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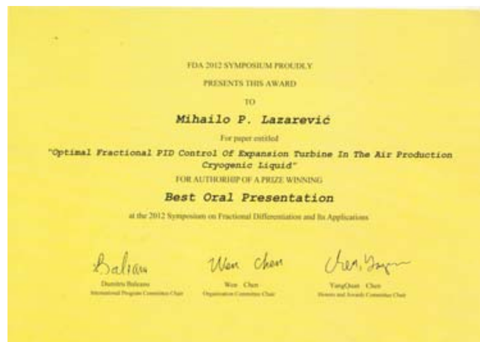
# FURTHER RESULTS ON APLICATIONS OF FRACTIONAL CALCULUS IN NONLINEAR DYNAMICS - STABILITY AND CONTROL ISSUES

**Mihailo Lazarević[1]**
[1] Department of Mechanics,Faculty of Mechanical Engineering,
The University of Belgrade, Kraljice Marije 16, 11120 Belgrade 35,Serbia
e-mail: mlazarevic@mas.bg.ac.rs

**ABSTRACT.** In this paper, they are presented recently obtained results which are related to applications of fractional calculus in dynamics- specially stability and control issues. Some of these results [1-4] are presented at the fifth symposium of fractional differentiation and its applications was held a t the Hohai University, Nanjing, China in the period of May 14-May 17, 2012. A number of 262 participants from 26 different countries have attended the symposium. A number of 12 symposia were organized, six plenary lecturers and 10 semi-plenary lecturers were presented. After a peer review 208 articles were accepted as oral presentations and 73 as posters. Also, I received awards *for the Best poster* for the paper *Finite Time Partial Stability of Fractional Order Time Delay Systems*,[1] as well as and *the Best oral presentation* for the *paper Optimal Fractional Order PID Control Of Expansion Turbine In The Air Production Cryogenic Liquid,* [3](*see 2page).*
In recent years**,** there have been extensive research activities related to applications of fractional calculus (FC), [5] in nonlinear dynamics, mechatronics as well as control theory. First, we propose sufficient conditions for finite time stability for the (non)homogeneous fractional order systems with time delay. Specially, the problem of finite time stability with respect to some of the variables (partial stability) is considered. Namely, along with the formulation of the problem of stability to all variables, Lyapunov also formulated a more general problem on the stability to a given part of variables (but not all variables) determining the state of a system,[6].The problem of the stability of motion with respect to some of the variables also known as partial stability arises naturally in applications. So, in this presentation, it will be proposed finite time partial stability test procedure of linear (non)autonomous unknown time delay fractional order systems. Time-delay is assumed to be unknown but its upper bound is assumed to be known. New stability criteria for this class of fractional order systems will be derived using a recently obtained generalized Gronwall inequality as well as "*classical"* Bellman-Gronwall inequality,[7]. Last,a numerical example is provided to illustrate the application of the proposed stability procedure.

Second, it is suggested a robust fractional-order sliding mode control of a 3-DOF`s robot system driven by DC motors. Primarily, a conventional sliding mode controller based on $PD^{\alpha}$ sliding surface is designed. Sliding-mode controller is a powerful tool to robustly control incompletely modeled or uncertain systems which has many attractive features such as fast response, good transient response and asymptotic stability, [8]. Numerical simulations have been carried out to verify and compare the significance of the proposed control which resulted in reducing high switching amplitude where fractional-order sliding mode controller has better performance in the comparison with the conventional sliding mode controller. Third, it will be presented here a new algorithms of PID control based on applying FC in control of given

mechatronic system for the producing of technical gases, i.e air production cryogenic. Objective is to find out optimum settings using genetic algorithms for a fractional $PI^{\alpha}D^{\beta}$ controller in order to fulfill different design specifications for the closed-loop system, taking advantage of the fractional orders and properties of liquid. This method allows the optimal design of all major parameters of a fractional PID controller and then enhances the flexibility and capability of the PID controller. Then, in simulations they are compared step responses of these two optimal controllers. Last, it will be shown that FOPID controller improves transient response as well as provides more robustness in than conventional *PID,* particularly in the case of disturbance rejection. Last, this paper addresses the problem of iterative learning control (ILC) for fractional (non)linear time delay system. Here, we extends results obtained in [9], to consider more general systems i.e. fractional time delay systems described in the form of state space and output equations with uncertain constant delay as well as time-varying delay. Sufficient convergent conditions of a proposed ILC will be derived in time-domain and formulated by the theorems.


**Keywords**: robust control, fractional calculus, sliding-mode, chattering-free, stability criteria, partial stability, time delay, finite-time stability, optimal settings, iterative learning control

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