11th INTERNATIONAL SCIENTIFIC CONFERENCE
NOVI SAD, SERBIA, SEPTEMBER 20-21, 2012

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KINEMATIC MODELING OF THE TRICEPT BASED 5-AXIS MACHINE TOOL

Abstract: *This paper is aimed in presenting a study on the kinematic modeling of the Tricept based five-axis vertical machine tool. Since the machine comprises 3-DOF parallel structure and 2-DOF serial wrist kinematic modeling also comprises serial and parallel part. As solution of direct and inverse kinematics of 2-DOF serial wrist is well known the study in this paper will focus on the parallel structure only.*

Key words: *Hybrid mechanism, kinematic modeling, machine tool*

1. INTRODUCTION

Compared with serial structured machine tools and robots, parallel kinematic machine tools and robots have many advantages. Basic knowledge about diverse aspects of parallel kinematic machines has already been published. Many different topologies of parallel mechanisms with 3-6 DOF has been used [1-3]. Considering that some limitations are indeed due to the use of parallel mechanisms, it is appealing to investigate architectures based on hybrid arrangements where serial and parallel concept are combined [3]. The Tricept robot or Tricept machine tool is based on parallel tripod combined with passive chain, and equipped with serial 3- or 2- DOF wrist. The inventor of this structure is K.-E. Neuman [4] while the mechanics has been constructed by Neos [5].

The primary application of commercially available Tricept robots was area of assembly where large insertion forces are required, e.g. as in the automobile industry.

Conceptual model of the Tricept based vertical five-axis machine tool considered in this paper, Fig. 1, is planed for HSC-milling of aluminium, steel as well as large model making, plastic and foam machining.

This paper is aimed in presenting a study on the kinematic modeling of the Tricept based five-axis vertical machine tool. Since the machine comprises 3-DOF parallel structure and 2-DOF serial wrist kinematic modeling also comprises serial and parallel part. As solution of direct and inverse kinematics of 2-DOF serial wrist is well known the study in this paper will focus on the parallel structure only.

2. KINEMATIC MODELING

Figure 2 represents a geometric model of the Tricept based vertical five-axis machine tool, Fig. 1, which comprises 3-DOF parallel structure and 2-DOF serial wrist. Parallel structure consists of four kinematic chains, including three variable length legs with identical topology and one passive leg connecting the fixed base B and the moving platform P. Three variable length legs with actuated prismatic joints $d_i,\ i=1,2,3$ are connected to the base B by Cardan joints and to movable platform P by spherical joints.

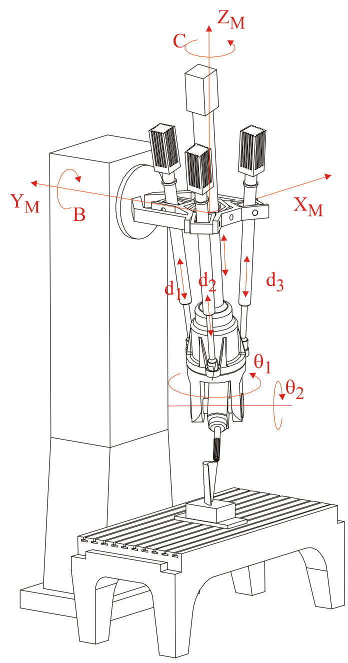

Fig. 1. Conceptual model of the Tricept based five-axis machine tool

The fourth chain (central leg) connecting the centre of the base B to the platform P is passive constraining leg. It consist of Cardan joint, a moving link, a prismatic joint and the second moving link fixed to the platform P. This fourth leg is used to constrain the motion of the platform to only 3-DOF. These 3-DOF are described by spherical coordinates i.e. by the axial translation $p_{Op}=\left|{}^{M}\mathbf{p}_{Op}\right|$ along the central leg and by two rotations Ψ and θ about two axes orthogonal to the central leg itself. Two-DOF serial wrist executes rotational motions i.e. tool orientation with actuated rotational joints θ_1 and θ_2 .

