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Original article

Verification of inverse kinematic equations for a five-axis machine tool with a spindle tilting configuration

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ABSTRACT

In this paper, the methodology for verifying inverse kinematic equations is presented. Analysis of the kinematic structure of the machine is shown, which results in the kinematic equations needed for the configuration of the machine's postprocessor. The machine proposed in this paper is a five-axis machine tool with two rotating axes on the spindle. The machine's configuration is X'OYZBA. The derived equations are used to transform the CL data for a test model into a G code used on a virtual model of the machine tool. A simplified model of the machine is developed using the PTC Creo software, in which the initial testing of the equations was performed. The inverse kinematic equations were implemented in a postprocessing program using the Matlab software. Generated G-code for multi-axis machining process was tested on a virtual machine tool in the Vericut software.

Key words: Verification; Inverse kinematics; Simulation; Virtual machine tools;

1. INTRODUCTION

Multi-axis machining is needed to achieve the manufacturing of complex, free-form surfaces. By definition, multi-axis machining utilises at least four axes of a machine simultaneously. The kinds of programs used on multi-axis machines need to provide the information for the movements of translational axes and the required angles of at least one rotary axis.

For this type of programming, CAD/CAM software is used. The tool path defined in a CAM software is typically transformed into a cutter location file (CLF), which contains the position and orientation of the tool regarding the workpiece, as shown in Fig1. Programming multi-axis machine tools is more complex than programming a 3-axis machine tool, as the necessary data cannot directly come from the CL data or CAM software [1].

CL data are generated without considering the structure of multi-axis machine tools. The part is assumed to be fixed, and the tool completes all motions. Multi-axis machines with different kinematic structures have the same CL data [2].

For the transformation of the CL data into G-code, the kinematic structure of the machine, in the form of inverse kinematic equations, must be defined. The kinematic model is integrated into a postprocessor and used to transform the CL data into the specific motions of the machine's components.

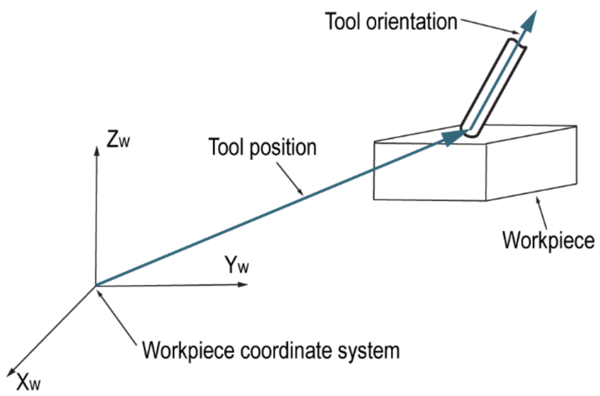

Fig. 1 *CL data representation*

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Lee and She [3] proposed a method of developing an inverse kinematic model for three types of five-axis machine tools based on the homogenous transformational matrix. Five-axis machine tools can be classified into three categories according to the disposition of the rotational and translational axes [3]:

1. Table tilting machines with two rotational axes placed on the table of the machine
2. Spindle tilting machines with two rotational axes on the spindle of the machine
3. Hybrid machines, with one rotational axis on both the table and spindle

Another approach, proposed by Che and Shang, is deriving the inverse kinematic model by using the generalised model of a multi-axis machine. The generalised model of a machine tool is composed of three translational axes and four rotational axes, two of which are on the table and two on the spindle. Excluding two or more of the rotational axes in the generalised model, the machine is transformed into an existing configuration of a multi-axis machine tool [4]. Zhimin, Jiangang, Yunjiang and Zexiang [2] propose a mathematical model for a serial five-axis machine tool using the product of the exponential formula of a serial robot. The analysed configurations of multi-axis machine tools are analysed in order to set up the forward and inverse kinematic model.
This paper analyses a kinematic structure of a five-axis spindle tilting machine tool. The mathematical model based on [3] and the final inverse kinematic equations are shown. The configuration of the analysed mechanism is X'OYZBA, meaning it has two rotational axes on the spindle, one rotating around the Y-axis and the other around the X-axis. In order to verify the kinematic model using a virtual simulation, a virtual model of the machine was made. G-code used in this simulation is generated using an automated process in the Matlab software. Lastly, a five-axis machining simulation was configured in the Vericut software, and the derived kinematic model was verified using two workpieces.

