot}^-(\xi) \frac{\partial}{\partial u} f(u) d\xi, \quad (11)$$

$$u = t' - \xi / v = t / A - \xi / v^*, \quad u \geq 0.$$

In a similar way it can be shown that the downward current pulse  $i_{0/d}^{(u)}$  at the channel base generated by the current source component  $di_Q^{(u)}$  is

$$i_{0/d}^{(u)}(t) = -(1-p) \sum_{v=0}^{N \rightarrow \infty} R^{v+1} \rho^v \int_0^{v A^v t} q_{tot}^-(\xi) \frac{\partial}{\partial u} f(u) d\xi, \quad (12)$$

where  $u = A(A^v t - \xi / v^*)$ . The total current at the channel-base can be calculated using (9), (10) and (12). The current at some altitude in the channel consists of the upward and the

downward moving components,  $i_u(z,t)$  and  $i_d(z,t)$ , respectively. Since the current pulse at some altitude is  $i_u(z,t) = i_{0/u}(t - z/c)$  and  $i_d(z,t) = i_{0/d}(t + z/c)$ , using (4) it follows

$$i(z,t) = [\rho i_0(t - z/c) + i_0(t + z/c)] / (1 + \rho), \quad (13)$$

where the channel-base current  $i_0$  is given by (9).

#### IV. CONCLUSION

The GTCS-model suggests that the lightning current is injected by a current source located along the increasing lightning channel. From there the current wave propagates up to the channel top and down to earth with the speed of light. Based on these assumptions, the GTCS-model is extended to take into account the current reflections at the channel top and the ground. The ground impedance is considered by a ground reflection factor  $\rho$  having a constant value. When the downward moving current wave arrives at ground, it is reflected and then this reflected current wave move up on the return stroke channel. When this upward moving current arrives at the top of the lightning channel, it is totally reflected. Otherwise, the charge carried by the current pulse would be stored in the lightning channel envelope, which seems to be unrealistic.

Due to this turn-on process, the top reflection factor  $R$  depends on the return stroke velocity  $v$ . The top reflection factor  $R$  just describes the property of a current wave arriving at the moving open end of a transmission line. Therefore, it is not restricted to the assumptions of the GTCS model and it is possible to use it in different "engineering" models.

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#### REFERENCES

- [1] J. Cvetic and B. Stanic, "An improved return stroke model with specified channel-base current and charge distribution along lightning channel", in Proc. Int. Conf. on Electromagnetics in Advanced Applications, Turin, Italy, 1995, 00.109-112.
- [2] J. Cvetic and P. Osmokrovic, "Dynamics of Lightning Discharge During Return Stroke", IEEE Transaction on Plasma Science, Vol.37, Issue 1, 2009.
- [3] F. Heidler, "Review and Extension of the TCS – Model to Consider the Current Reflections at Ground and at the Upper End of the Lightning Channel," Journal of Lightning Res., vol.1, 2007.