To adequately control the position and orientation of the tool during machining processes, kinematic model is required to establish mathematics description for the machine tool. Kinematic modeling of parallel structure involves solving of inverse kinematics, Jacobian matrix as the basis for numerical solution of direct kinematics, and direct kinematics. Based on machine inverse kinematics the workspace has been analyzed in order to select machine prototype design parameters.

2.1 Machine joint and world coordinates

As can be concluded from Fig. 1 i.e. Fig 2, Tricept based five-axis machine tool will be considered below as a specific configuration of the five-axis vertical milling machine (X, Y, Z, B, C) spindle-tilting type [6].

The machine reference frame {M} has been adopted according to the standard for this machine type [7]. Frame {P} is attached to the moving platform in a way that z_P axis coincides with the axis of the central leg and with the axis of joint θ_1. The tool frame {T} is attached to the milling tool at the tool tip T, so that the axis z_T coincides with tool axis, and the frame {W} is attached to the work piece. Vectors $\mathbf{v}$ referenced in frames {M}, {W}, {P} and {T} are denoted by ${}^{M}\mathbf{v}$, ${}^{W}\mathbf{v}$, ${}^{P}\mathbf{v}$ and ${}^{T}\mathbf{v}$.

To solve direct and inverse kinematics, joint and world coordinates will be defined first.

Joint coordinates vector for this 5-axis Tricept based machine tool is represented as

$$\mathbf{q} = \begin{bmatrix} d_1 & d_2 & d_3 & \theta_1 & \theta_2 \end{bmatrix}^T \tag{1}$$

where $d_i, i = 1,2,3$ and $\theta_i, i = 1,2$ are scalar joint variables controlled by actuators.

The description of world coordinates is based on tool path calculated by CAD/CAM systems defined by the set of successive tool positions and orientations in the work piece frame {W}, Fig. 2. The thus calculated tool path is machine independent and is known as a cutter location file (CLF). A tool pose is defined by the position vector of the tool tip T in the work piece frame {W} as ${}^{W}\mathbf{p}_T = \begin{bmatrix} x_{TW} & y_{TW} & z_{TW} \end{bmatrix}^T$ and tool orientation is defined by unit vector of the tool axis as ${}^{W}\mathbf{k}_T = \begin{bmatrix} k_{TWx} & k_{TWy} & k_{TWz} \end{bmatrix}^T$. In the general case, the tool tip position vector and tool axis vector in machine reference frame {M} can be expressed as

$$\begin{gathered} {}^{M}\mathbf{p}_T = \begin{bmatrix} X_M & Y_M & Z_M \end{bmatrix}^T = {}^{M}\mathbf{p}_{Ow} + {}^{M}_{W}R \cdot {}^{W}\mathbf{p}_T \\ {}^{M}\mathbf{k}_T = \begin{bmatrix} k_{Tx} & k_{Ty} & k_{Tz} \end{bmatrix}^T = {}^{M}_{W}R \cdot {}^{W}\mathbf{k}_T \end{gathered} \tag{2}$$