2. INVERSE KINEMATIC EQUATIONS FOR A FIVE-AXIS, SPINDLE TILTING MACHINE TOOL

In this approach, based on [3], machine tools are defined as open chains of serially connected links with joints, either revolute or prismatic. The inverse kinematic model of the machine is based on defining the relative positions between the joints. In Fig. 1. the cutting tool and the workpiece of the machine are shown, with their respective coordinate systems needed to define their relative positions. $O_W X_W Y_W Z_W$ and $O_T X_T Y_T Z_T$ are the workpiece and tool coordinate systems, and vectors [K_X K_Y K_Z] and [Q_X Q_Y Q_Z] define the tool's orientation and position, respectively. The pivot point (R) represents the intersection of the rotary axes and is introduced to simplify the derivation of the inverse kinematic equation. The pivot point's position is defined wits its distance from the tool point (L_t). The inverse kinematic model of the machine is based on defining the relative positions between the connected joints, and the relative positions are defined using the elementary transformation matrixes (1)-(4) [3]. These transformations represent either translational movements defined by the workpiece coordinate systems axes or a rotational movement around these axes. Multiplying these matrixes in an order defined by the machine's kinematic structure yields a complex transformation of the tool coordinate system into the workpiece coordinate system.

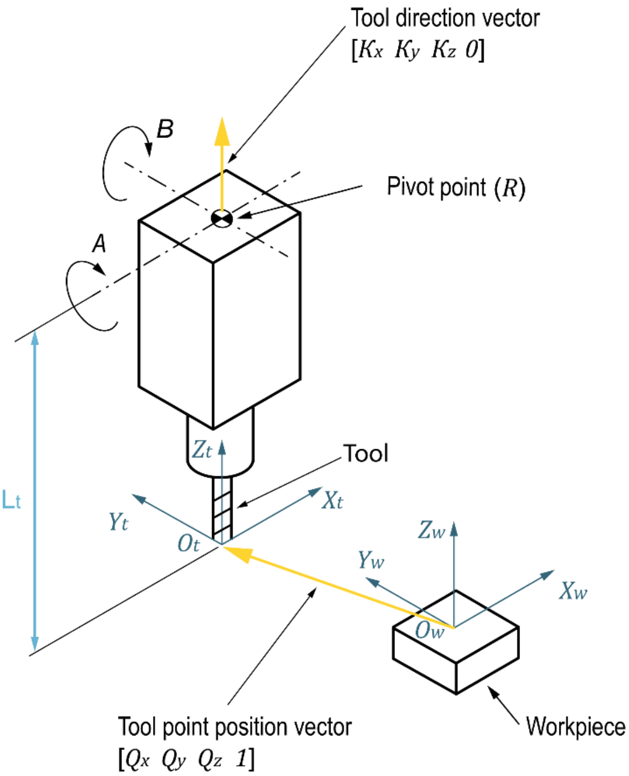

Fig. 2 *The vectors and coordinate systems of a spindle tilting machine [2]*

$$\boldsymbol{Trans}(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$\boldsymbol{Rot}(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta & -s\theta & 0 \\ 0 & s\theta & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$\boldsymbol{Rot}(y,\theta) = \begin{bmatrix} c\theta & 0 & s\theta & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta & 0 & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\boldsymbol{Rot}(z,\theta) = \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

In (1)-(4) **T**rans(a, b, c) represents the translation along the vector a**i** + b**j** + c**k**, whereas **R**ot(x, θ), **R**ot(y, θ), and

Rot(z, θ) define the rotary movements of θ around the x, y and z axes, respectfully. In these equations, “s” and “c” stand for sin and cos functions.

The purpose of defining the inverse kinematic equations is to calculate the necessary movements of the machine's axes to obtain the desired position of the tool point in regard to the workpiece coordinate system. The analysed mechanism is a five-axis spindle tilting machine. The rotary axes rotate around the X and Y axes of the machine coordinate system and therefore are given the characters A and B. The rotary axes A and B, alongside the translation axes X, Y and Z, define the NC data used in G code. The desired tool position and orientation are defined in the CLF and are presented in the form of two vectors, [KX KY KZ] and [QX QY QZ].