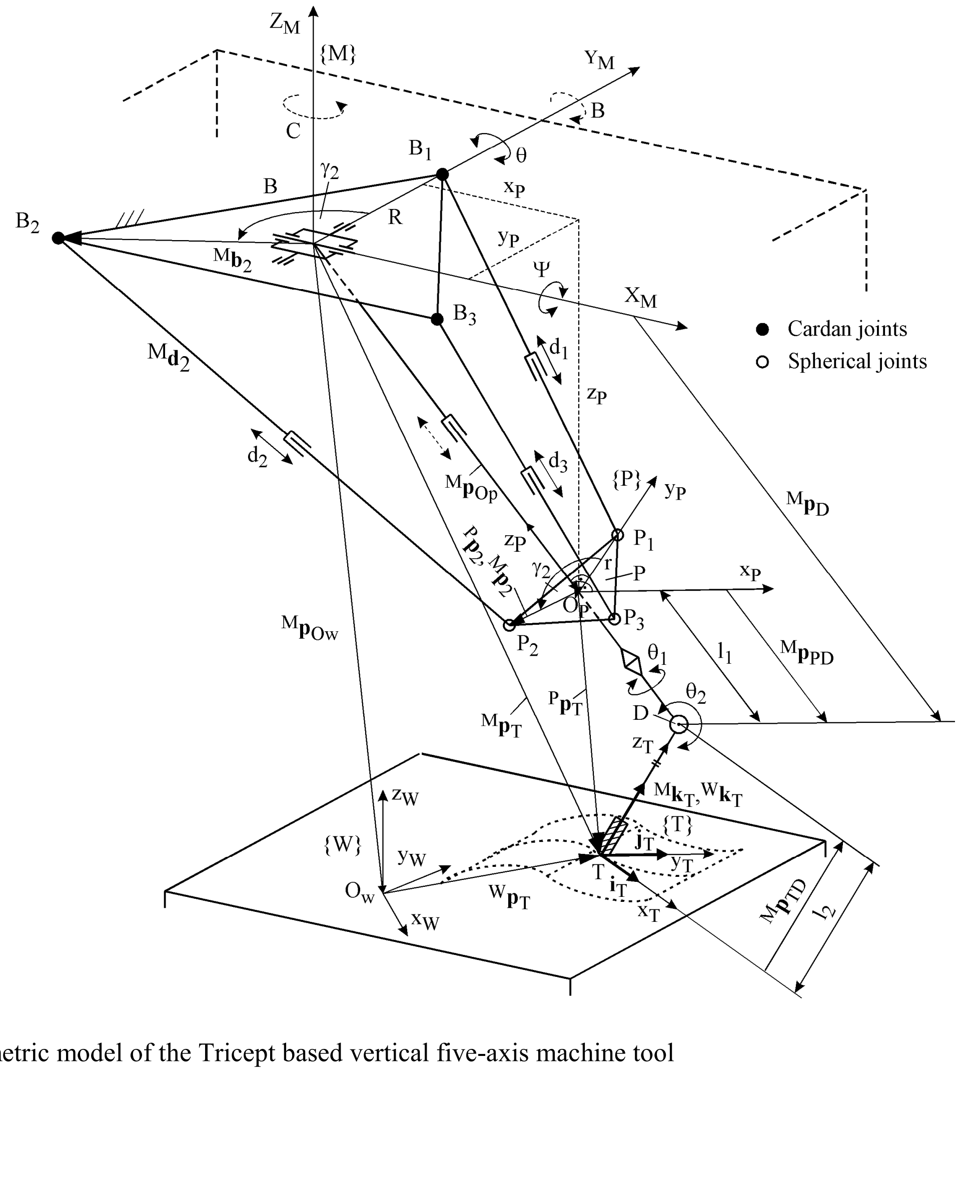

Fig. 2. Geometric model of the Tricept based vertical five-axis machine tool

where ${}^{M}\mathbf{p}_{Ow} = [x_{Ow} \quad y_{Ow} \quad z_{Ow}]^{T}$ is the position vector of the origin of work piece frame {W}. Determining the position vector ${}^{M}\mathbf{p}_{Ow}$ and the orientation of the work piece frame {W} is conducted according to the standard procedure for 5-axis CNC machine tools. It should be noted that determining the orientation matrix ${}^{M}_{W}R$ in equations (2) is determined and executed later in control system without changing G-code. To complete the vector of world coordinates, it is also needed to determine the tool orientation angles B and C which define direction of tool axis z_T that also coincides with axis of the last link, Fig. 2. Given that the machine has 5 DOF, only the direction of tool axis z_T is controllable, while axes x_T and y_T will have uncontrollable rotation about it. The position and orientation of the tool frame {T} relative to robot reference frame {M} can be described by homogenous coordinate transformation matrix 4x4 [8-10] as