The form-shaping function [3] connects the CL data and NC data in the form of the following equations:

$$[K_x, K_y, K_z, 0]^T = \boldsymbol{Trans}(P_x, P_y, P_z) \cdot \boldsymbol{Rot}(y, \phi_B) \cdot \boldsymbol{Rot}(x, \phi_A) \cdot [0\ 0\ 1\ 0]^T \quad (5)$$

$$[Q_x, Q_y, Q_z, 1]^T = \boldsymbol{Trans}(P_x, P_y, P_z) \cdot \boldsymbol{Rot}(y, \phi_B) \cdot \boldsymbol{Rot}(x, \phi_A) \cdot \boldsymbol{Trans}(P_x, P_y, P_z) \cdot [0\ 0\ -L_t\ 1]^T \quad (6)$$

$$[X\ \ Y\ \ Z\ \ 1]^T = [P_x\quad P_y\quad P_z - L_t\quad 1]^T \quad (7)$$

In Eq. (5)-(7), P_X, P_Y and P_Z are the corrections used to compensate for the translation displacements that occur with the movement of rotary axes for the desired angle. Two vector equations (8) and (9) are obtained by multiplying elementary transformation matrixes with the corresponding vectors in Eq. (5) and (6).

$$\begin{bmatrix} K_x \\ K_y \\ K_z \\ 0 \end{bmatrix} = \begin{bmatrix} s(\phi_B) \cdot c(\phi_A) \\ -s(\phi_A) \\ c(\phi_B) \cdot c(\phi_A) \\ 0 \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} Q_x \\ Q_y \\ Q_z \\ 1 \end{bmatrix} = \begin{bmatrix} -L_t \cdot s(\phi_B) \cdot c(\phi_A) + P_x \\ L_t \cdot s(\phi_A) + P_y \\ -L_t \cdot c(\phi_B) \cdot c(\phi_A) + P_z \\ 1 \end{bmatrix} \quad (9)$$

Required angles of the rotary axes A and B are calculated using Eq. (8) and are given in the form of Eq. (10) and (11).

$$\phi_A = A = arcsin(-K_y) \qquad \left(-\frac{\pi}{2} \le \phi_A \le \frac{\pi}{2}\right) \quad (10)$$

$$\phi_B = B = atan2(K_x, K_z) \quad (11)$$

Using Eq. (7) and (9), the transformations of the translational axes are derived, Eq. (12)-(14).

$$X = Q_x + L_t \cdot s(\phi_B) \cdot s(\phi_A) \quad (12)$$

$$Y = Q_y - L_t \cdot s(\phi_A) \quad (13)$$

$$Z = Q_z + L_t \cdot c(\phi_B) \cdot c(\phi_A) - L_t \quad (14)$$

3. CONFIGURATION OF A VIRTUAL FIVE-AXIS, SPINDLE TILTING MACHINE TOOL

For the verification of the equations (10)-(14), a virtual machine model is needed. Using the PTC Creo software, a simplified machine model was created. Only simplified models of the machine's axes needed to test the machine's motions accurately were made for the assembly of the machine. As previously stated, the chosen structural configuration of the machine is X'OYZBA, which means that the workpiece is moved using the X-axis. At the same time, the tool's motions are defined by the Y, Z, B and A axis, in that order. The model of the machine is presented in Fig. 3.

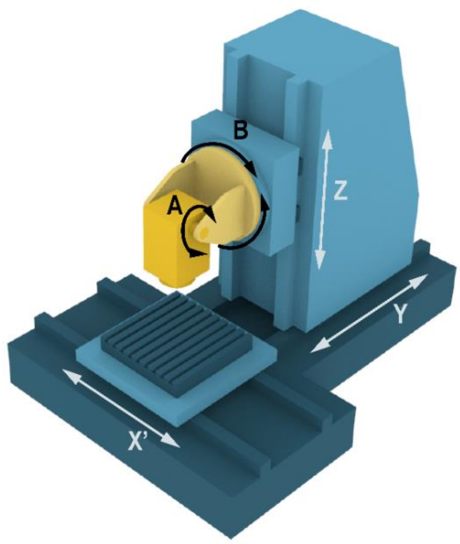

Fig. 3 *CAD model of the machine*

A machining simulation can be performed in the PTC Creo software, and it can be done within the NC check before creating a CLF for that machining operation. The NC check includes the tool path simulation and the material removal simulation. The simulation is run for two reasons. Firstly, the machining simulations verify that the kinematic connections between the machine's components within the assembly are correctly introduced. Second, the simulation is performed to check that the machine can perform the machining process and verify that there are no collisions. These tests are used primarily to verify the assembly of the virtual machine but cannot test the G-code, as the tool path in the software PTC Creo is performed according to the CLF. These machining simulations were performed twice to authenticate the tool path and machine motions for a concave and a convex surface. These tests are shown in Fig. 4. and Fig. 5.

4. DEFINING OF THE POSTPROCESSING PROGRAM IN MATLAB SOFTWARE

The CLF and G-code do not consist only of motion statements or motion commands but rather include all of the data needed for a manufacturing process. Since this testing aims to verify inverse kinematic equations, each motion statement defined in the CLF is directly translated into a motion command in the G-code. Given that there are no further transformations, the number of lines in the CLF is roughly the same as the number of lines in the G-code.

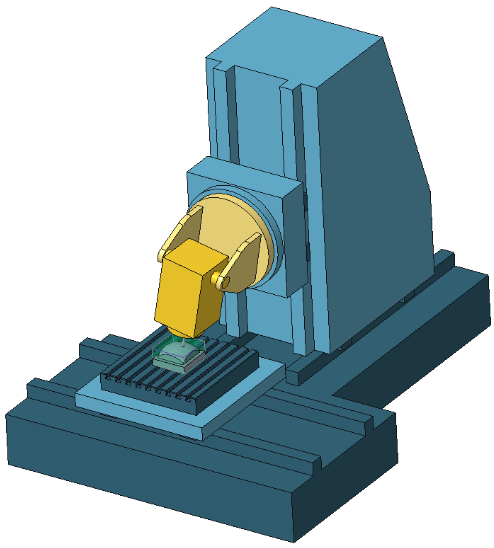

Fig. 4 *Machine motion simulation for a workpiece with a concave surface*

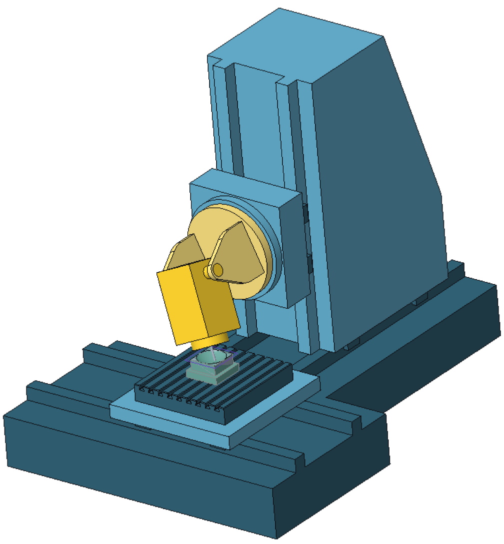

Fig. 5 *Machine motion simulation for a workpiece with a convex surface*

Firstly, the necessary data must be transformed to enable manipulation and necessary transformation using the kinematic equations. The beginning of G-code is designated for the initialisation parameters and defining the necessary, coordinate system regarding which the machining will occur. In order to transform the CL data into the machine's motions, the motion statements must be identified within the CLF. In this case, it is essential to differentiate CL data for 3-axis and multi-axis machining. The structure of CLF lines is different since 3-axis machining lacks data regarding the orientation of the tool. With the data collected, the translation into motion commands takes place. This is done with the (10)-(14) implemented into a Matlab function, where the input data are the tool point position vector and tool orientation vector. The output is a vector comprised of the machine's translational and rotational motions values.

The motions of the translational axes X, Y, and Z and the angles of rotational axes A and B are then presented in the form of a line in the G-code. Given the tool motions that occur with multi-axis machining, only the linear interpolation G1 is used while constructing the G code. The algorithm used to make a postprocessing program in the Matlab software is shown in Fig. 6.