$$
{}^{M}_{T}T = \left[\begin{array}{ccc|c} & {}^{M}_{T}R & & {}^{M}\mathbf{p}_T \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} i_{Tx} & j_{Tx} & k_{Tx} & X_M \\ i_{Ty} & j_{Ty} & k_{Ty} & Y_M \\ i_{Tz} & j_{Tz} & k_{Tz} & Z_M \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{3}
$$

where the rotation matrix ${}^{M}_{T}R$ represents the orientation, while vector ${}^{M}\mathbf{p}_T$ represents the position of the tool frame {T} with respect to the machine reference frame {M}. To bring the tool axis z_T to a desirable orientation with respect to frame {M}, the tool frame {T} must be rotated first about axis Y_M by angle B, and then about axis Z_M by the angle C, as prescribed by the convention for 5-axis vertical milling machine (X, Y, Z, B, C) spindle-tilting type. As it is known, the rotation matrix ${}^{M}_{T}R$ specifying the orientation of tool axis z_T can be derived as

$$
{}^{M}_{T}R = R_{ZM,C} \cdot R_{YM,B} \tag{4}
$$

where $R_{YM,B}$ and $R_{ZM,C}$ represents basic rotation matrices [10] and where "s" and "c" refer to sine and cosine functions. As it is of interest only orientation of the tool axis z_T specified by unit vector ${}^{M}\mathbf{k}_T = [k_{Tx} \quad k_{Ty} \quad k_{Tz}]^{T}$, by equating corresponding members of matrix ${}^{M}_{T}R$ from equation (4) the angles B and C can be determined [11]. This way, the world coordinates vector can be expressed as $\mathbf{x} = [X_M \quad Y_M \quad Z_M \quad B \quad C]^{T}$.

2.2. Kinematic modeling of parallel mechanism

As it was mentioned, the passive central leg is used to constrain the motion of the platform to only 3-DOF. According to Fig. 2 these 3-DOF can be described by spherical coordinates

$$
\mathbf{x}_{sp} = [p_{Op} \quad \Psi \quad \theta]^{T} \tag{5}
$$

where:

- $p_{Op} = \left|{}^{M}\mathbf{p}_{Op}\right|$ is axial translation along central leg, and
- Ψ and θ are the rotation angles of the central leg's Cardan joint about axes X_M and Y_M respectively.

Vector ${}^{M}\mathbf{p}_{Op} = [x_p \quad y_p \quad z_p]^{T} = \mathbf{x}_P$ is the position vector of origin O_p of the frame {P} attached to the moving platform with respect to machine reference frame {M}, and represents Cartesian world coordinates vector.

As noticeable from Fig. 2 joint axes of 2-DOF serial wrist intersect at point D (wrist centre). From this fact it is easy to conclude that the position of wrist centre D is influenced only by joint coordinates d_1, d_2 and d_3 of parallel mechanism.

For specified position vector of the tool tip ${}^{M}\mathbf{p}_T = [X_M \quad Y_M \quad Z_M]^{T}$ and for specified tool orientation angles B and C the rotation matrix ${}^{M}_{T}R$ from equation (4) is calculated first. Then by using only vector ${}^{M}\mathbf{k}_T$ from calculated rotation matrix ${}^{M}_{T}R$

$$
{}^{M}\mathbf{k}_T = [cC \cdot sB \quad sC \cdot sB \quad cB]^{T} \tag{6}
$$

the position vector of the wrist centre D ${}^{M}\mathbf{p}_D$ and its module p_D, according to Fig. 2 can be calculated as

$$
{}^{M}\mathbf{p}_D = \begin{bmatrix} x_D \\ y_D \\ z_D \end{bmatrix} = {}^{M}\mathbf{p}_T + {}^{M}\mathbf{p}_{TD} =
$$

$$
= {}^{M}\mathbf{p}_T + l_2 \cdot {}^{M}\mathbf{k}_T = \begin{bmatrix} X_M + l_2 \cdot cC \cdot sB \\ Y_M + l_2 \cdot sC \cdot sB \\ Z_M + l_2 \cdot cB \end{bmatrix}^{T} \tag{7}
$$

and it's module as

$$
\mathrm{p}_D = \left|{}^{M}\mathbf{p}_D\right| = \sqrt{x_D^2 + y_D^2 + z_D^2} \tag{8}
$$