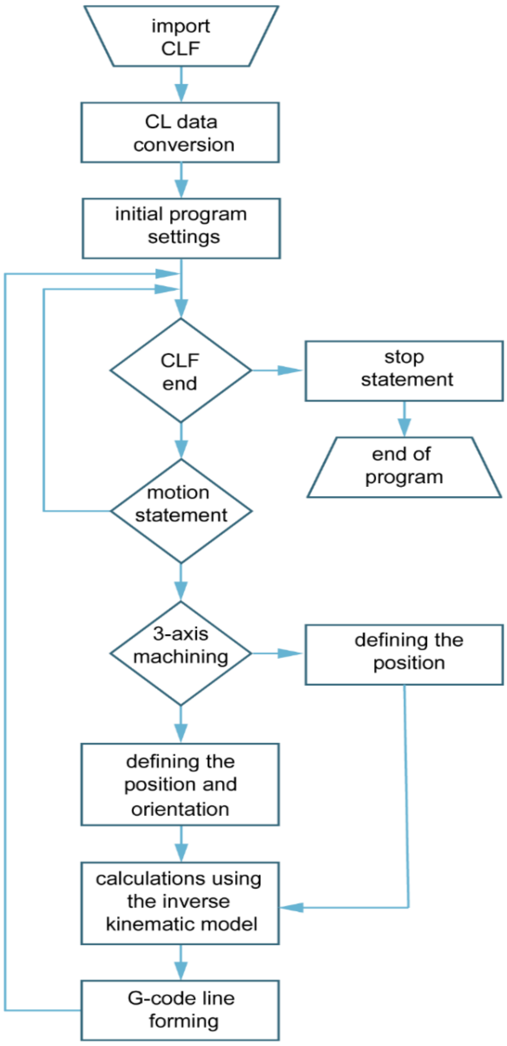

Fig. 6 *Postprocessing algorithm*

5. VERIFICATION OF INVERSE KINEMATIC EQUATIONS USING A VIRTUAL MACHINE TOOL

G-code generated by the Matlab code was verified using the Vericut software. In this software, it is possible to simulate a machining process based on G-code. It is possible to achieve a material removal simulation that includes the necessary parts of a machining system, including the machine's assembly, tool, workpiece and the finished part [5]. While configuring the machine's assembly, parts of the machine must be assigned the correct motions according to the machine's coordinate system. In order to achieve this, the machine's moving components are defined according to the assigned kinematic structure of the machine (X'OYZBA).

Initially, a fixed component of the machine is imported as a base in the simulation. Next, the components of the machine that conclude the motions of the tool and workpiece are divided into two groups and added to the virtual model. The first group consist of the components that define the motions of the workpiece. In this case, that is the machine's table, attaining the movements of the workpiece along the X-axis. The second group includes components that enable the movements of the tool. Next, components that define the motions along the Y, Z, B and A axes are added to the base of the machine. In order to conclude the simulation setting, a reference mode, workpiece and tool are defined. The tool is added to the end of the spindle, in this simplified version, at the end of the A-axis component, as its rotation directly impacts the tool's motion. The workpiece is added to the machine's table, and a reference model is attached to the same place used to check the material removal simulation. The previous process is illustrated in Fig. 7, which shows the hierarchical structure of the machine's kinematic model.

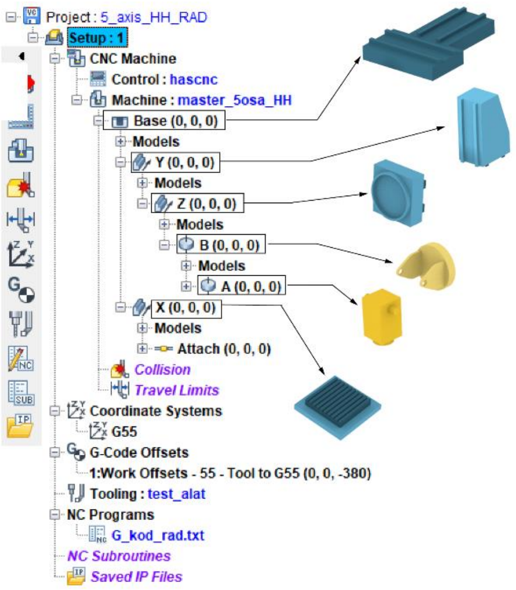

Fig. 7 *Hierarchical structure of the spindle tilting machine*

For this test, a machining simulation of a workpiece with a concave surface was executed to verify the engagement of A and B rotary axes simultaneously. The workpiece model, as well as its manufacturing program, were made in the PTC Creo software. For the purpose of testing only a 5-axis manufacturing program, it is assumed that the pre-machining has already taken place. The CLF for this simulation was created within the PTC Creo CAM software. Before processing the CL file, a tool path simulation was performed, as seen in Fig. 8. The aim of this test is to create the same tool path in the Vericut software using the G-code realised in the beforementioned postprocessing program. The simulation is shown in Fig. 9, showing that the workpiece was machined to the desired shape. The simulated tool path matches with the one defined by the CLF in Fig. 8.

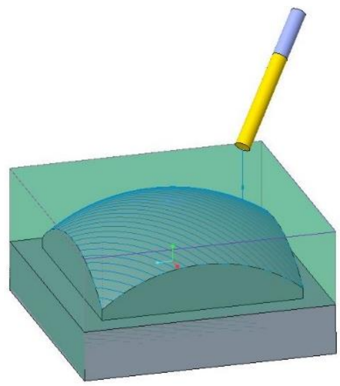

Fig. 8 *Tool path simulation in PTC Creo*

Another test was performed in order to justify the derived equations further. In this case, a machining simulation was performed for a convex surface. As with the previous test, a 5-axis manufacturing program was created using the postprocessing program. The model, workpiece, and the CLF for this machining simulation were made in the PTC Creo software. The tool path simulation based on the CLF can be seen in Fig. 10. The post-processed program was then tested in the Vericut program using the same machine assembly as the last part. Fig. 11 shows the simulation performed on the virtual spindle tilting machine.

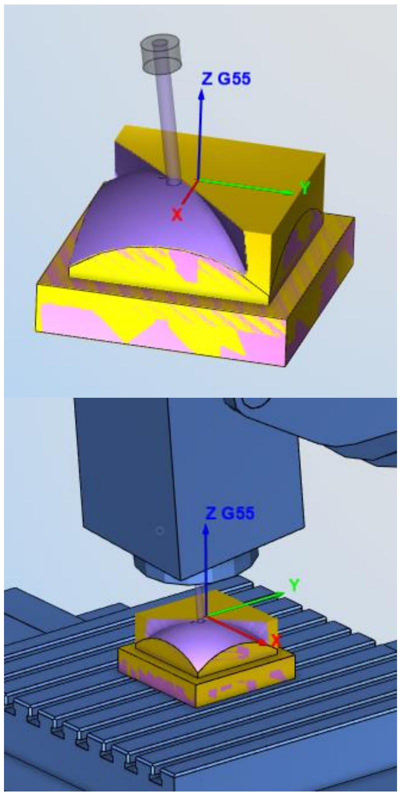

Fig. 9 *Testing the G-code in Vericut*

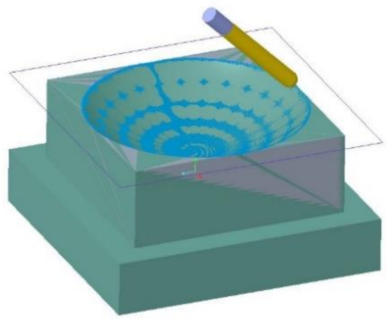

Fig. 10 *Tool path simulation in PTC Creo*

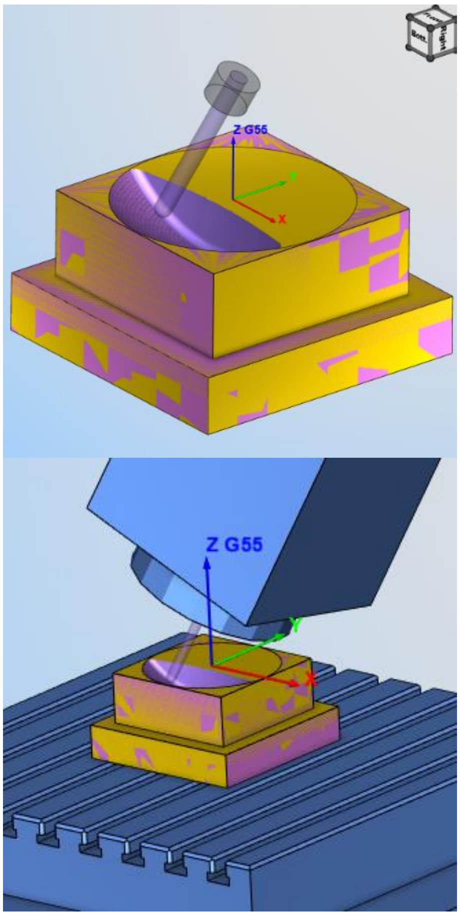

Fig. 11 *Testing the G-code in Vericut*

6. CONCLUSIONS

With the demands on the modern manufacturing system for producing parts with free-form surfaces, it relies significantly upon multi-axis machining. The choice of the kinematic structure of a multi-axis machine and its postprocessor highly impacts the machining process.

In this paper, a method of testing the mathematical model based on the machine's kinematic model was presented. The inverse kinematic model, presented as five equations, was tested using a postprocessing program written in the Matlab software. This program transformed the CL data into a G-code program for a specific 5-axis machining process. This machining program was tested in a virtual environment with a simplified model of a spindle tilting machine. The following steps for this research would be to create an original interpretation of a multi-axis machine's inverse kinematic model and test it with the proposed methodology.

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