As the position vectors ${}^{M}\mathbf{p}_{Op}$, ${}^{M}\mathbf{p}_D$ and ${}^{M}\mathbf{p}_{PD}$ are collinear and coincide with central leg, and as $\left|{}^{M}\mathbf{p}_{PD}\right| = l_1$ the module $p_{Op} = \left|{}^{M}\mathbf{p}_{Op}\right|$ can be calculated as

$$
p_{Op} = p_D - l_1 \tag{9}
$$

Now, the description of the position and orientation of the frame {P} attached to the moving platform with respect to machine reference frame {M} can be represented as

$$
{}^{M}_{P}T = \left[\begin{array}{c|c} {}^{M}_{P}R & {}^{M}\mathbf{p}_{Op} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \tag{10}
$$

where rotation matrix ${}^{M}_{P}R$ represents the orientation while vector ${}^{M}\mathbf{p}_{Op}$ represents the position of frame {P} with respect to the machine frame {M}. Frame

${}^{M}_{P}T$ can be further derived using homogenous transformation matrices 4x4 as

$$
{}^{M}_{P}T = Trot(X_M,\Psi)\cdot Trot(Y_M,\theta)\cdot Ttran(Z_M,-p_{Op}) =
\left[\begin{array}{ccc|c}
c\theta & 0 & s\theta & -p_{Op}\cdot s\theta \\
s\Psi\cdot s\theta & c\Psi & -s\Psi\cdot c\theta & p_{Op}\cdot s\Psi\cdot c\theta \\
-c\Psi\cdot s\theta & s\Psi & c\Psi\cdot c\theta & -p_{Op}\cdot c\Psi\cdot c\theta \\
\hline
0 & 0 & 0 & 1
\end{array}\right] \tag{11}
$$

where

$$
{}^{M}\mathbf{p}_{Op} = \begin{bmatrix} -p_{Op}\cdot s\theta \\ p_{Op}\cdot s\Psi\cdot c\theta \\ -p_{Op}\cdot c\Psi\cdot c\theta \end{bmatrix} = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} \tag{12}
$$

As the vectors ${}^{M}\mathbf{p}_D$ and ${}^{M}\mathbf{p}_{Op}$ are collinear, calculated components of vector ${}^{M}\mathbf{p}_D = [x_D \quad y_D \quad z_D]^T$ in equation (7) can also be described by spherical coordinates according to equation (12) as

$$
{}^{M}\mathbf{p}_{D} = \begin{bmatrix} -p_{D}\cdot s\theta \\ p_{D}\cdot s\Psi\cdot c\theta \\ -p_{D}\cdot c\Psi\cdot c\theta \end{bmatrix} = \begin{bmatrix} x_D \\ y_D \\ z_D \end{bmatrix} \tag{13}
$$

From equations. (13), (7) and (8) the platform's orientation angles Ψ and θ can be determined as

$$\theta = A\tan 2(x_D/-p_D, \sqrt{1-(x_D/-p_D)^2}) \tag{14}$$

$$\Psi = A\tan 2(y_D, -z_D) \tag{15}$$

As can be seen from equation (13), equation (14) is valid when $c\theta \neq 0$ i.e. $\theta \neq \pm 90^\circ$. This condition is always satisfied since angles Ψ and θ usually vary within the limits $\pm\pi/3$ specified by the ranges of passive joints motions.

This way, the spherical world coordinates vector of parallel mechanism $\mathbf{x}_{sp}$ in equation (5) or Cartesian world coordinates vector $\mathbf{x}_p$ in equation (12) are completed.

2.2.1. Inverse kinematics of parallel mechanism

The inverse kinematics of parallel mechanism from Fig. 2 deals with calculating the leg lengths d_i, i=1,2,3 when platform pose is given.

Observing geometric relations on the example of leg vector ${}^{M}\mathbf{d}_2$ shown in Fig. 2 the following equations can be derived

$$
{}^{M}\mathbf{d}_i = \begin{bmatrix} d_{ix} \\ d_{iy} \\ d_{iz} \end{bmatrix} = {}^{M}\mathbf{p}_{Op} + {}^{M}\mathbf{p}_i - {}^{M}\mathbf{b}_i \tag{16}
$$

where:

- ${}^{M}\mathbf{d}_i = [d_{ix} \quad d_{iy} \quad d_{iz}]^T$, i=1,2,3 are vectors of the actuated legs defined in the machine frame {M},
- ${}^{M}\mathbf{p}_{Op} = [x_p \quad y_p \quad z_p]^T$ is the position vector of the origin O_P of the frame {P} attached to the moving platform with respect to machine frame {M} and is given in equation (12),
- ${}^{P}\mathbf{p}_i = \begin{bmatrix} p_{ix} \\ p_{iy} \\ 0 \end{bmatrix} = \begin{bmatrix} r\cdot c\gamma_i \\ r\cdot s\gamma_i \\ 0 \end{bmatrix}$, i=1,2,3 are position vectors of the joint centers at the platform located on the circle of radius r with angular position $\gamma_i = \frac{2\pi}{3}(i-1)$, and are defined in the frame {P},
- ${}^{M}\mathbf{p}_i = {}^{M}_{P}R(\psi,\theta)\cdot{}^{P}\mathbf{p}_i$, i=1,2,3 are position vectors of the joint centers of the platform expressed in the machine frame {M},
- ${}^{M}\mathbf{b}_i = \begin{bmatrix} b_{ix} \\ b_{iy} \\ 0 \end{bmatrix} = \begin{bmatrix} R\cdot c\gamma_i \\ R\cdot s\gamma_i \\ 0 \end{bmatrix}$, i=1,2,3 are position vectors of the join centers at the base located on the circle of radius R with angular position $\gamma_i = \frac{2\pi}{3}(i-1)$ and are defined in the frame {M}.

By substituting corresponding vectors in equation (16) vectors ${}^{M}\mathbf{d}_i = [d_{ix} \quad d_{iy} \quad d_{iz}]^T$, i=1,2,3 can be obtained from which inverse kinematics equations

$$d_i == \sqrt{d_{ix}^2 + d_{iy}^2 + d_{iz}^2}\ , i=1,2,3 \tag{17}$$

are derived as

$$
\begin{aligned}
d_1 = (p_{Op}^2 + r^2 + R^2 - 2\cdot p_{Op}\cdot R\cdot c\theta\cdot s\Psi - \\
-2\cdot R\cdot r\cdot c\psi)^{1/2}
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
d_2 = [p_{Op}^2 + r^2 + R^2 + p_{Op}\cdot R\cdot(c\theta\cdot s\Psi - \sqrt{3}\cdot s\theta) + \\
+\frac{r\cdot R}{2}(-3\cdot c\theta - \sqrt{3}\cdot s\theta\cdot s\Psi - c\Psi)]^{1/2}
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
d_3 = [p_{Op}^2 + r^2 + R^2 + p_{Op}\cdot R\cdot(c\theta\cdot s\Psi + \sqrt{3}\cdot s\theta) + \\
+\frac{r\cdot R}{2}(-3\cdot c\theta + \sqrt{3}\cdot s\theta\cdot s\Psi - c\Psi)]^{1/2}
\end{aligned} \tag{20}
$$

This way, the joint coordinates vector of parallel mechanism can be expressed as

$$\mathbf{d} = [d_1 \quad d_2 \quad d_3]^T \tag{21}$$

where d_i, i = 1,2,3 are scalar variables controlled by actuators.

2.2.2. Jacobian matrix and direct kinematics of parallel mechanism

The direct kinematics problem for parallel mechanism consist of finding vector of world coordinates $\mathbf{x}_{sp}$ or $\mathbf{x}_P$ as a function of joint coordinates $\mathbf{d}$. Generally, such problem does not have analytical solutions and different numerical algorithms based on Jacobian matrix are used.

Differencing equations. (18) – (20) with respect to the time the Jacobian matrix is obtained as

$$J = \begin{bmatrix} \dfrac{\partial d_1}{\partial p_{Op}} & \dfrac{\partial d_1}{\partial \Psi} & \dfrac{\partial d_1}{\partial \theta} \\ \dfrac{\partial d_2}{\partial p_{Op}} & \dfrac{\partial d_2}{\partial \Psi} & \dfrac{\partial d_2}{\partial \theta} \\ \dfrac{\partial d_3}{\partial p_{Op}} & \dfrac{\partial d_3}{\partial \Psi} & \dfrac{\partial d_3}{\partial \theta} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \quad (22)$$

where:

$$J_{11} = (p_{Op} - R \cdot c\theta \cdot s\Psi) / d_1$$

$$J_{21} = [2 \cdot p_{Op} + R \cdot (c\theta \cdot s\Psi - \sqrt{3} \cdot s\theta)] / 2 \cdot d_2$$

$$J_{31} = [2 \cdot p_{Op} + R \cdot (c\theta \cdot s\Psi + \sqrt{3} \cdot s\theta)] / 2 \cdot d_3$$

$$J_{12} = (-p_{Op} \cdot R \cdot c\theta \cdot c\Psi + r \cdot R \cdot s\Psi) / d_1$$

$$J_{22} = [p_{Op} \cdot R \cdot c\theta \cdot c\Psi + r \cdot R \cdot (s\Psi - \sqrt{3} \cdot s\theta \cdot c\Psi)/2] / 2 \cdot d_2$$

$$J_{32} = [p_{Op} \cdot R \cdot c\theta \cdot c\Psi + r \cdot R \cdot (s\Psi + \sqrt{3} \cdot s\theta \cdot c\Psi)/2] / 2 \cdot d_3$$

$$J_{13} = p_{Op} \cdot R \cdot s\theta \cdot s\Psi / d_1$$

$$J_{23} = [p_{Op} \cdot R \cdot (-s\theta \cdot s\Psi - \sqrt{3} \cdot c\theta) + \\ + r \cdot R \cdot (3 \cdot s\theta - \sqrt{3} \cdot c\theta \cdot s\Psi)/2] / 2 \cdot d_2$$

$$J_{33} = [p_{Op} \cdot R \cdot (-s\theta \cdot s\Psi + \sqrt{3} \cdot c\theta) + \\ + r \cdot R \cdot (3 \cdot s\theta + \sqrt{3} \cdot c\theta \cdot s\Psi) / 2] / 2 \cdot d_3$$

This so called analytical Jacobian matrix [12] relates the spherical velocity vector $\dot{\mathbf{x}}_{sp}$ to the joint velocity vector $\dot{\mathbf{d}}$ and is used in this paper as a basis for simple numerical algorithm to solve direct kinematics for the purpose of simulation. The algorithm is based on constant Jacobian matrix calculated for the centre of workspace i.e. for the initial position [13].

At step (n+1), the estimated position of the platform is given by

$$\mathbf{x}_{spn+1} = \mathbf{x}_{spn} + J^{-1}(\mathbf{x}_{sp0}, \mathbf{d}_0) \cdot (\mathbf{d} - \mathbf{d}_n) \quad (23)$$

where:

- $\mathbf{x}_{spn+1} = [p_{Opn+1} \quad \Psi_{n+1} \quad \theta_{n+1}]^T$ is the estimated position of the platform at the step n+1,
- $\mathbf{x}_{spn} = [p_{Opn} \quad \Psi_n \quad \theta_n]^T$ is the estimated position of the platform at the step n,
- $\mathbf{d}_n = [d_{1n} \quad d_{2n} \quad d_{3n}]^T$ joint position (leg lengths) corresponding to the estimated platform position at the step n, result of the inverse kinematics of point $\mathbf{x}_{spn}$,
- $J^{-1}(\mathbf{x}_{sp0}, \mathbf{d}_0)$ is the inverse Jacobian matrix for the initial platform position $\mathbf{x}_{sp0}$ and joint position $\mathbf{d}_0$ as the result of inverse kinematics of point $\mathbf{x}_{sp0}$.

For the purpose of simulation, this algorithm converge in 1 to 5 steps depending on the distance between the initial position and actual position. This comes from the large workspace at the parallel mechanism on one hand and the other hand from the high accuracy provided by position sensors. The direct kinematics model takes almost twice as much time as the inverse model.

3. WORKSPACE ANALYSIS

Beside the selection appropriate kinematic topology the most important step in the parallel machine design is to select the right geometric dimensions [12].

Based on inverse kinematics, it is possible to determine the position and orientation workspace of the Tricept based five-axis milling machine. The applied approach proved to be very useful and is based on the definition of position and orientation workspace for parallel kinematic chains [14].

In the case of the Tricept based five axis machine tool considered in this paper, the position and orientation workspace are given by

$$W_S(X_M, Y_M, Z_M, B, C) = \{0,1\} \quad (24)$$

which represents a Boolean function whose value is equal to 1 if the tool pose-defined by the quintet (X_M, Y_M, Z_M, B, C) is reachable without exceeding the limited motion range of the joints. Starting from the selected point in the workspace volume, the estimation is made by specific step-by-step strategy that locates tool in a given pose in the workspace and that determines whether the pose is reachable or not by taking into account a limited motion range of the joints [6]. Based on selected design parameters: R = 350mm, r=100mm, l_1=300mm, l_2=150mm, d_{min}=934mm, d_{max}=1520mm the determined workspace for three-axis machining ($B = 0^\circ, C = 0^\circ$, i.e., spindle axis is perpendicular to the $X_M Y_M$ plane) is shown in Fig. 3.

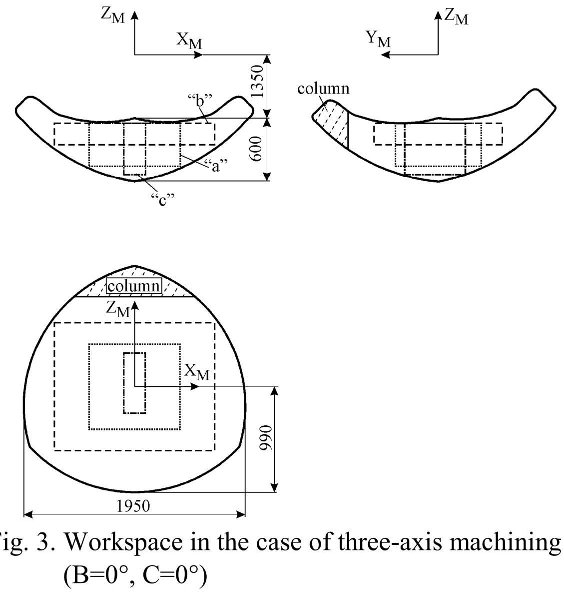

Fig. 3. Workspace in the case of three-axis machining (B=0°, C=0°)

For programmers and operators familiar with CNC machine tools, the determined workspace can be reduced to the parallelepiped “a” as indicated in Fig. 4. As it is known from practice, adopted portion of workspace in the form of parallelepiped “a” can be changed in form “b” or “c” depending on the workpieces’ shape and dimensions.

4. CONCLUSION

The results of a study on the kinematic modeling of the vertical Tricept based five-axis machine tool have been reported in this paper. For parallel structure inverse kinematics is solved analytically while direct kinematic is solved numerically based on constant Jacobian matrix calculated for the centre of workspace. Based on machine inverse kinematics workspace has been analyzed in order to select machine prototype design parameters. The focus of the current research, one part of the results being presented in this paper, is related to the prototype development of the Tricept based five-axis machine tool.

ACKNOWLEDGEMENT

The authors would like to thank the Ministry of Education and Science of Serbia for providing financial support that made this work possible.

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